PHYSICS

A Textbook for Grade 11

P11TB

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Foreword

Liberia, having gone through a period of utmost turmoil till 2003, due to the civil wars, is still reeling under its effect and the added trauma of Ebola in 2014 and effects of the COVID-19 outbreak in 2020. The Liberian government, in the past decade, has made valiant efforts to bring order to the lives of its people. In one such effort, the Ministry of Education (MoE) brought changes to the National Curriculum Framework which are relevant to the present generation, and which would prepare them to meet the challenges of the changing trends of the world. The National Curriculum Framework (NCF) 2018 recommends a change in basic assumptions in the teaching learning process from behaviorist to constructivist approach — moving from hardcore print material to the digital world. Keeping in consideration the sociocultural context and varied experiences of learners as laid down in the Framework, our Teaching Learning Materials are expected to be competent to use multiple methods and techniques like e-learning resources, energized textbooks, and readily available reference material to engage the learners.

As a first initiative, the MoE, through its World Bank-funded Improving Results in Secondary Education (IRISE) project, has adapted textbooks for Grades 10 to 12 in five subjects — English Language and Literature, Mathematics, Biology, Physics and Chemistry.

The National Curriculum Framework, 2018, recommends that children's learning at school is a reflection of their life outside the school and shows them the path to become a responsible citizen who makes knowledge-based choices.This principle marks a departure from the legacy of teacher centered learning to student centered learning. The syllabi and textbooks developed on the basis of the NCF indicate a serious attempt to implement the idea of Activity Base Learning (ABL). We hope these measures will take us ahead in the direction of building a system of education as outlined in the NCF.

Combined with the efforts by the school principals and teachers this will encourage children to reflect on their own learning and to pursue imaginative activities and questions.With this in mind, perhaps for the first time in our country, we are able to provide separate subject specific textbooks accompanied with guides for teachers for 10–12 grades. Not only have these been developed, adapted and modified to the Liberian context, each of the eight Minimum Learning Competencies (MLCs) have been included in each textbook. So as to reach every high school student, for the first time in the country's history we have included the digitized form of the textbook accessible by a Quick Response (QR) code given in each book. Not only does it have the digitized textbook, but it provides additional learning materials for use by students, teachers and interested persons. The links to these e-resources and digitized material is being made available on the MoE's website.

The Textbooks and Teacher Guides have reached the hands of the students after a rigorous quality evaluation by carefully handpicked subject specialists by the MoE, to whom the Ministry expresses gratitude. For the success of this project, I acknowledge the contributions of the IRISE Project Team in the World Bank, and in particular, the Task-Team Leaders; the Project Implementation Team in Liberia headed by its Coordinator Abraham A. Kiazolu II, supported by the Executive Director of the Center of Excellence for Curriculum Development and Textbooks Research, Mrs. Julia K. Sandiman-Gbeyai and her technical working group (TWG), and the International Textbook Consultant and Advisor, Dr Shveta Uppal engaged by the MoE. These notwithstanding would not have been possible without the guidance of the Senior Management Team (SMT) of the Ministry of Education, and in particular, the Deputy Ministers for Instructions, Administration, and Planning, Research and Development, respectively.

Professor Dao Ansu Sonii, Sr.
Minister of Education
Republic of Liberia

Monrovia, Republic of Liberia
January 24, 2023

Acknowledgments

The development of textbooks contributes to the quality of teaching and learning that go on in the classroom.

The Ministry of Education (MoE) has aligned its Curriculum for Grades 10–12 to the National Curriculum Framework (NCF) of 2018. To ensure the provision of Teaching Learning Materials (TLMs) that support the revised curriculum, the Ministry has sought, reviewed and adapted a new set of textbooks and teacher guides along with digitized contents and e-learning resources for the five core subjects taught at the Senior Secondary education level, namely English Language and Literature, Mathematics, Biology, Chemistry and Physics, through an internationally competitive bidding process from the market supported by the World Bank funded Improving Results in Secondary Education (IRISE) Project.

With profound gratitude and honor, we recognize the Senior Management Team of the Ministry, headed by the Coach, Professor D. Ansu Sonii, Sr., for the strategic decision to make teaching learning materials available and accessible to all in the Liberian Senior Secondary School System, and for providing directions through the process of securing these textbooks and other teaching learning materials for our students and teachers. Our special thanks and appreciation to the World Bank for the financial support towards this policy intervention, and its education task-team including Alonso Sanchez, Oni Lusk-Stover and Binta B. Massaquoi for all their technical inputs offered throughout the process to ensure the kind of quality TLMs the Liberian students deserve are made available for improved learning outcomes.

We would like to specifically recognize the invaluable contributions of the 15 subject experts selected by the MoE from across the various education systems and the West African Examinations Council (WAEC) to evaluate, review and sign off on these teaching learning materials. They didn't just deliver according to our expectations, but also ensured the contextual relevance of the materials to the Liberian Secondary Education Curriculum and its minimum learning competencies (MLCs). These subject experts include Professor Isaac Saye-Lakpoh Zawolo – *Superintendent* of the Monrovia Consolidated School System (MCSS), Mr. Matthew V.Z. Darblo, Sr. – *Mathematics Instructor* at the University of Liberia (UL), Mr. Charles Tieh Bropleh – *Mathematics Specialist* (MoE), Mrs. Linda Y. Dean – *English Specialist*, Mr. Hassan M. Bangura – *English Language and Literature Expert*, Mr. J. Emmanuel Milton – *English Specialist* (MoE), Mr. Moses K.M. Togbah – *Physics Specialist*, Mr. Prince A. Dossen – *Physics Specialist*, Mr. Benjamin Koryah – *Physics Instructor* at the University of Liberia (UL), Mr. Dominic Dugbe Doe – *Chemistry Specialist*, Mr. Patrick A. Anderson, Sr. – *Director o*f the Division of Technical and Vocational Education (MoE), Mr. Kandakai Massaquoi – *Chemistry Specialist*, Ms. Patricia N. Doe – *Head* of Biology Department, African Methodist Episcopal University (AMEU), Mr. Job Carpenter – *Biology Specialist* and Mr. Prince Philip K.A. Aderibigbe – *Biology Specialist*.

The MoE is sincerely grateful to Dr Shveta Uppal, the *International Textbook Consultant* engaged by the IRISE Project to provide technical guidance and quality assurance support to the revising of the Textbooks Management Guidelines (TMG) and the procurement process leading to the provision of textbooks, teacher guides, digital contents and e-learning resources for the Senior Secondary School System in Liberia in accordance with the revised TMG. Heartfelt thanks and appreciations also to the *Executive Director* for the Center of Excellence for Curriculum Development and Textbooks Research, Mrs. Julia K. Sandiman-Gbeyai, and members of her Technical Working Group (TWG) for taking up the responsibility to lead the process of making textbooks and other TLMs available to Liberian students and teachers.

Lastly, we acknowledge the IRISE Project Delivery Team led by Mr. Abraham A. Kiazolu, II – *Project Coordinator*, Mr. Fuseini A. Abu – *International Procurement Specialist* and Mr. Lawrence S. Taylor – *Project Control Specialist* who coordinated the entire process.

We remain grateful to you all!

Hon. Alexander N. Duopu, Sr.,
Deputy Minister for Instruction
Ministry of Education, Republic of Liberia
#The Teacher

Contents

P11CH01

CHAPTER 1

MOTION IN TWO DIMENSIONS

Chapter Contents

Chapter Outcome

Learners will able to:

- appreciate the use force in motion and its impacts in everyday activity

Chapter Objectives

Upon completion of this chapter, learners will:

- add /subtract vectors graphically;
- solve problems on simple Harmonic Motion;
- use the equation of motion in solving (one or two dimensional) problems;
- distinguish the types of motion (projectile, rotational, simple harmonic and oscillatory) and their application.

Introduction

In this unit, you will study motion in two dimensions using vectors to define quantities like position, velocity, acceleration and their relationship. The vector method you will learn will allow you to study more complex motion such as motion in two dimensions.

1.1 Vectors

These controls in the cockpit of a commercial aircraft assist the pilot maintaining control over the velocity of the aircraft how fast it is traveling and in what direction it is traveling allowing it to land safely. Quantities that are defined by both magnitude and direction, such as velocity, are called vector quantities.

Physical quantities that we encounter in our day to day activities may fall into two main categories: Scalar quantities and vector quantities. Scalar quantities can be completely described by a single number (with appropriate unit) giving only its magnitude or size. Common examples of scalars are mass, time, speed, distance, energy. Vectors are quantities that require their magnitude and direction for their description. Acceleration, velocity, displacement and force are some of the common examples of vectors.

Key terms

- Vector: a quantity with magnitude and direction.
- Scalar: a quantity with magnitude only
- Equal vectors: vectors of equal magnitude and the same direction representing the same quantity.

Representation of vectors

Vectors can be represented by Bold letters (e.g., V for velocity, S for displacement) or by placing an arrow above the letter (e.g., V for velocity, S for displacement). A vector can also be represented graphically by means of an arrow drawn to scale. The length of the arrow (drawn to scale) represents the magnitude of the vector and the arrowhead indicates the direction. Figure 1 shows the graphical representation of a displacement vector of 50 km, toward East.

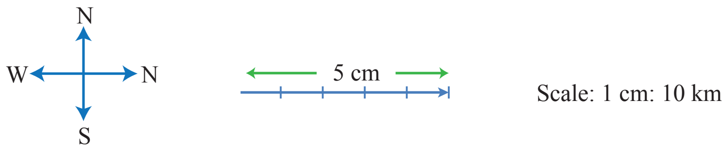
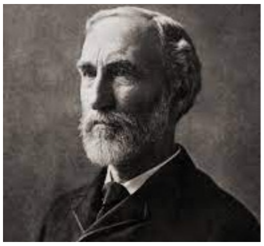


Figure 1. An arrow drawn to scale represents a vector

Did you know?



In 1881 vectors appeared in a publication called Vector analysis by the American J.W.Gibbs. They have been essential to physics and mathematics ever since. Lived 1839 - 1903, Willard Gibbs was a Mathematical Physicist who made enormous contributions to Science.

Equality of vectors

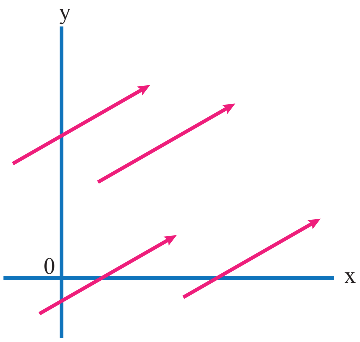


Figure 2. Equal vectors have equal magnitude and point in the same direction

For many purposes, two vectors **A** and **B** may be defined to be equal if they have the same magnitude and point in the same direction. That is, **A** = **B** only if $A = B$ and if **A** and **B** point in the same direction along parallel lines. For example, all the vectors in Figure 2 are equal even though they have different starting points. This property allows us to move a vector to a position parallel to itself in a diagram without affecting the vector.

Addition or Composition of Vectors

Unlike scalar quantities that can be added or subtracted the usual way like ordinary numbers, vector addition involves the magnitude and direction of the vectors. The result of addition of vectors is known as resultant vector and two or more vectors are added, only if they have the same unit. It does not make sense to add a velocity vector with a unit of meters/second with displacement vector with a unit of meter. Composition of vectors is another term for finding the resultant of vectors. Vectors can be added geometrically or algebraically.

Geometrical method of addition of vectors

Triangle law of vector addition: According to this law, if two vectors are represented by two sides of a triangle taken in order, then their vector total is represented by third side of triangle taken in the opposite order.

Two vectors **A** and **B** of Figure 3(a) can be added geometrically by first drawing **A** on a piece of graph paper to a given scale so that its direction is specified to a coordinate system. Then draw vector **B** to the same scale with the tail of **B** starting at the tip of **A**, as shown in Figure 3(b).

The resultant **R** = **A** + **B** is the vector drawn from the tail of **A** to the tip of **B**. This procedure is called Triangle method of vector addition.

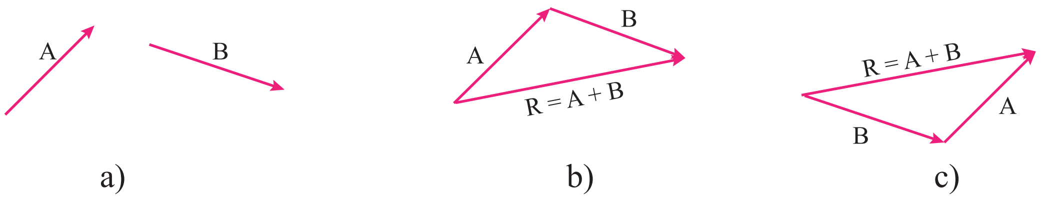


Figure 3. When vector **B** *is added to vector* **A**, *the resultant* **R** *is the vector that runs from the tail of* **A** *to the tip of* **B**.

The resultant does not change if **B** is taken first and then the tail of **A** starts at the tip of **B** and the resultant runs from the tail of **B** to the tip of **A**, as in Figure 3(c).

The same general approach can be used to add more than two vectors as shown in Figure 4. First one of the vectors is drawn on a graph paper beginning at any convenient point. Then the tail of the next vector is connected to the tip of the preceding vector. The procedure continues until all the given vectors are taken. The vector joining the tail of the first vector and the tip of the last vector represents the resultant. This is known as polygon method of addition of vectors.

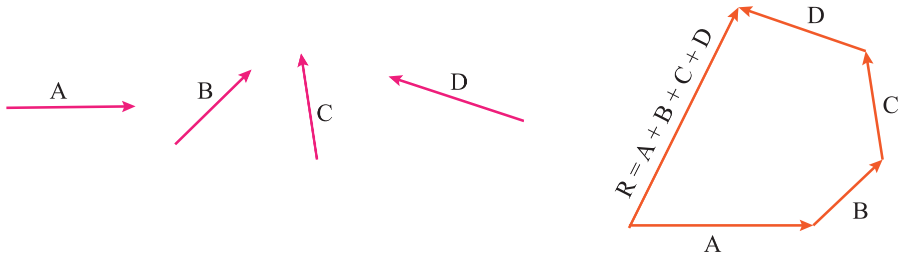
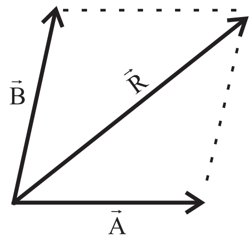


Figure 4. Geometric constructions for summing four vectors **A**, **B**, **C** *and* **D**. *The resultant* **R** *is the one that completes the polygon.*

Parallegram law of vector addition: According to this law, if two vectors are represented by two adjacent sides of a parallegram, then their vector total is represented by the diagonal of that parallegram starting from the same point.



Two vectors A and B with their tails brought to a common origion. By completing parallegram, the sum A + B will be diagonal of parallegram.

ACTIVITY 1

Draw on a graph paper and find the magnitude and direction of the following displacement vectors: S1 of 40 km, East, S2 of 30 km, 37o North of East, S3 of 10 km, South. Describe the scale used.

Negative of a Vector

The negative of the vector **A** is defined as the vector that when added to **A** gives zero for the vector sum. That is, **A** + (-**A**) = 0. The vectors **A** and - **A** have the same magnitude but point in opposite directions.

ACTIVITY 2

If force vector F = 35 N, toward 25o North of West, what is vector - F?

Subtraction of vectors

Subtraction of vectors makes use of the definition of negative of a vector. Subtracting vector **B** from vector **A** is the same as adding the negative of vector **B** to vector **A,** i.e., **A** – **B** = **A** + (–**B**), Figure 5(a).

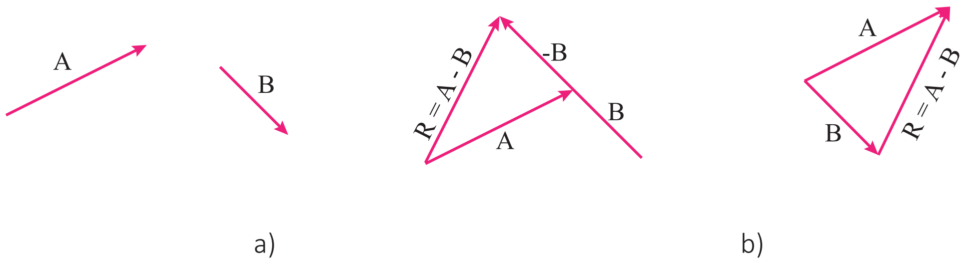


Figure 5. Substracting vector B from vector A.
*To subtract **B** from **A**, apply the rule of vector addition to the combination of **A** and -**B**: Draw **A** along some convenient axis, place the tail of -**B** at the tip of **A**, and **R** is the difference **A**-**B**. (Figure 1.5a) A second way of looking at vector subtraction. The difference vector **R** = **A** - **B** is the vector that we must add to **B** to obtain **A**. (Figure 5b).*

Key terms

- Resultant: The vector sum of two or more vectors.
- Negative of a given vector: a vector of the same magnitude but opposite directions

Another method of addition of two vectors of **A** and **B** involves a tail-to-tail connection of the vectors, Figure 6. A parallelogram is then constructed by taking the vectors as its sides and the diagonal of the parallelogram drawn from a point where the vectors are connected tail-to-tail to the opposite corner represents the resultant vector. The direction of the resultant can be expressed in terms of the angle between the resultant and one of the vectors.

$$\mathbf{R} = \mathbf{A} + \mathbf{B}$$

Figure 6. Parallelogram method of vector addition involves tail-to tail connection.

Activity 3

Draw the vectors on a graph paper and determine the resultant of two force vectors of F1 = 10 N, 37o above the positive x-axis and F2 = 6 N, 53o below the positive x-axis. What is the measure of the angle between the resultant and the positive x-axis?

Activity 4

In order to cross a river in the shortest possible time, is it better to aim yourself upstream so that you end up swimming straight across or to aim straight across and swim at an angle downstream? Discuss your answers in a small group.

Resolution of vectors, Component of a vector and Unit vectors

What is meant by resolution of vectors?

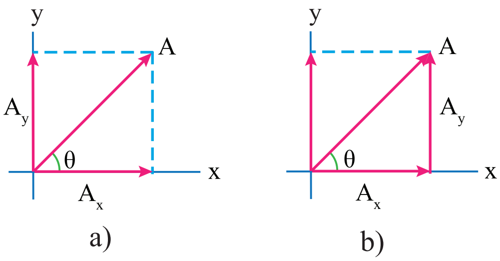


Figure 7. a) A vector **A** *lying in the xy plane can be represented by its component vectors A_x and A_y. b) The y component vector A_y can be moved to the right so that it adds to A_x.*

The component of a vector is projection of the vector on the axes of a rectangular coordinate system. A vector can be completely described by its components. Consider vector **A** in a rectangular coordinate system as shown in figure 7.

The projection of vector **A** on the x-axis labeled as A_x is the x-component of **A** and the projection of vector **A** on the y-axis labeled as A_y is the y-component of **A**.

$\mathbf{A} = \mathbf{A}_x + \mathbf{A}_y$ Eq 1.1

The process of finding the components of a vector is known as resolving the vector. From figure 7 (a) and the definition of sine and cosine, we have $\cos\theta = A_x/A$ and $\sin\theta = A_y/A$. Hence the components of A are

$$A_x = A\cos\theta$$

$$A_y = A\sin\theta$$

Vector **A** can be reconstructed by arranging the components A_x and A_y tip to tail. These components form two sides of a right triangle with a hypotenuse of length A and it follows that the magnitude and direction of **A** are related to its components through the expressions

$$A = \sqrt{A_x^2 + A_y^2} \quad \text{and} \quad \theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) \qquad \text{Eq 1.2}$$

Note that component of a vector can be positive or negative, Table 1.1

Table 1 The signs of the components of a vector A depend on the quadrant in which the vector is located.

	y (left of axis)	y (right of axis)
Above x-axis	A_x negative A_y positive	A_x positive A_y positive
Below x-axis	A_x negative A_y negative	A_x positive A_y negative

ACTIVITY 5

Think about which side of the triangle containing the components is adjacent to the angle and which side is opposite, and assign the cosine and sine accordingly.

Unit vectors

Vector quantities often are expressed in terms of unit vectors. A unit vector is a dimensionless vector having a magnitude of exactly 1. Unit vectors are

KEY TERMS

- Unit vector: a vector of magnitude one. $\hat{i}$, $\hat{j}$, and $\hat{k}$ are unit vectors along the x, y, and z axis, respectively.
- Resolving a vector: Finding the components of the vector.

used to specify a given direction and have no other physical significance. They are used solely as a convenience in describing a direction in space. We shall use the symbols $\hat{i}$, $\hat{j}$, and $\hat{k}$ to represent unit vectors pointing in the positive x, y, and z directions, respectively. (The "hats" on the symbols are standard notations for unit vectors.) The unit vectors $\hat{i}$, $\hat{j}$, and $\hat{k}$. form a set of mutually perpendicular vectors in a right-handed coordinate system, as shown in Figure 8 (a). The magnitude of each unit vector equals 1.

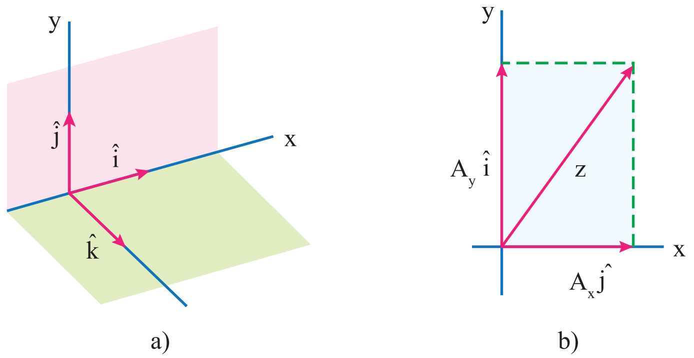


Figure 8. (a) Unit vector along the x, y, and z = axis (b) Unit vectors, $\hat{i}$ and $\hat{j}$ are used to represent components of a plane vectors.

Consider a vector **A** lying in the xy plane, as shown in Figure 8 (b). The product of the component A_x and the unit vector $\hat{i}$ is the vector $A_x\ \hat{i}$, which lies on the x axis and has magnitude A_x. Likewise, $A_y\ \hat{j}$ is a vector of magnitude A_y lying on the y axis. Thus, the unit–vector notation for the vector A is

$$\mathbf{A} = A_x\ \hat{i}\ +\ A_y\ \hat{j} \qquad \text{Eq 1.3}$$

A unit vector in the direction of a given vector is defined as the ratio of the vector to its magnitude.

$$\mathbf{U}_A = \frac{\mathbf{A}}{|A|} \qquad \text{Eq 1.4}$$

Position vector

If a point lying in the xy plane has Cartesian coordinate (x ,y), then the position vector r can be used to specify the position of the point in terms of its components x and y as (Figure 9).

$$\mathbf{r} = x\,\hat{i} + y\,\hat{j} \qquad \text{Eq 1.5}$$

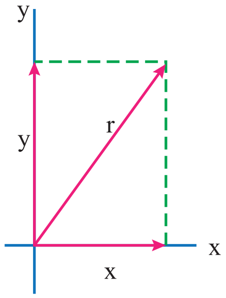


Figure 9. Position vector representing the position of point p in the x-y plane

Concurrent vectors

Concurrent vectors are those types of vectors that pass the same point.

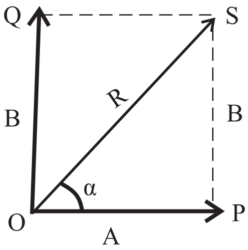


Figure 10.

Let OP and OQ represent two concurrent vectors A and B acting at right angle.

The sum of A and B,

$R = A + B$

So from geometry of figure,

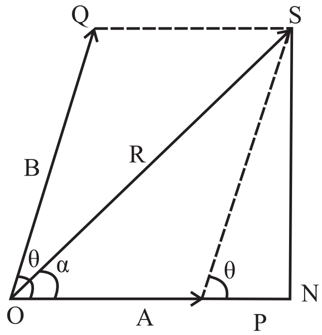
$OS^2 = OP^2 + PS^2$

$R^2 = A^2 + B^2$

if the α is the angle resultent R makes with vector A than,

$$\tan\alpha = \frac{PS}{PO} = \frac{B}{A}$$

Addition of two concurrent vectors acting at an angle (Anayltical Method)



Let OP and OQ represent two concurrent vectors A and B making and angle θ, Then sum of A and B

$R = A + B$

SN is normal to OP, So from the geometry of figure, $OS^2 = ON^2 + SN^2$

but $ON = OP + PN = A + BCos\theta$

$SN = BSin\theta$

$OS^2 = (A + BCos\theta)^2 + (BSin\theta)^2$

$R^2 = A^2 + B^2Cos^2\theta + 2ABCos\theta + BSin^2\theta$

$R^2 = A^2 + B^2(Cos^2\theta + Sin^2\theta) + 2ABCos\theta$

$R^2 = A^2 + B^2 + 2ABCos\theta$

$$R = \sqrt{A^2 + B^2 + 2AB\,Cos\theta}$$

The above relation gives the magnitude of resultent vector, if α is the angle resultent makes with vector A. Than Tan

$$\alpha = \frac{SN}{ON} = \frac{SN}{OP + PN}$$

$$Tan\,\alpha = \frac{B\,Sin\theta}{A + B\,Cos\theta}$$

Case 1 If $\theta = 0°$ i.e. two vectors are parallel than magnitude of resultant $R = A+B$ and $\tan\alpha = 0$ and $\alpha = 0°$ i.e. resultant vector is also parallel to vector A and vector B.

Case 2 If $\theta = 90°$ i.e. two vectors are perpendicular than magnitude of resultant

$$R = \sqrt{A^2 + B^2} \text{ and } \tan\alpha = \frac{B}{A}$$

Case 3 If θ = 180° i.e. two vectors are anti parallel than magnitude of resultant R = A - B or B - A and α = 0°

Addition of vectors algebraically

Algebraically by means of their components. If the x- and y- components of vector **A** are $\mathbf{A}_x$ and $\mathbf{A}_y$, and those of vector **B** are $\mathbf{B}_x$ and $\mathbf{B}_y$, respectively, then their resultant will be

$$\mathbf{R} = \mathbf{A} + \mathbf{B}$$

And the components of the resultant are

$$R_x = A_x + B_x \text{ and } R_y = A_y + B_y$$

The magnitude and direction of the resultant are given by

$$R = \sqrt{R_x^2 + R_y^2} \quad \text{and} \quad \theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

To add vector **A** to vector **B** with the vectors expressed in terms of the unit vectors as $\mathbf{A} = A_x\,\hat{i} + A_y\,\hat{j}$ and $\mathbf{B} = B_x\,\hat{i} + B_y\,\hat{j}$, all we do is add the x and y components separately. The resultant **R** = **A** + **B** is therefore

$$\mathbf{R} = (A_x\,\hat{i} + A_y\,\hat{j}) + (B_x\,\hat{i} + B_y\,\hat{j})$$

$$\mathbf{R} = (A_x + B_x)\,\hat{i} + (A_y + B_y)\,\hat{j} \qquad \text{Eq 1.7}$$

We see that the components of the resultant vector are

$$\mathbf{R_x} = A_x + B_x$$

$$\mathbf{R_y} = A_y + B_y$$

We obtain the magnitude of **R** and the angle it makes with the x axis from its components, using the relationships

$$R = \sqrt{R_x^2 + R_y^2} \quad \text{and} \quad \theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

For situations involving motion in three component x, y and z directions, the extension of our methods to three-dimensional vectors is straightforward. If **A** and **B** both have x, y, and z components, we express them in the form

$$\mathbf{A} = A_x\,\hat{i} + A_y\,\hat{j} + A_z\,\hat{k}$$

$$\mathbf{B} = B_x\,\hat{i} + B_y\,\hat{j} + B_z\,\hat{k}$$

The vector sum (resultant) of A and B is

$\mathbf{R} = \mathbf{A} + \mathbf{B}$

$\mathbf{R} = (A_x\,\hat{i} + A_y\,\hat{j} + A_z\,\hat{k}) + (B_x\,\hat{i} + B_y\,\hat{j} + B_z\,\hat{k})$

$\mathbf{R} = (A_x + B_x)\,\hat{i} + (A_y + B_y)\,\hat{j} + (A_z + B_z)\,\hat{k}$ Eq 1.8

Where $R_x = A_x + B_x$, $R_y = A_y + B_y$ and $R_z = A_z + B_z$

The magnitude of the resultant is $R = \sqrt{R_x^2 + R_y^2 + R_z^2}$, and the angle θ_x that **R** makes with the x axis is found from the expression $\cos\theta_x = \frac{R_x}{R}$.

Similarly the angle θ_y that **R** makes with the y axis is found from the expression $\cos\theta_y = \frac{R_y}{R}$, and the angle θ_z that **R** makes with the z axis is found from the expression $\cos\theta_z = \frac{R_z}{R}$. The cosine of the angles is also known as direction cosine.

Examples

Vector A has x-component of 2 units and y-component of -3 units. Vector B has x-component of -5 units and y-component of 7 units. Find

(a) the x- and y- components of the resultant R = A + B,

(b) the magnitude of the resultant, and

(c) The angle that the resultant forms with the x-axis.

Solution

(a) The x-and y- components of the resultant is vector are Rx= Ax + Bx = 2 units + (-5) units = -3 unit and Ry = Ay + By = -3 units + 7 units = 4 units

(b) The magnitude of the resultant is $R = \sqrt{R_x^2 + R_y^2} = \sqrt{(-3\text{ units})^2 + (3\text{ units})^2} = 5$ units

(c) The angle between the resultant and the x-axis is $\theta = \tan^{-1}\left(\frac{4}{-3}\right) = -53^o$. With R_x negative and R_y positive, **R** is in the second quadrant and therefore, the resultant makes an angle of 53^o above the negative x axis.

Examples

A child walks 5 m north and then 4 m east on a horizontal field. How far and in what direction is she from the starting point?

Solution

Use a graph paper to draw each of the displacement vectors to a given scale. Join the vectors tip to tail and then construct the resultant, as in Figure 10.

Taking displacement S_1 = 5m, East, S_{1x} = 5 m and S_{1y} = 0.

Displacement S_2 = 4 m North, S_{2x} = 0 and S_{2y} = 4 m.

The x- and y-components of the resultant displacement are

Rx = $S_{1x} + S_{2x}$ = 5 m, and $R_y = S_{2x} + S_{2y}$ = 4 m.

The magnitude of the resultant displacement is $R = \sqrt{R^2 + R^2} = \sqrt{(5m)^2 + (4m)^2}$ $= \sqrt{41}$ m = 6.4m.

The angle $\theta = \tan\text{-}1\left(\frac{R_y}{R_x}\right) = \tan\text{-}1\left(\frac{4}{5}\right) =$ 38.6o. Both R_x and Ry are positive and the resultant is in the first quadrant making an angle of 38.6o above the positive x direction.

The resultant displacement is therefore, R = 6.4m, 38.6o North of East.

Multiplication of a vector by a scalar

When a vector is multiplied by a scalar n, the resulting vector will have a magnitude equal to n times that of the original vector. If n is positive, the direction of the new vector will be the same as that of the original vector, and if n is negative, the new vector will be in opposite direction to the original vector.

Examples

If force vector A = 25 N, toward 37° N of E, then what is vector (a) 3A? (b) -3A?

Solution

(a) vector 3 A = 75 N, toward 37° N of E, and

(b) vector -3 **A** = 75 N, toward 37° S of W.

Multiplication of vectors

Vectors can be multiplied either by way of scalar (Dot) product or vector (cross product).

Scalar (Dot) product of two vectors

Given two vectors **A** and **B** such that $\mathbf{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$, $\mathbf{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$, the dot product **A . B** is defined as

A . B = AB cos θ, where θ is the angle between **A** and **B**.

Activity 6

Apply the definition of dot product to show each of the following. Note that each of the unit vectors $\hat{i}$, $\hat{j}$, and $\hat{k}$ have magnitude 1.

$\hat{i}.\hat{i} = |\hat{i}||\hat{i}|\cos(0°) = 1$ $\hat{j}.\hat{i} = |\hat{j}||\hat{i}|\cos(90°) = 0$ $\hat{k}.\hat{i} = |\hat{k}||\hat{i}|\cos(90°) = 0$

$\hat{i}.\hat{j} = |\hat{i}||\hat{j}|\cos(90°) = 0$ $\hat{j}.\hat{j} = |\hat{j}||\hat{j}|\cos(0°) = 1$ $\hat{k}.\hat{j} = |\hat{k}||\hat{j}|\cos(90°) = 0$

$\hat{i}.\hat{k} = |\hat{i}||\hat{k}|\cos(90°) = 0$ $\hat{j}.\hat{k} = |\hat{j}||\hat{k}|\cos(90°) = 0$ $\hat{k}.\hat{k} = |\hat{k}||\hat{k}|\cos(0°) = 1$

Activity 7

Using the properties obtained in activity 1.6 show that the dot product of A . B can also be expressed as

$A . B = A_x B_x + A_y B_y + A_z B_z$

Did you know?

The scalar product of two vectors is a scalar (a number).

Vector (cross) product of two vectors

Given two vectors **A** and **B** such that $\mathbf{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$, $\mathbf{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$, the cross product **A x B** is defined as **A x B** = (AB sin θ) $\hat{\boldsymbol{n}}$, where θ is the angle between **A** and **B, and** $\hat{\boldsymbol{n}}$ is a unit vector in the direction of **A x B**. $\hat{\boldsymbol{n}}$ is perpendicular to the plane formed by **A** and **B**, and the best way to determine this direction is to use the right-hand rule illustrated in Figure 11.

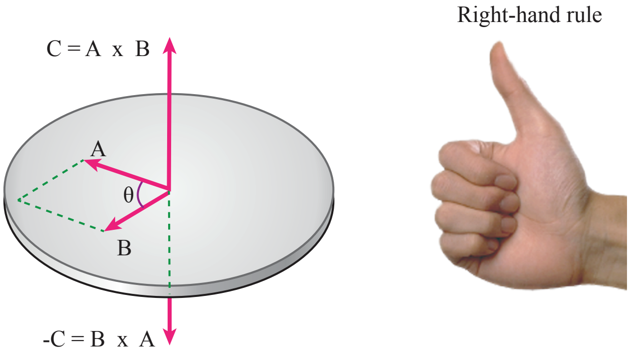


Figure 11. The vector product **A x B** *is a third vector* **C** *having a magnitude AB sin θ equal to the area of the parallelogram shown. The direction of* **C** *is perpendicular to the plane formed by* **A** *and* **B***, and this direction is determined by the right-hand rule.*

Activity 8

Apply the right-hand rule to show that A x B = - B x A

Activity 9

Apply the definition of cross product to show each of the following.

$\hat{i}\times\hat{i}=0$	$\hat{j}\times\hat{i}=\hat{k}$	$\hat{i}\times\hat{k}=\hat{j}$
$\hat{i}\times\hat{j}=\hat{k}$	$\hat{j}\times\hat{j}=0$	$\hat{k}\times\hat{j}=\hat{i}$
$\hat{i}\times\hat{k}=\hat{j}$	$\hat{j}\times\hat{k}=\hat{i}$	$\hat{k}\times\hat{k}=0$

Activity 10

Using the properties obtained in activity 1.9, show that the cross product A x B can be written as

$$A\times B=(A_yB_z-A_zB_y)\hat{i}+(A_zB_x-A_xB_z)\hat{j}+(A_xB_y-A_yB_x)\hat{k}$$

Examples

When force **F** applied on a body displaces the body through displacement **S**, the work done by the applied force is calculated as W = **F . S**. If **F** = $(2\hat{i}-\hat{j}+3\hat{k})$ N and S = $(-\hat{i}+3\hat{j}+2\hat{k})$ m, what is the work done by the force?

Solution

Using W = **F . S** = $F_xS_x+F_yS_y+F_zS_z$ = [(2)(-1) + (-1)(3) + (3)(2)] = 1 Nm = 1 J

Examples

Given two vectors **A** and **B** such that $\mathbf{A} = \hat{i} - 4\hat{j} - 2\hat{k}$, $\mathbf{B} = 3\hat{i} + 2\hat{j} - 2\hat{k}$, find (a) the magnitude and direction of the cross product **C** = **A x B**.(b) a unit vector along vector **C**.

Solution

(a) $A \times B = (A_yB_z - A_zB_y)\hat{i} + (A_zB_x - A_xB_z)\hat{j} + (A_xB_y - A_yB_x)\hat{k}$

$= ((-4)(-2) - (-2)(2))\ \hat{i} + ((-2)(3) - (1)(-2))\ \hat{j} + ((1)(2) - (-4)(3)\ \hat{k} = 12\ \hat{i} - 4\hat{j} + 14\ \hat{k}$

Therefore $C = 12\ \hat{i} - 4\hat{j} + 14\ \hat{k}$

The magnitude of **C** is $C = \sqrt{C_x^2 + C_y^2 + C_z^2} = \sqrt{(12)^2 + (-4)^2 + (14)^2} = 18.9$

(b) The unit vector of a given vector is obtained by dividing the vector by its magnitude.

$$\mathbf{U}_C = \frac{C}{|C|} = \frac{12\hat{i} - 4\hat{j} + 14\hat{k}}{18.9} = 0.63\ \hat{i} - 0.21\ \hat{j} + 0.74\ \hat{k}$$

Examples

A particle starting at a point travelled through the following displacements in the order indicated: $\mathbf{S}_1$ = 7 m, due East, $\mathbf{S}_2$ = 5 m, due 53° N of W, $\mathbf{S}_3$ = 6m due West, and finally $\mathbf{S}_4$ = 6 m, due South, Figure 12. Find

(a) the x- and y-components of the resultant displacement, and

(b) the magnitude and direction of the resultant displacement.

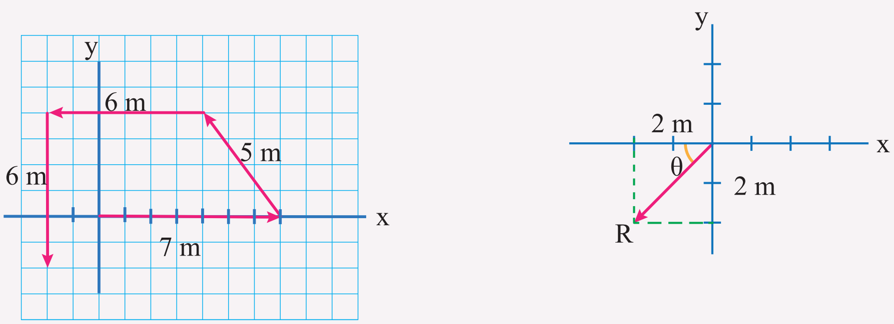


Figure 12. (a) Geometrical representation of the displacements. The displacements are joined tip to tail. (b) the resultant R makes angle θ below the negative x- axis.

Solution

The key idea here is finding the components of each of the displacement vectors.

(a) The x-and y- components of each of the displacements are listed in Table 1.2 . Components that are directed toward the negative axis are taken to be negative.

Table 2

	x-component	y-component
S_1	$S_{1x} = 7m$	$S_{1y} = 0$ m
S_2	$S_{2x} = -5$ m (cos 53°) = -5 m (0.6) = -3 m	$S_{2y} = 5$ m (sin 53°) = 5 m (0.8) = 4 m
S_3	$S_{3x} = -6$ m	$S_{3y} = 0$ m
S_4	$S_{4x} = 0$ m	$S_{4y} = -6$ m
Sum	$R_x = 7$ m + (-3 m) + (-6 m) + 0 = -2 m	$R_y = 0$ m + 4 m + 0 m + (-6 m) = -2 m

(b) The magnitude of the resultant is $R = \sqrt{R_x^2 + R_y^2} = \sqrt{(-2m)^2 + (-2m)^2} = 2\sqrt{2}$ m, and

the resultant makes angle $\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{2}{-2}\right) = \tan^{-1}(-1) = -45^0$. R is in the second quadrant and we say that it forms an angle of 45o above the negative x-axis.

Examples

Find the sum of two vectors A and B lying on the xy plane such that $A = -3\hat{i} - 5\hat{j}$ and $B = 6\hat{i} + \hat{j}$.

Solution

Comparing the expressions of **A** and **B** with the general expressions we see that

$A_x = -3, A_y = -5, B_x = 6$, and $B_y = 1$

$\mathbf{R} = \mathbf{A} + \mathbf{B}$

$R_x = A_x + B_x = (-3) + 6 = 3$ and $R_y = A_y + B_y = -5 + 1 = -4$

$\mathbf{R} = R_x\hat{i} + R_y\hat{j}$

$\mathbf{R} = 3\hat{i} - 4\hat{j}$

The magnitude of **R** is $R = \sqrt{R_x^2 + R_y^2} = \sqrt{(3)^2 + (-4)^2} = 5$, and $\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{-4}{3}\right) = -53^o$. The resultant is in the fourth quadrant making an angle of 53° below the positive x axis.

Examples

An object undergoes the following displacements: $S_1 = 20\hat{i} - 10\hat{j} - 12\hat{k}$, $S2 = -8\hat{i} + 15\hat{j} + 10\hat{k}$, $S_3 = -18\hat{i} - 10\hat{j} + 7\hat{k}$. Find the components of the resultant displacement and its magnitude.

Solution

Knowing the components of the displacement, the resulting displacement will have its x, y, and z components as

$R_x = \mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3$

The x component of **R** is $R_x = S_{1x} + S_{2x} + S_{3x} = 20 + (-8) + (-18) = -6$

The y component of **R** is $R_y = S_{1y} + S_{2y} + S_{3y} = -10 + 15 + (-10) = -5$

The z component of **R** is $R_z = S_{1z} + S_{2z} + S_{3z} = -12 + 10 + 7 = 5$

The magnitude of the resultant displacement is

$$\sqrt{R_x{}^2 + R_y{}^2 + R_z{}^2} = \sqrt{(-6)^2 + (-5)^2 + 5^2} = 9.3\text{m}$$

Examples

Two vectors **A** and **B** are given by $\mathbf{A} = (4\hat{i} - 3\hat{j} + \hat{k})$ m, and $\mathbf{B} = (-\hat{i} + \hat{j} + 4\hat{k})$ m. In unit vector notation find

(a) A + B

(b) A – B

(c) **A** third vector **C** such that **A** – **B** + **C** = 0

Solution

Here the vectors are expressed in terms of their components and what we do is add the respective components.

(a) $A + B = (Ax + Bx)\hat{i} + (Ay + By)\hat{j} + (Az + Bz)\hat{k}$

$= (4 + (-1))\hat{i} + (-3 + 1)\hat{j} + (1 + 4)\hat{k}$

$= (3\hat{i} - 2\hat{j} + 5\hat{k})$ m

(b) $\mathbf{A} - \mathbf{B} = (A_x - B_x)\hat{i} + (A_y - B_y)\hat{j} + (A_z - B_z)\hat{k}$

$= (4 - (-1))\hat{i} + (-3 - 1)\hat{j} + (1 - 4)\hat{k}$

$= (5\hat{i} - 4\hat{j} - 3\hat{k})$ m

(c) **A** + B – **C** = 0

Taking $C = C_x\hat{i} + C_y\hat{j} + C_z\hat{k}$

$$\mathbf{A} - \mathbf{B} + \mathbf{C} = (A_x - B_x + C_x)\hat{i} + (A_y - B_y + C_y)\hat{j} + (A_z - B_z + C_z)\hat{k}$$
$$= (4 - (-1) + C_x)\hat{i} + (-3 - 1 + C_y)\hat{j} + (1 - 4 + C_z)\hat{k}$$
$$= (5 + C_x)\hat{i} + (-4 + C_y)\hat{j} + (-3 + C_z)\hat{k}$$

For $\mathbf{A} - B + \mathbf{C} = 0$, we write

$$(5 + C_x)\hat{i} + (-4 + C_y)\hat{j} + (-3 + C_z)\hat{k} = 0$$

$5 + C_x = 0$, $-4 + C_y = 0$, and $-3 + C_z = 0$

$C_x = -5$ $C_y = 4$ $C_z = 3$

Therefore, vector $C = -5\hat{i} + 4\hat{j} + 3\hat{k}$

Exercises

1. Does the order in which the vectors are drawn affect the resultant? Is **A** + **B** + **C** different from **B** + **C** + A?
2. Vector **A** has a magnitude of 6 units and makes an angle of 45° with the positive x axis. Vector **B** also has a magnitude of 6 units and is directed along the negative x axis. Using graphical methods, find (a) the vector sum **A** + **B** and (b) the vector difference **A** - **B**.
3. Vector **A** has magnitude of 10 units and makes an angle of 45° above the positive x-axis. Vector **B** has a magnitude of 10 units and is directed along the negative x- axis. Using algebraic method of vector addition, find
   (a) the vector sum **A** + **B**,
   (b) the vector difference **A** – **B**
4. What are the x- and y- components of vector B of Figure 13, where the angle θ is measured with respect to the y-axis?

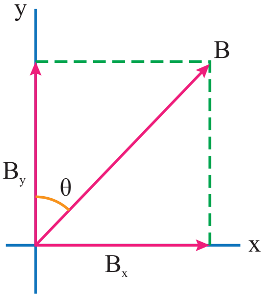


Figure 13.

5. If vector P is added to vector **C** = $6\hat{i} + 2\hat{j} - 3\hat{k}$, the result is a vector in the positive direction of the y axis, with a magnitude equal to that of C. What is the magnitude of P?

1.2 Projectile Motion

Key terms

- Uniform motion: Motion at a constant speed.
- Uniformly accelerated motion: Motion with a constant acceleration where the rate of change of velocity is constant.

Projectile motion is the motion of objects thrown or projected into the air and subjected to only the acceleration due to gravity. The object is called a projectile and the path it follows is its trajectory. In this section, you will learn the theory behind projectile motion and how to calculate the values you need to know about the motion of a projectile.

Projectile motion can be observed in many everyday situations. Throwing a baseball or volleyball, shooting a rifle or an arrow are obvious examples of a projectile. Galileo studied projectiles and found that they moved in two directions at the same time. He determined that the trajectory of a projectile, neglecting air resistance, is a parabolic path (Figure 14).

A boy shoot a basketball

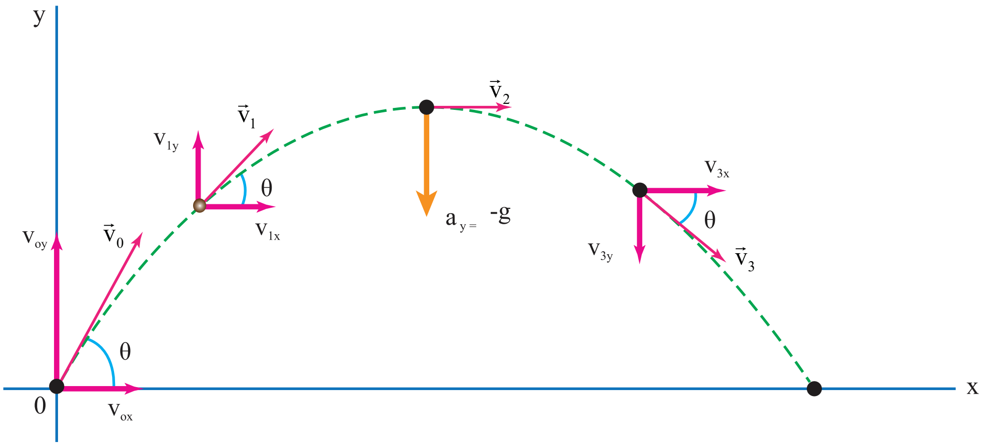


Figure 14. Parabolic path of a projectile that leaves the origin with velocity v_o.

Parabolic path of a projectile that leaves the origin with velocity Gravity influences the vertical motion of a projectile by accelerating it downward while the horizontal motion is not affected by gravity. So, a projectile undergoes a uniform horizontal motion and vertical uniformly accelerated motion. The horizontal component of acceleration of the projectile, $a_x = 0$ and the vertical acceleration of the projectile $a_y = g = 9.8\ m/s^2$.

Thus, when analyzing projectile motion, consider it to be the superposition of two motions: (1) constant-velocity motion in the horizontal direction and (2) free-fall motion in the vertical direction. The horizontal and vertical components of a projectile's motion are completely independent of each other and can be handled separately, with time t as the common variable for both components.

The Horizontal Motion

Because there is no acceleration in the horizontal direction, the horizontal component v_x of the projectile remains v_{ox} throughout the motion.

The horizontal displacement x from the starting point with $a_x = 0$ is

$$x = v_{ox}t$$

With $v_{ox} = v_o \cos\theta$, we have

$x = (v_o \cos\theta)t$ Eq 1.9

The Vertical Motion

The vertical motion is similar with the motion of a body thrown vertically upward where the initial velocity u is replaced by the y-component of the initial velocity,

$$v_{oy} = v_o\sin\theta.$$

$$y = v_{oy}t - ½ gt^2$$

$y = (v_o \sin\theta)t - ½ gt^2$ Eq 1.10

The vertical component of the velocity behaves just as for a ball thrown vertically upward. It is directed upward initially, it decreases as the projectile moves upward, becomes zero at the maximum height and then increases as the projectile moves downward.

The vertical component of the velocity at time t is

$v_y = v_{oy} - gt$

$v_y = v_o\sin\theta - gt,$ Eq 1.11

The maximum height

The vertical component of velocity in terms of the displacement h from the point of projection are related as

$v_y^2 = v_{oy}^2 - 2gh$

$v_y^2 = (v_o \sin\theta)^2 - 2gh$ Eq 1.12

The magnitude of the vertical velocity steadily decreases to zero, which marks the maximum height of the path. As the projectile reaches the maximum height, the vertical component of the velocity $v_y = 0$ and Eq. 1.12 is written as

$$0 = (v_o \sin\theta)^2 - 2gh_{max}$$

$$h_{max} = \frac{v_{oy}^2}{2g} = \frac{(v_o \sin\theta)^2}{2g} = \frac{v_o^2 \sin^2\theta}{2g}$$

The time of flight

Equation 1.11 with $v_y = 0$ can be used to determine the time taken by the projectile to reach the maximum height of its trajectory as

$$v_y = v_o \sin\theta - gt$$

$$0 = v_o \sin\theta - gt$$

Solving for t we get time taken to reach the maximum height $t = \frac{v_o \sin\theta}{g}$

The projectile takes the same time to fall back to the starting point and the total time of flight is twice t.

$$T = \frac{2v_o \sin\theta}{g}$$ Eq 1.13

Activity 11

Show that a body projected at an angle from the ground takes equal time to rise to the maximum height as it takes to fall from the maximum height.

The Horizontal Range

The horizontal range R of the projectile is the horizontal distance the projectile has traveled when it returns to its initial height (the height at which it is launched).

$R = Vo_x T = (vo \cos\theta)T$ Eq 1.14

Using Eq. 1.13 in Eq. 1.14, gives $R = \dfrac{2v_o^{\,2}\sin\theta\cos\theta}{g}$

Using the identity $2v_o^{\,2}\sin\theta\cos\theta = \sin(2\theta)$,

$$R = \frac{v_o^{\,2}\sin(2\theta)}{g} \qquad \text{Eq 1.15}$$

Caution: This equation does not give the horizontal distance traveled by a projectile when the final height is not the launch height.

Note that R in Eq 1.15 has its maximum value when $\sin(2\theta) = 1$, which corresponds to $2\theta = 90^o$ or $\theta = 45°$.

The horizontal range R is maximum for a launch angle of 45°.

$$R_{max} = \frac{v_o^{\,2}}{g} \qquad \text{Eq 1.16}$$

Activity 12

Show that for the same initial speed of projection, complementary values of angles of projection (i.e., θ and 90 - θ) result in the same value of range. What can you say about the time of flight and the maximum height attained for complementary angles of projection? See Figure 15.

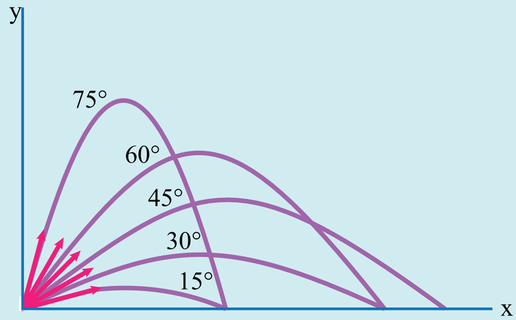


Figure 15. A projectile launched from the origin with the same initial speed but at various angles of projection.

The Equation of the Path

We can find the equation of the projectile's path (its trajectory) by solving for time t in Eq 1.9 and using it in Eq 1.10. We obtain

$$y = (\tan\theta)\,x - \left(\frac{g}{2v_o^{\,2}\cos^2\theta}\right)x^2 \qquad \text{Eq 1.17}$$

Velocity of the projectile

After time t, the x- component of velocity is $v_x = v_{ox}$, the y-component of velocity $v_y = v_{oy} - gt$, and the magnitude of velocity of the projectile at time t is has a magnitude of

$$v = \sqrt{v_x^{\,2} + v_y^{\,2}}$$

$$v = \sqrt{v_{ox}^{\,2} + v_y^{\,2}} \qquad \text{Eq 1.18}$$

Examples

A long-jumper leaves the ground at an angle of 22° above the horizontal and at a speed of 10 m/s.

(a) How far does he jump in the horizontal direction?

(b) What is the maximum height reached?

Solution

We consider the motion of the athlete as equivalent to that of a body projected at an angle.

(a) The horizontal distance covered is given by the equation $x = (v_o\cos\theta)t$

The total time for which the jumper stays in the air is

$$T = \frac{2v_o\sin\theta}{g} = \frac{2\times 10\,m/s\times\sin(22°)}{10\,m/s^2} = 0.75s$$

The horizontal distance is therefore, x = (10 m/s x cos(22°)) = 0.75 s = 6.95 m

(b) We find the maximum height by using $h_{max} = \dfrac{v_o^{\,2}\sin^2\theta}{2g}$

$$= \frac{(10\,m/s^2)\ \sin(22°)}{2(10\,m/s^2)} = 0.7m$$

Examples

A ball is kicked upward from the ground at an angle of projection of 30°. If it takes 3.2s to hit the ground, then find

(a) the initial speed of projection,

(b) the maximum height attained, and

(c) the horizontal range.

Solution

(a) Given the time of flight T = 3.2 s, we use the equation $T = \frac{2v_o \sin\theta}{g}$ and solving for initial speed vo we write

$$V_O = \frac{gT}{2\times\sin 30^\circ} = 32\ m/s$$

(b) The maximum height is $h_{max} = \frac{v_o^{\ 2}\sin^2\theta}{2g} = \frac{(32\,m/s)^2 \sin(30°)}{(10\,m/s^2)} = 12.8m$

(c) The range is $R = \frac{v_o^{\ 2}\sin(2\theta)}{g} = 89m$

Activity 13

Projectile-Target demonstration

In a popular lecture demonstration, a projectile is fired at a target T in such a way that the projectile leaves the gun at the same time the target is dropped from rest, as shown in Figure 16. Show that if the gun is initially aimed at the stationary target, the projectile hits the target.

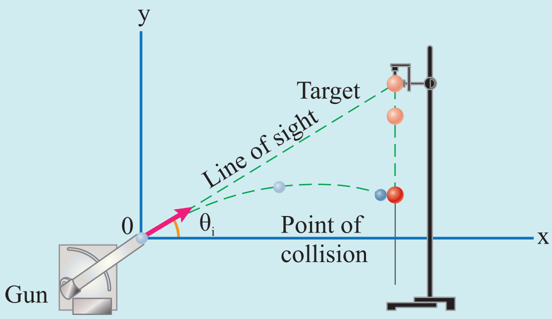


Figure 16. Schematic diagram of motion of projectile and target

An object projected at an angle from a certain height above the ground

A projectile projected at some height above the ground performs two dimensional motion (horizontal uniform motion and vertical uniformly accelerated motion) as the one projected at an angle from the ground. As the body falls beyond the level at which it was launched Eq. 1.13 and 1.15 that are used to determine the time of flight and the range of projectile, respectively cannot be applied here. Example 1.12 below analyses the case where a body is projected at a certain height above the ground.

Examples

A stone is thrown from the top of a cliff upward at an angle of 37° to the horizontal with an initial speed of 20 m/s, as shown in Figure 17. If the height of the cliff is 65 m,

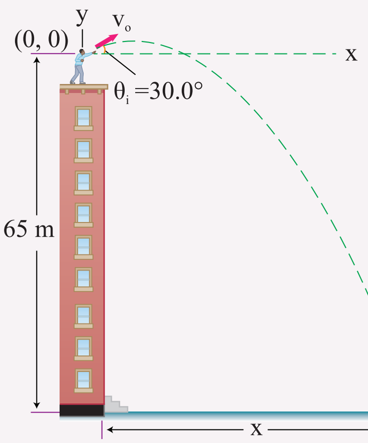


Figure 17. A stone is thrown from the top of a tower.

(a) how long does it take the stone to reach the ground?

(b) Where from the bottom of the height will the stone strike the ground?

(c) What is the speed of the ball just before it strikes the ground?

Solution

(a) Taking vertical displacement below the point of throwing to be negative we see that the displacement of the stone as it reaches the ground is -65 m.

Using the equation $y = (vo \sin\theta)t - ½gt^2$

$$-65\ m = (20\ m/s \times \sin37^o)t - ½\ (10\ m/s^2)t^2$$

$$(10\ m/s)t - (5\ m/s^2)t^2 + 65 = 0$$

Solving the quadratic equation for t gives, for the positive root, t = 5 s.

Note that the equation t = cannot be used to find the time of flight.

(b) The horizontal distance covered at time t is $x = (v_o \cos\theta)t$

$x = (20\ m/s\ x\ \cos 37^o)\ x\ 5\ s = 80\ m$

Note that the equation R = cannot be used to find the horizontal distance covered.

(c) of the stone at any time t is v =

$v_{ox} = v_o\cos\theta = 20\ m/s\ x\ \cos 37^o = 16\ m/s$, and

$v_y = v_{oy} - gt = v_o\sin\theta - gt = (20\ m/s\ x\ \sin 37^o) - 10\ m/s^2\ x\ 5\ s = -38\ m/s$. The minus sign indicates that the vertical component of velocity as the stone just strikes the ground is directed vertically downward.

v = = 41.2 m/s

Activity 14

A ball is kicked from the ground with a velocity of 20 m/s, 37o above the horizontal. Use the equations of motion to work out the components of the ball's velocity and the x and y- coordinates of its position every 0.5 s. Plot graphs of velocity versus time and displacement versus time of the motion of the ball from the time it is projected to the time it hits the ground.

An object projected horizontally from a certain height above the ground

The motion of body projected horizontally from a given height is also influenced by gravity. Its initial velocity has a horizontal component only ($v_{ox} = v_o$, and $v_{oy} = 0$). Its horizontal motion is uniform ($a_x = 0$) while the vertical motion is uniformly accelerated ($a_y = 9.8$ m/s^2), starting with $v_{oy} = 0$. Example 1.13 analyzes the motion of a horizontally projected body.

Activity 15

One ball is released from rest at the same instant that another ball is shot horizontally to the right, as in Figure 18.

Compare

(a) the time taken by the balls to reach the ground

(b) the speed with which the balls strike the ground Hint: Their vertical motions are identical

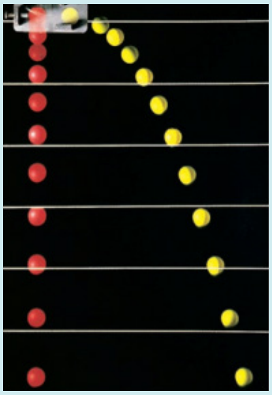


Figure 18. A stroboscopic photograph of two balls

Examples

A plane drops a package of supplies to a target on the ground, as shown in Figure 19. If the plane is traveling horizontally at 30 m/s and is 125 m above the ground, at what prior horizontal distance from the target must the package be released in order to hit the target?

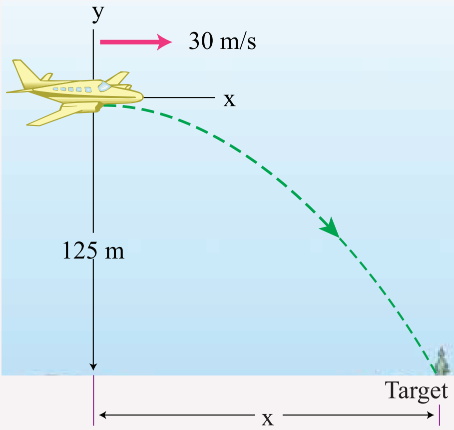


Figure 19. A package released from the plane to reach the target on the ground

Solution

The plane is traveling horizontally when it drops the package. The package is considered to be projected horizontally at the given height and it is in free fall while moving in the horizontal direction.

The package has initial velocity v_o which is equal to the velocity of the plane at the instant the package is released. The horizontal distance travelled by the package in time t will be $x = v_o t$, where v_o is the velocity of the plane at the instant of release.

The initial vertical component of velocity of the package is zero because at the instant of release, the package has only a horizontal component of velocity, i.e., $v_{oy} = 0$.

The vertical displacement of the package as it hits the target is

$$y = v_{oy}t - \frac{1}{2}gt^2$$

$$-125\text{m} = -\frac{1}{2} \times 10\ \text{m/s}^2 \times t^2$$

Solving for t, we get, t = 5 s.

The horizontal distance covered by the package in this time is x = 30 m/s x 5 s = 150 m.

Therefore, the package should be released at a horizontal prior distance of 150 m in order to reach the target.

Exercises

1. A fly ball is hit to the outfield. During its flight (ignore the effects of the air), what happens to its (a) horizontal and (b) vertical components of velocity? What are the (c) horizontal and (d) vertical components of its acceleration during ascent, during descent, and at the topmost point of its flight?

2. A player kicks a ball upward with speed v_o at an angle θ above the ground. With what sped and at what angle below the horizontal will the ball hit the ground?
3. A stone and a ball are thrown upward at an angle of projection of 30° and 60°, respectively.
   (a) What is the ratio of the maximum height attained by the stone to that attained by the ball?
   (b) What is the ratio of time of flight of the stone to that of the ball?
4. A ball is thrown upward in the air by a passenger on a train that is moving with constant velocity. Describe the path of the ball as seen by the passenger. Describe the path as seen by an observer standing by the tracks outside the train.

1.3 Circular Motion

A particle is in a circular motion if it travels around a circle or a circular arc. There are numerous cases of object moving in a curve about some fixed point. Rounding a curve in a car, and orbiting planet, spacecraft orbiting the Earth are examples of such a motion. The objects may have a constant or varying speed while moving along a circular path.

A spacecraft orbiting the Earth

Uniform Circular Motion

A particle in uniform circular motion travels around a circle or a circular arc at constant (uniform) speed. Although the speed does not vary, the particle is accelerating because the velocity changes in direction. Figure 20 shows the relationship between the velocity and acceleration vectors at various locations during uniform circular motion.

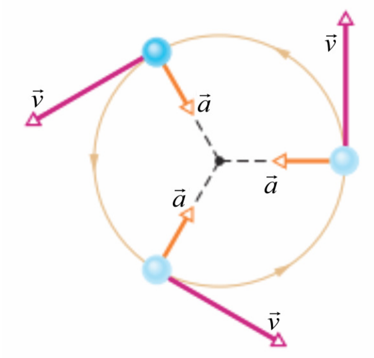


Figure 20. a uniform circular motion.

In a uniform circular motion, both the velocity and the acceleration vectors have constant magnitude, but their directions change continuously. The velocity is always directed tangent to the circle in the direction of motion and the acceleration is always directed radially inward. Because of this, the acceleration associated with uniform circular motion that arises due to the change in direction of velocity is called centripetal (meaning "center seeking") acceleration. The magnitude of this acceleration is

$$a_c = \frac{v^2}{r} \qquad \text{Eq. 1.19}$$

Derivation of the equation of centripetal acceleration

In Δ OAB and Δ oab of Figure 21 (a) and Figure (b), we see that $\frac{\Delta v}{\Delta s} = \frac{v}{r}$

Using the definition of acceleration that with $\Delta s = v\Delta t$ and $\frac{\Delta v}{v\Delta t} = \frac{v}{r}$, we write ac $= \frac{v^2}{r}$

Rearranging we get $\frac{\Delta v}{\Delta t} = \frac{v}{r}$, $a = \frac{\Delta v}{\Delta t}$

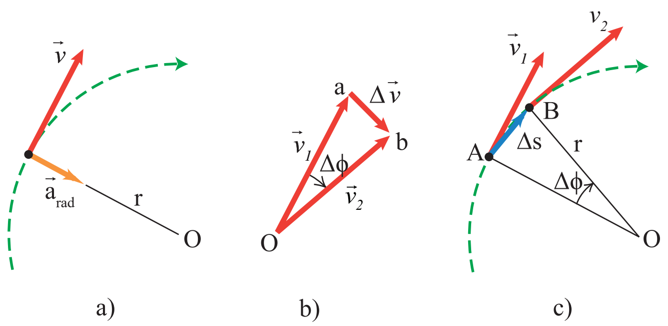


Figure 21 (a) A particle moves a distance as at constant speed along a circular path. (b) The corresponding change in velocity. (c) The instantaneous acceleration in uniform circular motion always points toward the center of the circle.

In many situations it is convenient to describe the motion of a particle moving with constant speed in a circle of radius r in terms of the period T, which is defined as the time required for one complete revolution. In the time interval T the particle moves a distance of 2πr, which is equal to the circumference of the particle's circular path. Therefore, because its speed is equal to the circumference of the circular path divided by the period, or $v = \frac{2\pi r}{T}$, it follows that

$$T = \frac{2\pi r}{v} \qquad \text{Eq.1.20}$$

Non uniform circular Motion

We have assumed in a uniform circular motion that the particle's speed is constant as it goes around the circle. If the speed varies, we call the motion non-uniform circular motion. In non-uniform circular motion, Eq 1.19 still gives the radial component of acceleration which is always perpendicular to the instantaneous velocity and directed toward the center of the circle. But since the speed v has different values at different points in the motion, the magnitude of the centripetal acceleration is not constant. The radial (centripetal) acceleration is greatest at the point in the circle where the speed is greatest. In non-uniform circular motion there is also a component of acceleration that is parallel to the instantaneous velocity. This component of the acceleration that

KEY TERMS

- Uniform circular motion: motion along a circle at a constant speed.
- Centripetal acceleration: acceleration that arises due to change in direction of tangential velocity.
- Tangential acceleration: acceleration that arise due to change in magnitude of tangential velocity.

arises due to change in magnitude of the velocity is tangent to the circle and it is known as tangential acceleration (a_t).

Consider the motion of a car along circular path as in Figure 22. The centripetal (radial) component of acceleration is toward the center of the circle and the tangential acceleration is parallel/anti parallel to the velocity (depending on whether the body moves with increasing/ decreasing speed).

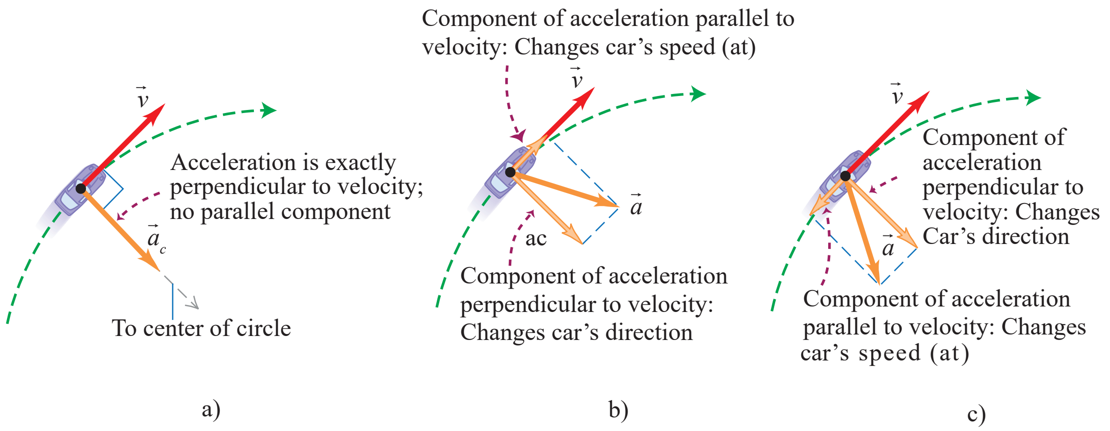


Figure 22. (a) Uniform circular motion: Speed is constant (b) Car speeding up along a circular path (c) Car slowing down along a circular path

The tangential component is in the same direction as the velocity if the particle is speeding up, and in the opposite direction if the particle is slowing down. If the particle's speed is constant, $a_t = 0$.

The total acceleration is the resultant of the centripetal acceleration (a_c) and tangential acceleration (a_t).

$$a = \sqrt{a_c^{\,2} + a_t^{\,2}} \qquad \text{Eq. 1.21}$$

Examples

An airplane is flying in a horizontal circle of radius 1.5 km with a speed of 450 km/h. What is the magnitude of the centripetal acceleration of the plane?

Solution

Given the speed v = 450 km/h = 125 m/s, and radius of the circular path r = 1.5 km = 1500 m, the centripetal acceleration is $a_c = \frac{v^2}{r} = \frac{(125\,m/s)^2}{1500m} = 10.4\,m/s^2$

Examples

What is the centripetal acceleration of the Earth as it moves in its orbit around the Sun?

Solution

We will simplify the problem by modeling the Earth as a particle and approximating the Earth's orbit as circular. In order to apply the equation for the centripetal acceleration, we need to know the orbital speed of the Earth. The time period of the Earth is T = 1 year = 3.15×10^7 s, and the radius of the circular path (Earth-Sun distance) r = 1.5×10^{11} m.

The speed of motion of the Earth is $\frac{2\pi r}{T} = \frac{2\times 3.14\times 1.5\times 10^{11}\text{m}}{3.15\times 10^{7}\text{s}} = 3\times 10^4\ \text{m/s}$

$$a_c = \frac{v^2}{r} = \frac{(3\times 10^4\ \text{m/s})}{1.5\times 10^{11}\text{m}} = 6\times 10^{-3}\ \text{m/s}^2$$

Examples

Suppose a race car slows down uniformly from 60 m/s to 30 m/s in 4.5 s to avoid accident while still traversing a circular path of 400 m in radius. What are the car's (a) centripetal acceleration, (b) angular speed, (c) tangential acceleration, and (d) total acceleration when the speed is 40 m/s?

Solution

(a) The centripetal acceleration when moving at 40 m/s round a curve of radius 40 m

is $a_c = \frac{v^2}{r} = \frac{(40\text{m/s})^2}{400\text{m}} = 4\text{m/s}^2$

(b) Tangential acceleration at $a_t = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t} = \frac{30\text{m/s} - 60\text{m/s}}{4.5\text{s}}$ = -30/4.5 = -6.67 m/s²

(c) Total acceleration is $a = \sqrt{a_c^{\ 2} + a_t^{\ 2}} = 7.77\text{m/s}^2$

Centripetal acceleration and force

Consider a ball of mass m that is tied to a string of length r and is being whirled at constant speed in a horizontal circular path over a smooth horizontal table as shown in Figure 23 (a). Its weight is supported by a frictionless table. Why does the ball move in a circle?

According to Newton's first law, the ball tends to move in a straight line; however, the string prevents motion along a straight line by exerting on the ball a radial force F_c that makes it follow the circular path. This force is directed along the string toward the center of the circle, and it is known as centripetal (radial) force. If we apply Newton's second law along the radial direction, we find that the net force causing the centripetal acceleration, i.e., the centripetal force, is expressed as

$F_{net} = ma$

$F_c = ma_c$

Using $a_c = \frac{v^2}{r}$, $F_c = \frac{mv^2}{r}$

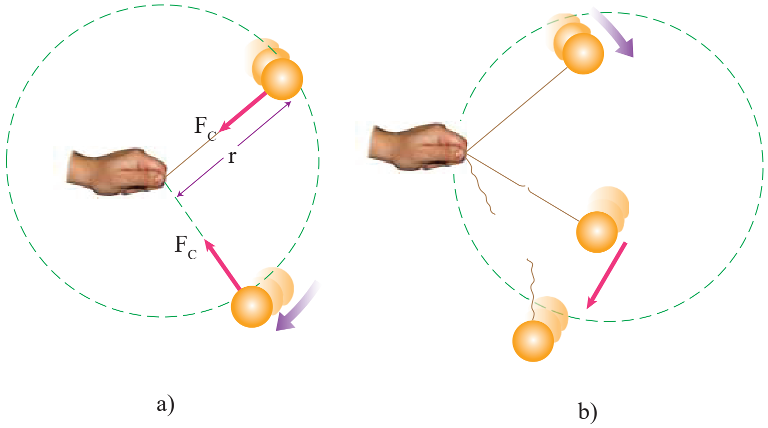


Figure 23. a) Overhead view of a ball moving in a circular path in a horizontal plane. b) When the string breaks, the ball moves in the direction tangent to the circle.

Think About It

A common misconception is that centripetal force acts radially outward from the center of a circle. This is not what happens. The change in the velocity of the object is inward, and therefore, so is the centripetal acceleration and force. What causes confusion is that when you spin an object in a circle at the end of a rope, you feel the force pulling outward on your hand (Figure 24). This outward pull is mistakenly thought to be the centripetal force.

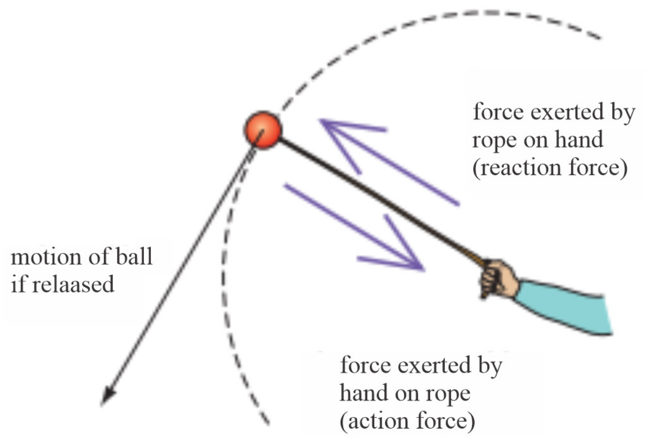


Figure 24.

Examples

A 1500 kg automobile (Figure 25) is traveling along a flat curved road of radius 100 m. What frictional force is required between tires and the road if the car is to take the curve safely at a speed of 20 m/s?

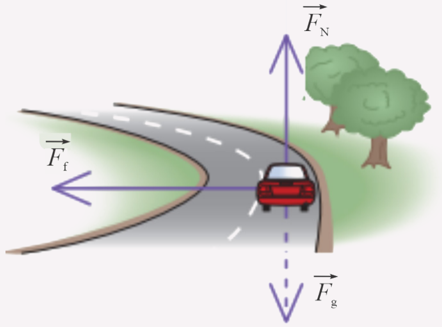


Figure 25. A car rounding a flat curve

Solution

As the car rounds the curve the frictional force between the tires and the road provides the centripetal force that keeps the car moving along the curve.

Frictional force = Centripetal force

$F_f = F_c$

$$F_c = \frac{mv^2}{r} = \frac{1500\text{kg} \times (20\,\text{m/s})^2}{100\text{m}} = 6000\text{N}$$

Examples

Determine the maximum speed at which a 1200 kg vehicle can round a flat curve of radius 50 m, if the static friction between the tires and the rod is 0.4 ?

Solution

The frictional force between the tires and the road provides the centripetal force.

$F_f = F_c$

Frictional force $F_f = \mu$ FN, μ = coefficient of friction, F_N is the normal force, for the case where an object rests on a horizontal surface F_N = weight

$F_N = mg$

$F_f = \mu\, mg$

$$\mu\, mg = \frac{mv^2}{r}$$

Solving for speed v

$$v = \sqrt{\mu rg} = \sqrt{0.4 \times 50m \times 10m/s^2} = 14.1m/s$$

Motion in a vertical circle

When studying the motion of a body in a vertical circle we have to consider the effect of gravity. Due to the influence of gravity, the magnitude of the velocity of the body and the tension in the string change continuously. The following example shows the nature of motion of a body in vertical circle.

Examples

A small sphere of mass m is attached to the end of a cord of length R and set into motion in a vertical circle about a fixed point O, as illustrated in Figure 26. Determine the tension in the cord at any instant when the speed of the sphere is v and the cord makes an angle θ with the vertical.

Unlike the situation in a uniform circular motion, the speed is not uniform in this example because, at most points along the path, a tangential component of acceleration arises from the gravitational force exerted on the sphere. From the free-body diagram in Figure 26a, we see that the only forces acting on the sphere are the gravitational force F_g = mg exerted by the Earth and the force T exerted by the cord. Now we resolve F_g into a tangential component mg sin θ and a radial component mg cosθ. Applying Newton's second law to the forces acting on the sphere in the tangential direction yields.

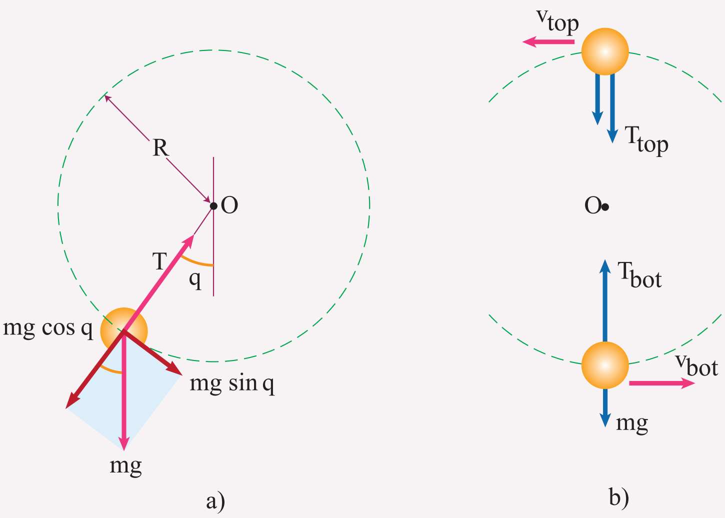


Figure 26. a) Forces acting on a sphere of mass m connected to a cord of length r and rotating in a vertical circle centered at O. b) Forces acting on the sphere at the top and bottom of the circle. The tension is a maximum at the bottom and a minimum at the top.

Net force along the tangent = mass x tangential acceleration

$F_t = ma$

$mg\sin\theta = ma_t$

$a_t = mg\sin\theta$ .... tangential acceleration Eq 1.22

Applying Newton's second law to the forces acting on the sphere in the radial direction and noting that both T and a_c are directed toward the center O, we obtain

Net force in the radial direction = Centripetal force

$F_r = F_c$

In Figure 1.26a, $T - mg\cos\theta = \frac{mv^2}{r}$

$$T = \frac{mv^2}{r} + mg\cos\theta \qquad \text{Eq 1.23}$$

In Figure 26 b, where the mass is at the top most point of the circle,

$F_r = F_c$

$$T_{top} + mg = \frac{mv^2}{r}$$

$$T_{top} = \frac{mv^2}{r} - mg \qquad \text{Eq 1.24}$$

In Figure 1.26b, where the mass is at the lowest point of the circle

$$T_{low} - mg = \frac{mv^2}{r}$$

$$T_{low} = \frac{mv^2}{r} + mg \qquad \text{Eq 1.25}$$

Examples

A mass tied to a string is whirled in a vertical circle. What minimum speed is required by the mass in order to move past the upper most point of the circle?

Solution

When the mass attains the minimum possible speed in order to be in circular motion at the top of the circle, the tension in the string becomes zero.

$$T_{top} = \frac{mv^2}{r} - mg$$

$$0 = \frac{mv^2}{r} - mg$$

Solving for v we obtain $v = \sqrt{rg}$

ACTIVITY 16

Based on the result obtained in Worked Example 1.20, discuss what happens if we set the ball in motion such that the speed at the top is less than $\sqrt{rg}$?

ACTIVITY 17

Pour water in a glass until the glass is half filled. Now hold the glass firmly and rotate it in a vertical circle. Did you see that the glass is inverted as it reaches the top of the circle? What minimum speed must you give to the glass so that the water does not spill at the instant the glass is inverted? What if the speed is greater than the minimum speed? Less than the minimum speed?

Examples

A passenger on a carnival Ferris wheel moves in a vertical circle of radius r with constant speed v. The seat remains upright during the motion. Find expressions for the force the seat exerts on the passenger when at the top of the circle and when at the bottom, Figure 27.

Solution

The variables we want to determine are the upward normal force NT the seat exerts on the passenger at the top of the circle and NB, the normal force at the bottom. For this we apply Newton's second law.

In Figure 27b, the acceleration is downward and the resultant of the weight mg and the normal force NT is equal to the centripetal force

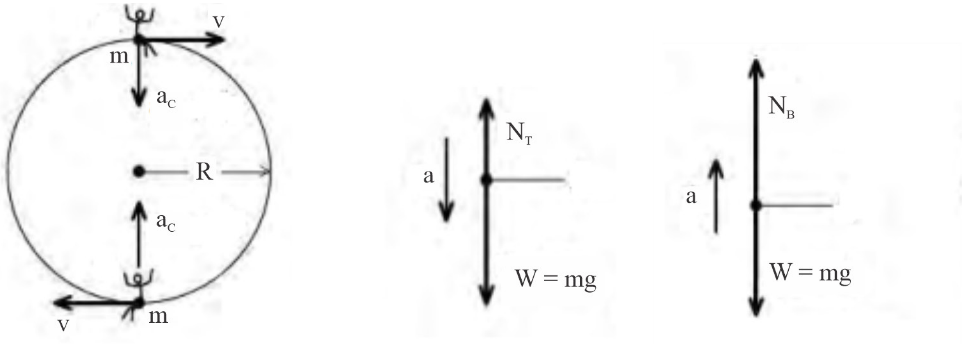
$$mg - NT = \frac{mv^2}{r}$$

$$NT = mg - \frac{mv^2}{r}$$

In Fig. 27c, the acceleration is upward and the resultant of the weight mg and the normal force NT is equal to the centripetal force.

$$NB - mg = \frac{mv^2}{r}$$

$$NB = \frac{mv^2}{r} + mg$$



(a) Sketch of two positions

(b) Free-body diagram for passenger at top

(b) Free-body diagram for passenger at bottom

Figure 27. Sketch the problem

ACTIVITY 18

The roller coaster often has a vertical loop somewhere along its track (Figure 28). Why is it that when you reach the top of the loop, where the car is inverted, you don't fall out?

Figure 28. Passengers on 'corkscrew' roller coaster experience a radial force toward the center of the circular track and a tangential force due to gravity

Forces Affecting an Object Moving in a Vertical Circle

In summary, an object moving in a vertical circle is affected by the following forces:

- The centripetal force is the net force on the object in any position.
- The centripetal force is determined by the object's mass, speed, and radius.

In the case of the roller coaster and bucket of water, their mass and radius of curvature are constant so only their speed affects the centripetal force.

- The force of gravity is one of the forces that may contribute to the centripetal force.
- The force of gravity remains constant regardless of the position of the object.

Exercises

1. Which of the following is impossible for a car moving in a circular path?
   (a) the car has tangential acceleration but no centripetal acceleration.
   (b) the car has centripetal acceleration but no tangential acceleration.
   (c) the car has both centripetal acceleration and tangential acceleration.
2. How would you explain the force that pushes a rider toward the side of a car as the car rounds a corner?
   (a) A car makes a turn without skidding.
   (b) A ball is tied to the end of a rope and spun in a horizontal circle.
   (c) The Moon moves in a circular orbit around Earth
3. A 200-g object is tied to the end of a cord and whirled in a horizontal circle of radius 1.20 m at a constant 3.0 rev/s. Assume that the cord is horizontal—that is, that gravity can be neglected. Determine (a) the centripetal acceleration of the object and (b) the tension in the cord.
4. A certain light truck can go around a flat curve having a radius of 150 m with a maximum speed of 32 m/s. With what maximum speed can it go around a curve having a radius of 75 m?
5. A 600 N roller coaster car full of people goes around a vertical loop that has a radius of 25 m. What minimum speed must the roller coaster car have at the top of the vertical loop to stay on the track?

1.4 Rotational Motion

The motion of a body about a fixed axis is termed as Rotational motion. All the particles constituting the body undergo circular motion about a common axis. Rotation of Earth about its own axis, motion of wheel, gears, motors, motion of the blades of a helicopter, a door swiveling on its hinges as we open or close it are some of common examples of rotational motion.

Key terms

- Rotational motion: the motion of a body about a fixed axis where all the particles constituting the body undergoes circular motion about a common axis.
- Rigid body: a body with a perfectly defined and unchanging shape.

When an extended object such as a wheel rotates about its axis, the motion cannot be analyzed by treating the object as a particle because at any given time different parts of the object have different linear velocities and linear accelerations. We can, however, analyze the motion by considering an extended object to be composed of a collection of particles, each of which has its own linear velocity and linear acceleration.

A Ferris wheel

In order to simplify the analysis of rotational motion of an object, the object is assumed to be a rigid body. A rigid body is a body that can rotate with all its parts locked together and without any change in its shape. No matter how the body moves, the distance between two points in a rigid body remains constant (non deformable).

All real objects are deformable to some extent; however, our rigid-object model is useful in many situations in which deformation is negligible.

Angular Position, Velocity, and Acceleration

Figure 29 shows a rigid body in rotation about a fixed axis, called the axis of rotation or the rotation axis. In pure rotation (angular motion), every point in the body moves in a circle whose center lies on the axis of rotation, and every point moves through the same angle during a particular time interval.

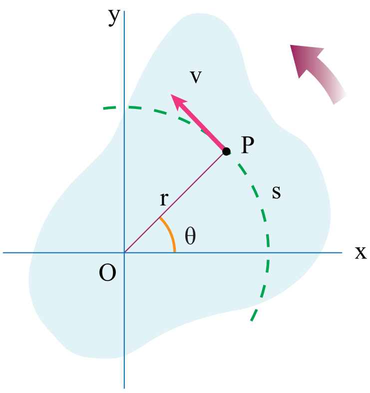


Figure 29. A rigid body in pure rotation about the z axis of a coordinate system.

In the study of linear motion, the important concepts are displacement Δx, velocity v, and acceleration a. Each of these concepts has its analog in rotational motion: angular displacement $\Delta\theta$, angular velocity ω, and angular acceleration α.

The angle displacement θ in radians is defined as

$\theta = \frac{s}{r}$, where S is the arc length Eq 1.26

If the rigid object spins rapidly, the angular displacement changes in a short time interval. If it rotates slowly, this displacement changes in a longer time interval. These different rotation rates can be quantified by introducing a quantity known as Angular velocity. We define the average angular velocity ω_{av} (Greek omega) as the ratio of the angular displacement $\Delta\theta$ of a rigid object to the time interval Δt during which the displacement changes.

$$\omega_{av} = \frac{\Delta\theta}{\Delta t} = \frac{\theta_f - \theta_i}{t_f - t_i} \qquad \text{Eq 1.27}$$

In analogy to linear speed, the instantaneous angular speed is defined as the limit of the ratio $\frac{\Delta\theta}{\Delta t}$ as Δt approaches zero:

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} \qquad \text{Eq 1.28}$$

Angular speed has units of radians per second (rad/s), which can be written as per second (1/s) because radians are not dimensional.

If the instantaneous angular speed of an object changes from ω_i to ω_f in the time interval Δt, the object is said to have angular acceleration. The average angular acceleration (Greek alpha) of a rotating rigid object is defined as the ratio of the change in the angular speed to the time interval Δt during which the change in the angular speed occurs:

$$\alpha_{av} \equiv \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t} \qquad \text{Eq 1.29}$$

KEY TERMS

- Angular displacement: the angle through which a point revolves around a center or specified axis in a specified sense.
- Instantaneous velocity: velocity at a particular instant of time.
- Instantaneous acceleration: acceleration at a particular instant of time.

In analogy to linear acceleration, the instantaneous angular acceleration is defined as the limit of the ratio, $\frac{\Delta\theta}{\Delta t}$ as Δt approaches zero:

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t} \qquad \text{Eq 1.30}$$

Angular acceleration has units of radians per second squared (rad/s^2).

When a rigid object is rotating about a fixed axis, every particle on the object rotates through the same angle in a given time interval and has the same angular speed and the same angular acceleration.

Rotational Kinematics: Rotational Motion with Constant Angular Acceleration

The simplest form of accelerated motion to analyze is motion under constant linear acceleration. Likewise, for rotational motion about a fixed axis, the simplest accelerated motion to analyze is motion under constant angular acceleration. Therefore, we next develop kinematic relationships for this type of motion.

From Eq 1.29, we get $\alpha = \Delta\omega/\Delta t$, the angular velocity of the body at time t is

$$\omega_f = \omega_i + \alpha t \qquad \text{Eq 1.31}$$

where ω_i is the angular speed of the rigid object at time t = 0

If the angular position of the rigid object at time t = 0 is θ_i, the angular position of the object at any later time t is

$$\theta_f = \theta_i + \omega_i t + ½\, \alpha t^2 \qquad \text{Eq 1.32}$$

The angular speed ωf of the rigid object for any value of its angular position θ_f is

$$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i) \qquad \text{Eq 1.33}$$

The angular displacement can also be expressed as

$$\theta_f = \theta_i + ½\,(\omega_f + \omega_o)t \qquad \text{Eq 1.34}$$

Notice that these kinematic expressions for rotational motion under constant angular acceleration are of the same mathematical form as those for linear motion under constant linear acceleration. They can be generated from the equations for linear motion by making the substitutions $x \to \theta$, $v \to \omega$ and $a \to \alpha$ (Table 3) compares the kinematic equations for rotational and linear motion.

Table 3

Kinematic Equations for Rotational and Linear Motion Under Constant Acceleration	
Rotational Motion About Fixed Axis	**Linear Motion**
$\omega_f = \omega_i + \alpha t$	$\upsilon_f = \upsilon_i + at$
$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$	$x_f = x_i + \upsilon_i t + \frac{1}{2}at^2$
$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$	$\upsilon_f^2 = \upsilon_i^2 + 2a(x_f - x_i)$
$\theta_f = \theta_i + \frac{1}{2}(\omega_i + \omega_f)t$	$x_f = x_i + \frac{1}{2}(\upsilon_i + \upsilon_f)t$

Examples

A wheel rotates with a constant angular acceleration of 2.5 rad/s^2.

(a) If the angular speed of the wheel is 6 rad/s at t = 0, through what angular displacement does the wheel rotate in 2 s?

(b) Through how many revolutions has the wheel turned during this time interval?

(c) What is the angular speed of the wheel at t = 2 s?

Solution

(a) Rearranging the equation $\theta_f = \theta_i + \omega_i t + ½\, \alpha t^2$

$$\Delta\theta = \theta_f - \theta_i = \omega_i t + ½\, \alpha t^2$$

$$= (6 \text{ rad/s})(2 \text{ s}) + ½\,(2.5 \text{ rad/s}^2)(2\text{s})^2 = 17 \text{ rad}$$

(b) To convert radians into revolutions and degrees, we use the following conversion factors: 1 rev = 2π rad = 360^o

$$\Delta\theta = 17 \text{ rad} = 2.7 \text{ rev}$$

(c) The angular speed of the wheel at *t* = 2 s, can be calculated using

$$\omega_f = \omega_o + \alpha t = 6 \text{ rad/s} + (2.5 \text{ rad/s}^2)(2\text{s}) = 11 \text{ rad/s}$$

Examples

A centrifuge in a medical laboratory rotates at an angular speed of 3 600 rev/min. When switched off, it rotates 50 times before coming to rest. Find the constant angular acceleration of the centrifuge.

Solution

Given initial angular velocity $\omega_a = 3600 \text{ rev/min} = 3600 \left(\frac{2\pi}{60}\right) \text{rad/s} = 3.77 \text{ x } 10^2 \text{ rad/s}$,

θ = 50 rev = 50(2π) rad = 314 rad, final angular velocity = 0, we find the angular acceleration using

$$\omega_f^2 = \omega_i^2 + 2\alpha\theta$$

Solving for α, we get $\alpha = \dfrac{w_f^{\ 2} - w_i^{\ 2}}{2\theta}$

Substituting the given values we obtain α = -226 rad/s^2. The minus sign indicates deceleration.

Activity 19

Use the equations of translational analogs of those used in rotational motion to solve the following problem. Suppose a particle moves along a straight line with a constant acceleration of 2 m/s^2. If the velocity of the particle is 6 m/s at t= 0, through what displacement does the particle move in 2 s? What is the velocity of the particle at t =2 s?

Angular and Linear Quantities

In this section we derive some useful relationships between the angular speed and acceleration of a rotating rigid object and the linear speed and acceleration of a point in the object. To do so, When a rigid object rotates about a fixed axis, as in Figure 30, every particle of the object moves in a circle whose center is the axis of rotation.

Because point P in Figure 30 moves in a circle, the linear velocity vector v is always tangent to the circular path and hence is called tangential velocity.

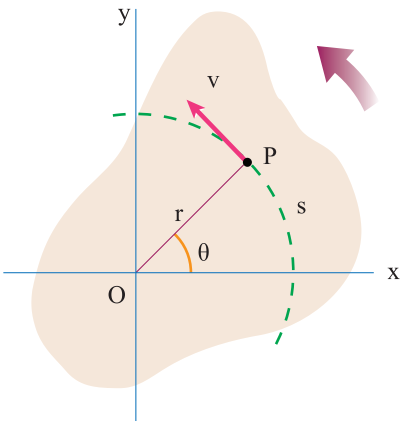


Figure 30. As a rigid object rotates about a fixed axis through O*, the point* P *experiences a tangential component of linear acceleration* a_t *and a radial component of linear acceleration* a_r .

The magnitude of the tangential velocity of the point P is by definition the tangential speed v = Δs/Δt, where s is the distance traveled by this point measured along the circular path.

Recall that s = rθ

$$v = \frac{v\Delta(r\theta)}{\Delta t}$$

Since r is constant, we write $v = \frac{\Delta(r\theta)}{\Delta t} = \frac{r\Delta\theta}{\Delta t}$

Because $\frac{\Delta\theta}{\Delta t} = \omega$, we see that

$$v = r\omega$$

That is, the tangential speed of a point on a rotating rigid object equals the perpendicular distance of that point from the axis of rotation multiplied by the angular speed. Therefore, although every point on the rigid object has the same angular speed, not every point has the same tangential speed because r is not the same for all points on the object.

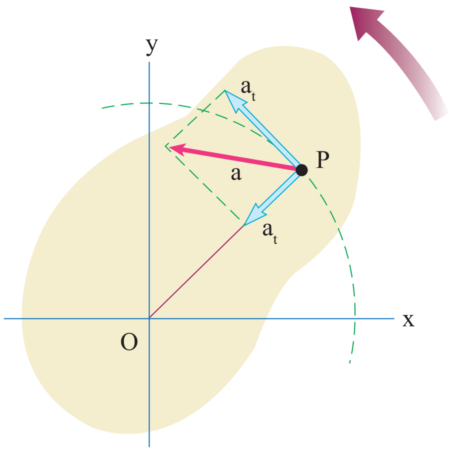


Figure 31. As a rigid object rotates about the fixed axis through O.

We can also relate the angular acceleration of the rotating rigid object to the tangential acceleration of the point P of Figure 31 using the relations

$$a_t = \frac{\Delta\theta}{\Delta t}$$

Using $v = r\omega$, we write $a = \dfrac{\Delta(r\omega)}{\Delta t}$

$$a_t = r\frac{\Delta\omega}{\Delta t}$$

Because $\dfrac{\Delta\omega}{\Delta t} = \alpha$, we see that

$a_t = r\alpha$ Eq 1.36

A body in circular motion undergoes a radial acceleration a_c of magnitude v^2/r directed toward the center of rotation. Because $v = \omega r$ for a point P on a rotating object, we can express the centripetal acceleration at that point in terms of angular speed as

$$a_c = \frac{v^2}{r} = \frac{(\omega r)^2}{r}$$

$a_c = \omega^2 r$ Eq 1.37

The total linear acceleration vector at the point is $\mathbf{a} = \mathbf{a}_t + \mathbf{a}_c$.

The linear acceleration has radial components and tangential component accelerations and the magnitude of **a** at the point P on the rotating rigid object is

$a = \sqrt{a_t^2 + a_c^2}$ Eq 1.38

Activity 20

Show that the magnitude of linear acceleration at the point P on the rotating rigid object is

$a = \sqrt{\alpha^2 + \omega^4}$

Examples

A racing car travels on a circular track of radius 250 m. If the car moves with a constant linear speed of 45 m/s, find (a) its angular speed and (b) the magnitude and direction of its acceleration.

Solution

Given the tangential speed v = 45 m/s and radius r = 250 m,

(a) The angular speed $\omega = \dfrac{v}{r} = \dfrac{45\text{m}}{250\text{m}} = 0.18\,\text{rad/s}$

(b) The acceleration is centripetal acceleration $a_c = \frac{45\,m/s}{250m} = 1.8\,m/s^2$ toward the center of the track. We can also use $a_c = \omega^2 r$ to get the same result.

Examples

A car accelerates uniformly from rest and reaches a speed of 20 m/s in 10 s. If the diameter of a tire is 50 cm, find (a) the number of revolutions the tire makes during this motion, assuming that no slipping occurs. (b) What is the final angular speed of a tire in revolutions per second?

Solution

Given initial velocity $v_i = 0$, final velocity $v_f = 20$ m/s, time taken t = 10 s and diameter D = 50 cm = 0.5 m, we find

(a) the number of revolution as $\theta = \frac{s}{r}$.

But distance $s = v_{av}\, t = \left(\frac{v_f + v_i}{2}\right) t = \left(\frac{20\,m/s + 0\,m/s}{2}\right) \times 10s = 100m$

Therefore, $\theta = \frac{s}{r} = \frac{100m}{0.5} = 200$ rad = 200 rev = 31.8 rev

(b) Using v = rω, the final angular velocity in terms of the final tangential velocity can be written as $\omega_f = \frac{v_f}{r} = \frac{20\,m/s}{0.5m}$ 40 rad/s = 40 $\left(\frac{1}{2\pi}\right)$ rev/s = 6.37 rev/s

Exercises

1. A flywheel rotates with constant angular velocity. Does a point on its rim have a tangential acceleration? A radial acceleration? Are these accelerations constant in magnitude? In direction? In each case give your reasoning.
2. A disc rotates counterclockwise in the xy plane. What is the direction of ω? What is the direction of α if the magnitude of the angular velocity is decreasing in time? Increasing in time?
3. A disc is initially rotating at 27.5 rad/s slows down to a stop at a constant angular acceleration of – 10 rad/s^2. A line PQ on the disc's (Figure 32) surface lies along the +x axis at t = 0. (a) What is the disc's angular velocity at t = 0.3 s? (b) What angle does the line PQ make with the +x axis at this time?

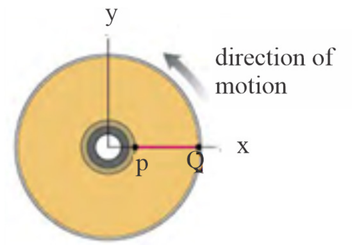
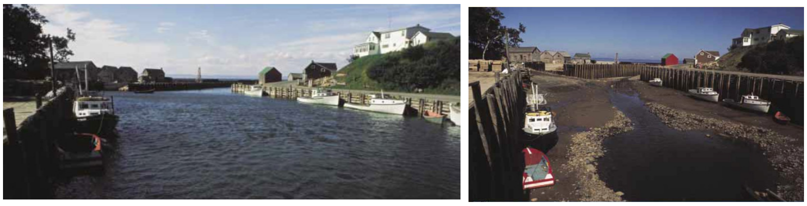


Figure 32. A disc rotating about an axis perpendicular through its center

4. A wheel starts from rest and rotates with constant angular acceleration to reach an angular speed of 12 rad/s in 3 s. Find (a) the magnitude of the angular acceleration of the wheel and (b) the angle in radians through which it rotates in this time.

1.5 Simple Harmonic or Oscillatory Motion

Many kinds of motion repeat themselves over and over: the sound vibrations produced by a clarinet or an organ pipe, a plucked guitar string, molecules vibrating within a solid, water waves, the back-and-forth motion of the pistons in a car engine, are just a few Figure 33. This kind of motion, called periodic motion or oscillation, is the subject of this section.

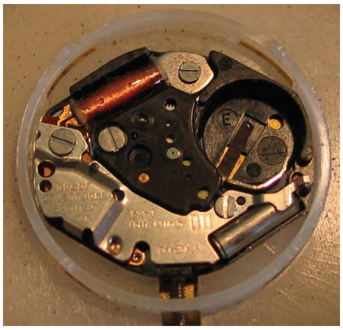


Oscillatory Motion

Oscillation is a repetitive or periodic motion that regularly repeats – the object returns to a given position after a fixed time interval. The molecules in solids oscillate about their equilibrium position, electromagnetic waves such as light waves, radar and radio waves, are characterized by oscillating electric and magnetic fields.

Familiar examples of oscillation are the motion of a swinging pendulum and the motion of a spring-mass system, Figure 34.

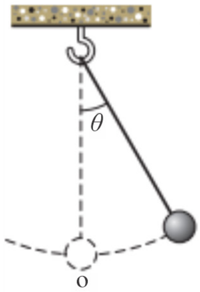
A body that undergoes periodic motion always has a stable equilibrium position, point o of Figure 34. When it is moved away from this position and released, a force or torque comes into play to pull it back toward equilibrium. But by the time it gets there, it has picked up some kinetic energy, so it overshoots, stopping somewhere on the other side, and is again pulled back toward equilibrium.

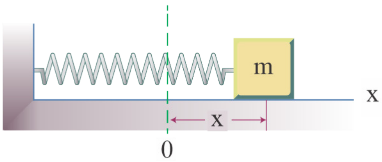


a) b)

Figure 33. a) The drive wheel mechanism of an old locomotive. b) The quartz crystal in a wristwatch is enclosed in the small metal cylinder (lower right).

A special kind of periodic motion occurs in mechanical systems when the force acting on an object is proportional to the position or displacement of the object relative to the equilibrium position. If the force is always directed toward the equilibrium position, the motion is called Simple Harmonic Motion (SHM).





a) Pendulum b) mass-spring system

Figure 34.

Did you know?

Christian Huygens (1629 – 1695), a Dutch mathematician, astronomer and physicist invented the first accurate clock in 1657. It used a swinging pendulum and was a revolution in clock making (Figure 35). This technology reduced the loss of time by clocks from about 15 minutes to about 15 seconds per day.

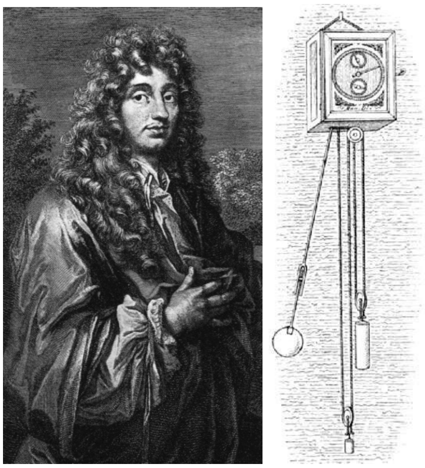


Figure 35. Christian Huygens (1629 – 1695) and his pendulum clock

Motion of a spring-mass system

As a model for a simple harmonic motion, let us consider a block of mass m attached to the end of a spring, with the block free to move on a horizontal, frictionless surface (Figure 36).

When the spring is neither stretched nor compressed, the block is at the position called equilibrium position of the system, which is labeled as x = 0. The system will oscillate back and forth if disturbed from its equilibrium position.

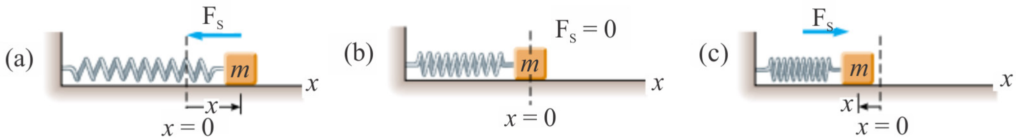


Figure 36. A block attached to a spring moving on a frictionless surface.

How does a pendulum perform oscillation?

If the bob of a pendulum is pulled to one side and released, it accelerates back toward its equilibrium position, Figure 37a. When the pendulum bob gets back to the equilibrium position as in Figure 39b, it is moving relatively fast and although there is no resultant force at the equilibrium position, its inertia keeps it moving. In Figure 37c the pendulum bob keeps moving slowing down until it is at the same height as it started and then it accelerates back to the equilibrium position. In Figure 37d the pendulum bob passes through the equilibrium position again, going back the other way. Then the pendulum bob gets back to where it started completing one cycle, Figure 37e. The process goes on again and again.

We use common terms such as Amplitude, Period and Frequency to describe simple harmonic motion. Amplitude is the maximum displacement from the equilibrium position, Period of oscillation is the time taken to make one complete oscillation, and Frequency is the number of oscillations in a unit time.

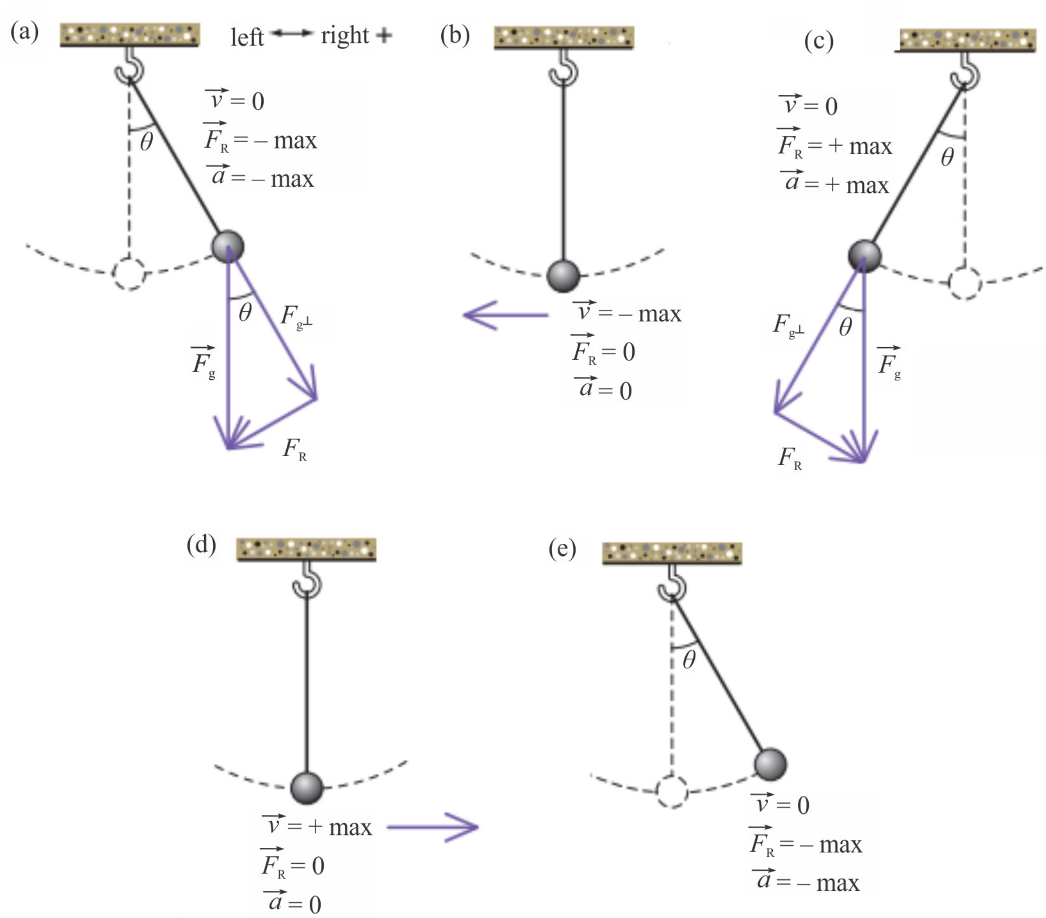


Figure 37. Oscillation of a pendulum

The period T, and frequency f of the oscillation are related as

$$T = \frac{1}{f} \qquad \text{Eq 1.39}$$

Angular frequency is defined as the rate of change of angular displacement, $\omega = \dfrac{\Delta\theta}{\Delta t}$

For one complete cycle, the angular displacement $\theta = 2\pi$ rad and the time taken for one complete oscillation is $\Delta t = T$.

$$\omega = \frac{2\pi}{T} \qquad \text{Eq 1.40}$$

Using $T = \frac{1}{f}$, we have $\omega = 2\pi f$

We identify two kinds of frequency for a simple harmonic oscillator—f, called simply the frequency, is measured in hertz, and ω, the angular frequency, is measured in radians per second.

Did you know?

Bio. Application Wing Frequency. The ruby-throated hummingbird normally flaps its wings at about 50 Hz, producing the sound that gives hummingbirds their name. Insects can flap their wings at even faster rates, from 330 Hz for a house fly and 600 Hz for a mosquito to amazing 1040 Hz for the tiny biting midge.

Key terms

- Oscillation: the back-and-forth motion of a body about equilibrium position in equal time interval.
- Simple Harmonic Motion: an oscillation in which the acceleration is directly proportional to the displacement and directed toward the equilibrium position.
- Amplitude: Maximum displacement from the equilibrium position.
- Period: time of one complete oscillation
- Frequency: number of oscillations per unit time.

The motion of the spring- mass system shown in Figure 1.36 can be discussed qualitatively by first recalling that when the block is displaced to a position x, the spring exerts on the block a force that is proportional to the position given by Hooke's law.

$$F_s = -kx$$

The force F_s is known as a restoring force and the minus sign is used because it is always directed toward the equilibrium position and therefore opposite to the displacement from the equilibrium. When the block is displaced to the right of the equilibrium position, the restoring force is directed to the left tending to get it back to the equilibrium position. When the block is displaced to the left of the equilibrium position, the position is negative and the restoring force is directed to the right.

The acceleration of the mass can be determined by applying Newton's second law of motion.

$$F_{net} = ma$$

The net force is the restoring force and we write

$$a = -\frac{k}{m}x \qquad \text{Eq 1.41}$$

That is, the acceleration is directly proportional to the displacement of the mass from the equilibrium position, and it is directed along the restoring force, i.e., toward the equilibrium position.

A simple harmonic motion is a motion in which the acceleration of the mass is directly proportional to the displacement and directed toward the equilibrium position.

$$a \sim -x$$

The mass attains its maximum acceleration when it is at the maximum displacement position where x = A, and its acceleration is minimum or zero at the equilibrium position, where x = 0.

$$a_{max} = \frac{-k}{m}A \text{ and } a_{min} = \frac{-k}{m}(0) = 0$$

The above discussion is also valid for an object hanging from a vertical spring Figure 38, as long as we recognize that the weight of the object will stretch the spring to a new equilibrium position x = 0.

Figure 38 a shows a spring without a mass attached, anchored to a ceiling. Assume that the spring itself is massless, so it will not experience any displacement. When a mass is attached, the spring is pulled down and deforms as predicted by Hooke's law. The mass will come to rest when the downward force of gravity is equal to the upward pull (tension) of the spring.

In Figure 38 b, the net force (or restoring force) acting on the mass is zero. It is the result of the upward tension exerted by the spring balancing the downward force of gravity. This position is considered the equilibrium position and the displacement is zero.

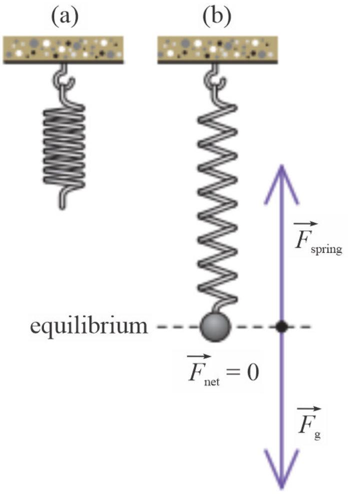


Figure 38. An object suspended from a vertical spring is set at its new equilibrium position

If the mass is lifted to the position shown in Figure 39a and released, it will begin oscillating with simple harmonic motion. Its amplitude will equal its initial displacement. Regardless of the position of the mass, the force of gravity remains constant but the tension of the spring varies. In the position shown in Figures 39b and (d), the net (restoring) force is zero. This is where the spring's tension is equal and opposite to the force of gravity. In the position shown in Figure 39c, the displacement of the spring is equal to the amplitude, and the tension exerted by the spring is at its maximum. The mass experiences the greatest restoring force, which acts upward.

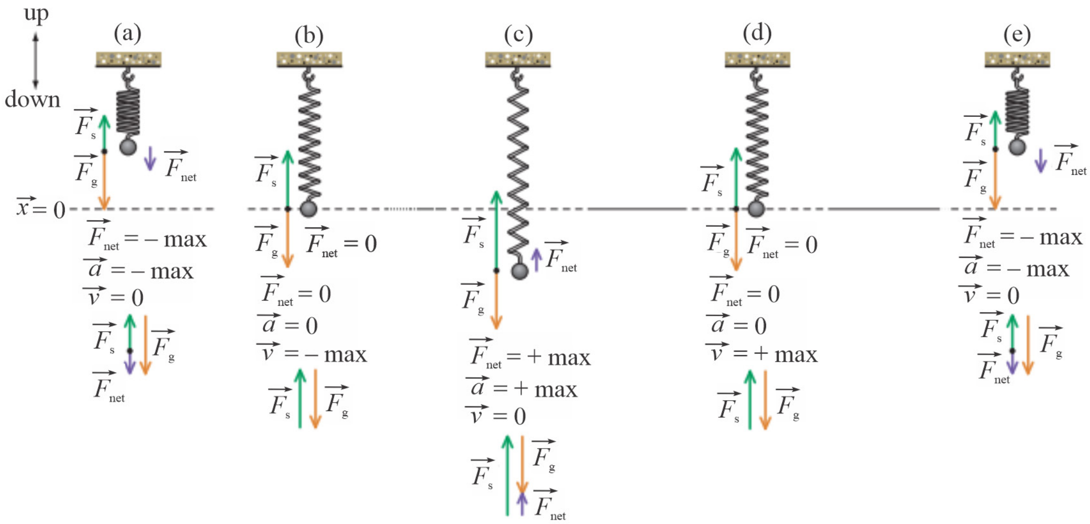


Figure 39. Oscillation of a mass-spring system along the vertical

Did you know?

Robert Hooke was an English polymath active as a scientist and architect, who, using a microscope, was the first to visualize a micro-organism.

Robert Hooke (1635 – 1703)

Experiment: HOOKE'S LAW

Objectives

The main objective of this experiment is to show Hooke's Law of spring.

Equipment

Tripod Base, Barrel Base, Support Rod , Right Angle Clamp, Cursor, 1 pair, Weight Holder for Slotted Weights, Slotted Weight, 10g, black, Slotted Weight, 10g, silver bronze, Slotted Weight, 50g, black, Slotted Weight, 50g, silver bronze, Helical Spring , Meter Scale

Figure 40. Experimental setup

Theory

Measuring the stretching produced by different loads, added to the spring, tests the elasticity of a spring. When a spring is stretched by an applied force, a restoring force is produced. Due to the restoring force, simple harmonic motion is caused in a straight line in which the acceleration and the restoring force are directly proportional to the displacement of the vibrating load from the equilibrium position. The relation between the force F and displacement x is $F = -kx$. The force is opposite in direction to the displacement. The constant k is known as the force constant of the spring. This is the force, expressed in Newton, which will produce an elongation of one meter in the spring.

Experimental Procedure (Method) and Calculations

(i) Set up the spring as Figure 41.
(ii) Measure the length of the spring before the masses are added
(iii) Hang m_1 on spring and record the elongation.

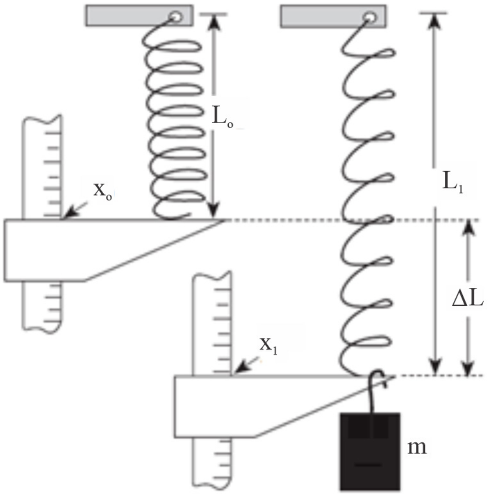


Figure 41. The spring stretches when mass is added

(iv) Repeat the step iii for different masses.(m_2, m_3, m_4 etc.)
(v) Calculate the applied force for different masses.
(vi) Calculate k using and find the average value k_{av} for the spring.

Mass(kg)	Length of the spring(m)	Elongation (m)	Force (m x g)	k = (N / m)	Average value k_{av}

Results and Discussion: Discuss the results

List down the possible source of error in this experiment.

Activity 21

1. Determine whether or not the following quantities can be in the same direction for a simple harmonic oscillator:
   (a) position and velocity,
   (b) velocity and acceleration,

(c) position and acceleration.

2. Is a simple harmonic motion a uniformly accelerated motion? Discuss your answer briefly.
3. Is it possible for a body to have acceleration while it is at rest? If your answer is yes, give examples of such cases?

Group discussion

4. Is a bouncing ball an example of simple harmonic motion? Is the daily movement of a student from home to school and back simple harmonic motion? Why or why not?

Figure 42 shows plot of the Restoring force versus displacement and Acceleration versus Dis-placement of a simple harmonic motion.

KEY TERMS

- Restoring force: the force which acts to bring the mass back to its equilibrium position
- Inertia: property of a mass to retain its state of motion or rest.

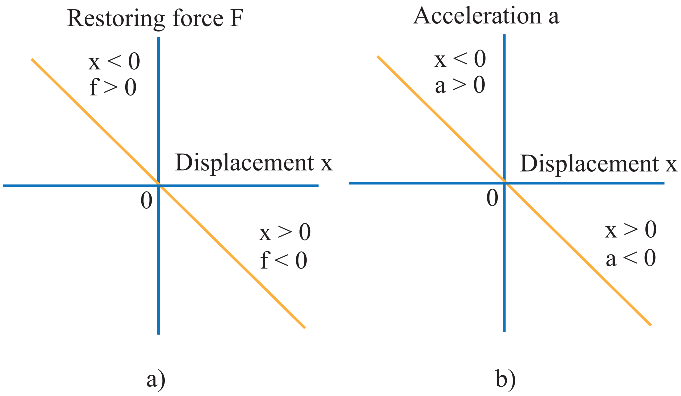


Figure 42 a) The graph of restoring force versus displacement x is a straight line. (b) The graph of acceleration versus displacement x.

Examples

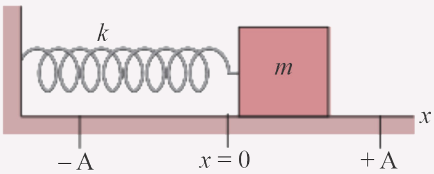


Figure 43. A spring-mass system oscillating between x = -A and x = +A

An object undergoing simple harmonic motion takes 0.25 s to travel from one point of zero velocity to the next such point. The distance between those points is 36 cm. Calculate the (a) period, (b) frequency, and (c) amplitude of the motion.

Solution

The time taken from a zero velocity point (x = -A) to another zero velocity point (x = -A) is one half of the period, and the distance between these points is twice the amplitude, Figure 1.43.

(a) T = 2(0.25 s) = 0.5 s

(b) Frequency f = = = 2 Hz

(c) Amplitude A = (36 cm) = 18 cm

Examples

A mass of 2 kg is attached to a spring with a spring constant of 40 N/m on a horizontal frictionless surface. Determine the restoring force acting on the mass when the spring is compressed to a displacement of 15 cm.

Solution

Restoring force $F_s = -kx$.

With k = 40 N/m and x = -15 cm = -0.15 m, we have Fs = - (40 N/m)(-0.15 m) = 6 N. Assuming displacement to the left of the equilibrium position negative, the restoring force is to the right hence positive.

Concepts of Oscillation Rates in Simple Harmonic Motion

The most essential physical concepts in simple harmonic motion are those pertaining to the rate of oscillation. Those concepts are the period, frequency, and angular frequency. Understanding simple harmonic oscillations and measures of oscillation rates can be facilitated by a comparison of simple harmonic motion with uniform circular motion.

Comparing Simple Harmonic Motion with uniform circular Motion

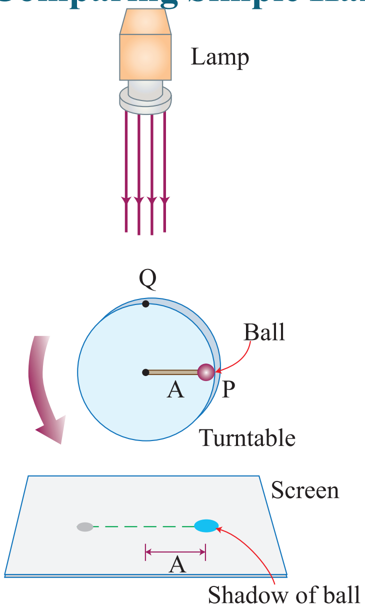


Figure 44. Uniform circular motion.

We can better understand and visualize many aspects of simple harmonic motion along a straight line by looking at its relationship to uniform circular motion. Figure 44 is an overhead view of an experimental arrangement that shows this relationship. A ball is attached to the rim of a turntable of radius A, which is illuminated from the side by a lamp. The ball casts a shadow on a screen. We find that as the turntable rotates with constant angular speed, the shadow of the ball moves back and forth in simple harmonic motion.

Consider a particle located at point P on the circumference of a circle of radius A, as in Figure 45a, with the line OP making an angle ϕ (also known as phase constant) with the x axis at t = 0. We call this circle a reference circle for comparing simple harmonic motion with uniform circular motion, and we take the position of P at t = 0 as our reference position. If the particle moves along the circle with constant angular speed ω until OP makes an angle θ with the x axis, as in Figure 45b, then at some time t > 0, the angle between OP and the x axis is $\theta = \omega t + \phi$. As the particle moves along the circle, the projection of P on the x axis, labeled point Q, moves back and forth along the x axis between the limits $x = \pm A$.

Note that points P and Q always have the same x coordinate. We conclude that simple harmonic motion along a straight line can be represented by the projection of uniform circular motion along a diameter of a reference circle.

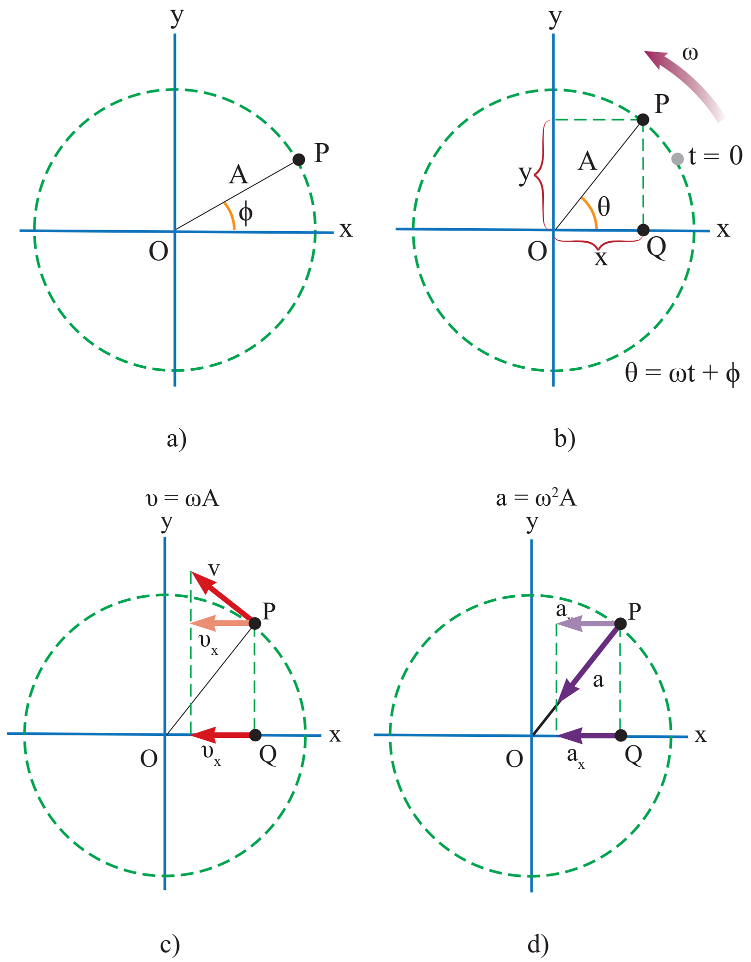


Figure 45. (a) A reference circle showing the position of P at t = 0. (b) The x coordinates of points P and Q are equal and vary in time according to the expression $x = A\cos(\omega t + \varphi)$. (c) The x component of the velocity of P equals the velocity of Q. (d) The x component of the acceleration of P equals the acceleration of Q

From the right triangle OPQ, we see that this x coordinate is

$$x(t) = A\cos(\omega t + \phi) \qquad \text{Eq. 1.42}$$

The linear speed and angular speed in circular motion are related as $v = \omega A$. As the particle moves with constant speed v in counterclockwise direction as in Figure 5.15c, the x component of the velocity will be $\omega A\sin(\omega t + \phi)$.

$$v = -\omega A\sin(\omega t + \phi) \qquad \text{Eq. 1.43}$$

The acceleration of P on the reference circle is directed radially inward toward O and has a magnitude $\frac{v^2}{A} = \omega^2 A$. From the geometry in Figure 5.15d, we see that the x component of this acceleration is $-\omega^2 A\cos(\omega t + \phi)$. This value is also the acceleration of the projected point Q along the x axis.

$$a = -\omega^2 A\cos(\omega t + \phi) \qquad \text{Eq. 1.44}$$

Think About It

Given the expression for the displacement x of a body, the equation of velocity can be obtained by using differential calculus as $v = \frac{dx}{dt}$. Acceleration is the derivative of velocity $a = \frac{dv}{dt}$

Comparing Eq. 1.42 and Eq.1.44 we see that

$$a = -\omega^2 x \qquad \text{Eq. 1.45}$$

This expression tells us that the acceleration is proportional to the displacement and directed toward the equilibrium position, which is the typical feature of a simple harmonic motion.

We can make a similar argument by noting from Figure 45b that the projection of P along the y axis also exhibits simple harmonic motion. The displacement, velocity and acceleration of the simple harmonic motion will be

$$x(t) = A\sin(\omega t + \phi)$$

$$v = \omega A\cos(\omega t + \phi)$$

$$a = -\omega^2 A\sin(\omega t + \phi)$$

Once again we see that $a = -\omega^2 x$

From $a = -\frac{k}{m}x$ and $a = -\omega^2 x$, we see that

$$\omega = \sqrt{\frac{k}{m}} \qquad \text{Eq. 1.46}$$

Activity 22

Using the expressions $\omega = \frac{2\pi}{T}$ and $\omega = \sqrt{\frac{k}{m}}$, show that the time period of A simple harmonic oscillation is $T = 2\pi\sqrt{\frac{m}{k}}$ and the frequency of oscillation is $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

The equations are summarized in Table 1.4 below.

Table 4 Equations of SHM

	Where at $t = 0, x = A$	Where at $t = 0, x = 0$
Displacement	$x = A\cos(\omega t + \phi)$	$x = A\sin(\omega t + \phi)$
Velocity	$v = -\omega A\sin(\omega t + \phi)$	$v = \omega A\cos(\omega t + \phi)$
Acceleration	$a = -\omega^2 A\cos(\omega t + \phi)$	$a = -\omega^2 A\sin(\omega t + \phi)$

Think about it

The acceleration of a particle in simple harmonic motion is not constant and we cannot apply the equations of uniformly accelerated motion for the displacement, velocity and acceleration of the motion.

Did you know?

Pendulum Motion is not Simple Harmonic. Remember that the pendulum does not exhibit true simple harmonic motion for any angle. If the angle is less than about 15°, the motion can be modeled as approximately simple harmonic.

Activity 23

Show that for a pendulum set in SHM the period and frequency are given by $T = 2\pi\sqrt{\frac{\ell}{g}}$ and $f = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}}$ respectively.

Did you know?

Use of Pendulum in Prospecting

Geologists often make use of the simple pendulum and equation $T = 2\pi\sqrt{\frac{\ell}{g}}$

whenprospecting for oil or minerals. Deposits beneath the Earth's surface can produce irregularities in the free-fall acceleration over the region being studied. A specially designed pendulum of known length is used to measure the period, which in turn is used to calculate g. Although such a measurement in itself is inconclusive, it's an important tool for geological surveys.

Practical Experiment

Acceleration Due To Gravity Using Simple Pendulum

Aim

To measure the acceleration due to gravity using a simple pendulum

Apparatus Required

Retort stand, pendulum bob, thread, meter scale, stop watch.

Theory

The period of simple pendulum is depends on the length of the pendulum the value of acceleration due to gravity at that location as $T = 2\pi\sqrt{\frac{\ell}{g}}$ from which the value of acceleration due to gravity is

$$g = 4\pi^2\left(\frac{L}{T^2}\right)$$

where

$T \rightarrow$ Time period of simple pendulum (second)

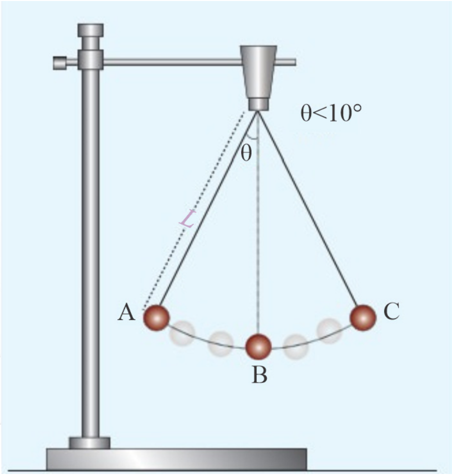
$g \rightarrow$ Acceleration due to gravity (meter/sec^2)

$L \rightarrow$ Length of the pendulum (meter)

Diagram

Procedure

- Attach a small brass bob to the thread
- Fix this thread on to the stand



- Measure the length of the pendulum from top to the middle of the bob of the pendulum. Record the length of the pendulum in the table below.
- Note the time (t) for 10 oscillations using stop watch
- The period of oscillation T = t/10
- Repeat the experiment for different lengths of the pendulum 'L'. Find acceleration due to gravity g using the given formula.

Observations

To find the acceleration due to gravity 'g'

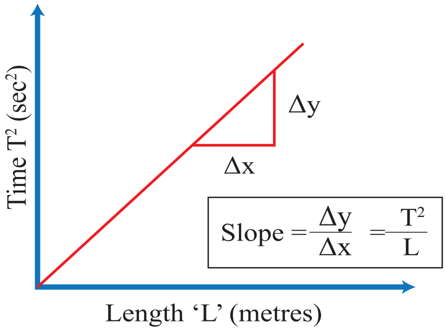
Length of the pendulum L (metre)	Time taken for 10 oscillations t (s)			Period of oscillation $T = \frac{t}{10}$ (s)	T^2 (s^2)	$g = \frac{4\pi^2 L}{T^2}$ ms^{-2}
	Tril 1	Tril 2	Average			
					Mean g =	

Model Graph

Result

The acceleration due to gravity 'g' determined using simple pendulum is

By calculation = ... m s^{-2}



$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{T^2}{L}\ ;\ 1\ \text{slope} = L/T^2$$

Discuss any possible source of error in your experiment.

Velocity as a Function of Position

Conservation of energy provides a simple method of deriving an expression for the velocity of an object undergoing periodic motion as a function of position. The object in question is initially at its maximum extension A (Figure. 46 a) and is then released from rest. The initial energy of the system is entirely elastic potential energy stored in the spring, $\frac{1}{2}kA^2$. As the object moves toward the origin to some new position x (Figure. 46 b), part of this energy is transformed into kinetic energy, and the potential energy stored in the spring is reduced to $\frac{1}{2}kx^2$. Because the total energy of the system is equal to $\frac{1}{2}kA^2$ (the initial energy stored in the spring), we can equate this quantity to the sum of the kinetic and potential energies at the position x:

KEY TERMS

- Kinetic energy; Energy of a body due to its motion
- Elastic potential energy: energy of a body due to its shape.

$$\frac{1}{2}kA^2 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

Solving for v we get, $v = \sqrt{\frac{k}{m}\left(A^2 - x^2\right)}$ Eq. 1.47

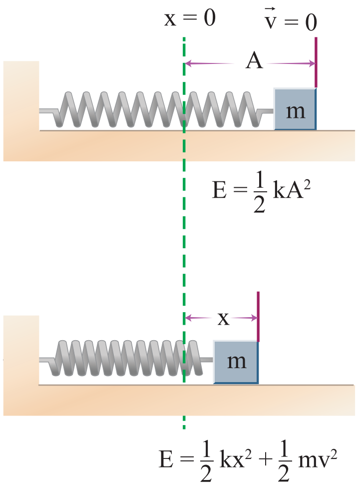


Figure 46. Energy consideration in SHM

This expression shows that the object's speed is a maximum at x = 0 and is zero at the extreme positions x±A. The right side of Equation 1.47 is preceded by the ± sign because the square root of a number can be either positive or negative. If the object in Figure 46 is moving to the right, v is positive; if the object is moving to the left, v is negative.Using

$\omega = \sqrt{\frac{k}{m}}$, we have $v = \pm\omega\sqrt{A^2 - x^2}$ Eq. 1.48

Graphical representation of simple harmonic motion

Using the equations of position, velocity and acceleration with respect to time of SHM, the position versus time, velocity versus time and acceleration versus time graphs can be plotted as in Figure 47.

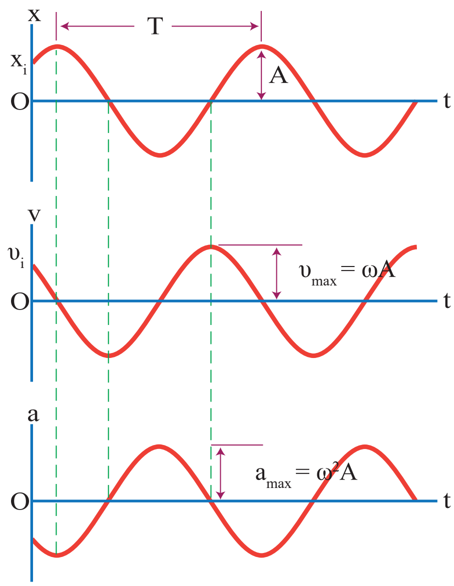


Figure 47. Graphical representation of simple harmonic motion. a) Position versus time. b) Velocity versus time. c) Acceleration versus time. Note that at any specified time the velocity is 90° out of phase with the position and the acceleration is 180° out of phase with the position. T is period of oscillation, A = Amplitude

ACTIVITY 24

Group activity

Experimental apparatus for demonstrating simple harmonic motion.

Figure 1.48 illustrates an experimental arrangement that demonstrates the sinusoidal nature of simple harmonic motion. An object connected to a spring has a marking pen attached to it. While the object vibrates vertically, a sheet of paper is moved horizontally with constant speed. The pen traces out a sinusoidal pattern.

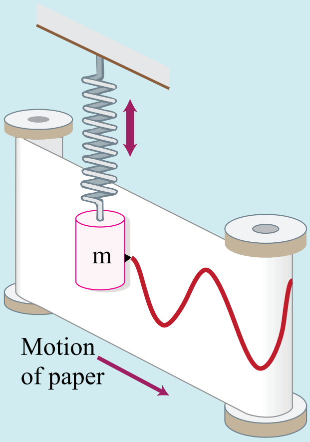


Figure 48. Experimental apparatus for demonstrating simple harmonic motion

Examples

An object oscillates with simple harmonic motion along the x axis. Its position varies with time according to the equation

$$x(t) = (2\text{ m})\cos\left(\pi t + \frac{\pi}{4}\right)$$

(a) Determine the amplitude, frequency, and period of the motion.

(b) Determine the maximum speed and the maximum acceleration.

Solution

(a) Comparing the given expression $x(t) = (2\text{ m})\cos\left(\pi t + \frac{\pi}{4}\right)$ with Eq 1.42, we see

that A = 2m, and ω = π rad/s, and $\phi = \frac{\pi}{4}$, $f = \frac{\omega}{2\pi} = 0.5\text{Hz}$, $T = \frac{1}{T} = 2\text{sec}$.

(b) The speed of the SHM whose displacement is $x(t) = (2\text{ m})\cos\left(\pi t + \frac{\pi}{4}\right)$ is

$$v = -\omega A\sin(\omega t + \varphi) = -(\pi\text{rad/s})(2\text{m})\sin\left(\pi t + \frac{\pi}{4}\right)$$

The maximum speed is $v_{max} = \omega A = (\pi\text{rad/s})(2\text{m}) = 6.28\text{ m/s}$

The acceleration of the SHM whose displacement is $x(t) = (2\text{ m})\cos(\pi t + \pi/4)$ is

$a = -\omega^2 A\cos(\omega t + \varphi) = -(\pi\text{rad/s})^2(2\text{m})\cos(\pi t + \pi/4)$.

The maximum acceleration $a_{max} = (\pi\text{rad/s})^2(2\text{m}) = 19.7\text{ m/s}^2$

Examples

(a) Find the amplitude, frequency, and period of motion for an object vibrating at the end of a horizontal spring if the equation for its position as a function of time is

$$x = (0.25\text{ m})\cos\left(\frac{\pi}{8}t\right)$$

(b) What are the position, velocity, and acceleration of the object after 1s has elapsed?

Solution

Given: $x = (0.25\text{ m})\cos\left(\frac{\pi}{8}t\right)$

(a) Comparing the given function with the defining function for the displacement of SHM, i.e., $x = A\cos(\omega t + \phi) = A\cos(\omega t)$, with phase angle $\phi = 0$, we see that

$A = 0.25\text{ m}$, $\omega = \left(\frac{\pi}{8}\right)\text{rad/s} = 0.39\text{ rad/s}$, and from $\omega = 2\pi f$, we have

$$f = \frac{\omega}{2\pi} = \frac{0.38\text{rad/s}}{2\pi\text{ rad}} = 0.063\text{Hz}$$

The period of motion $T = \frac{1}{f} = \frac{1}{0.63\text{Hz}} = 15\text{s}$

(b) As listed in Table 1.4, for the case where position of SHM is given as $x = A\cos(\omega t)$, the velocity and acceleration are given by $v = -\omega A\sin(\omega$, and $a = -\omega^2 A\cos(\omega t)$, respectively. With A = 0.25 m, ω = rad/s, and t = 1s, we have

$$x = (0.25\ \text{m})\cos\quad \omega = \frac{\pi}{8}\text{rad} = 0.23\,\text{m/s}$$

$$v = -(0.39\ \text{rad/s})(0.25\ \text{m})\ \sin = \frac{\pi}{8}\text{rad} = -0.037\,\text{m/s}$$

$$a = \left(\frac{-\pi}{8}\text{rad/s}\right)^2 (0.25\text{m})\cos\frac{\pi}{8}\text{rad} = -0.037\,\text{m/s}^2$$

Examples

A 200 g mass vibrates horizontally without friction at the end of a horizontal spring for which k = 7 N/m. The mass is displaced 5 cm from equilibrium and released. Find (a) its maximum speed and (b) its speed when it is 3 cm from equilibrium. (c) What is its acceleration in each of these cases?

Solution

(a) The maximum speed is where the mass is at the equilibrium position, x = 0

$$v = \pm\sqrt{\frac{k}{m}\left(A^2 - x^2\right)}, v_{max} = \pm A\sqrt{\frac{k}{m}} = \pm 0.05\text{m}\sqrt{\frac{7\,\text{N/m}}{0.2\text{kg}}} = \pm 0.3\,\text{m/s}$$

(b) The speed at any displacement x is $v = \pm\sqrt{\frac{k}{m}\left(A^2 - x^2\right)}$.

With x = 3 cm, $v = \pm\sqrt{\frac{7\,\text{N/m}}{0.2\text{kg}}\left((0.05\text{m})^2 - (0.05)^2\right)} = \pm 0.24\,\text{m/s}$

(c) At the equilibrium position where x = 0, the speed is maximum and the acceleration is zero.

$$a = -\frac{k}{m}\times = 0$$

At x = 3 cm = 0.03 m, the magnitude of acceleration is

$$a = \left(\frac{7\,\text{N/m}}{0.2\text{kg}}\right)\times(0.03\ \text{m}) = 1.05\ \text{m/s}^2$$

Examples

A 50 g mass vibrates in SHM at the end of a spring. The amplitude of the motion is 12 cm, and the period is 1.7 s. Find: (a) the frequency, (b) the spring constant, (c) the maximum speed of the mass, (d) the maximum acceleration of the mass, (e) the speed when the displacement is 6 cm and the mass is moving to the right, and (f) the acceleration when x = 6 cm and the mass is moving to the right.

Solution

(a) Frequency f = 1/T = 1/1.7s = 0.58 Hz

(b) Solving for the spring constant k of the equation $T = 2\pi\sqrt{\frac{m}{k}}$

$$k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2 (0.05\text{kg})}{(1.7\text{s})^2} = 0.68 \text{ N/m}$$

(c) The maximum speed of the mass $v_{max} = A\sqrt{\frac{k}{m}} = 0.12\text{m}\sqrt{\frac{0.68\text{ N/m}}{0.05\text{kg}}} = 0.44\text{ m/s}$

(d) The maximum acceleration of the mass is when x = ±A. The magnitude of the maximum acceleration is

$$\text{amax} = \frac{k}{m} A = \frac{0.68\text{ N/m}}{0.05\text{m}} (0.12 \text{ m}) = 1.63 \text{ m/s}^2$$

(e) The speed as the mass is at x = 6 cm = 0.06 m is

Speed is always positive and we use

$$v = \sqrt{\frac{k}{m}\left(A^2 - x^2\right)} = \sqrt{\frac{0.68\text{ N/m}}{0.05\text{kg}}\left((10.12\text{m})^2 - (0.06\text{m})^2\right)} = 0.38\text{ m/s}$$

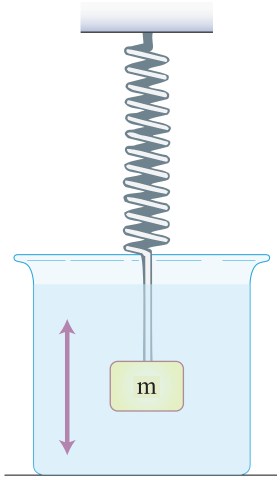
(f) Here we want the acceleration. Since x = 6 cm, the force on the mass is to the left and negative. Likewise the mass is accelerating to the left even as it is moving to the right. Hence the acceleration must be negative; the mass is slowing down.

$$a = \frac{-k}{m}x = \frac{-0.68\text{ N/m}}{0.05\text{kg}} \times (0.06\text{m}) = -0.82\text{ m/s}^2$$

Damped Oscillations

The vibrating motions we have discussed so far have taken place in ideal systems that oscillate indefinitely under the action of a linear restoring force. In all real mechanical systems, forces of friction retard the motion, so the systems don't oscillate indefinitely. The friction reduces the mechanical energy of the system as time passes, and the motion is said to be damped.

a) A swinging bell



b) A mass oscillating in a viscous liquid

Figure 49.

Examples of damped oscillations are a swinging bell (Figure 49a) and an object attached to a spring and submersed in a viscous liquid. The swinging bell left to itself will eventually stop oscillating due to damping forces (air resistance and friction at the point of suspension). An object attached to a spring and set in oscillation in a viscous liquid (Figure 49b) will come to rest due to the drag force it experiences in the liquid.

Figure 50 shows the position as a function of time for an object oscillating in the presence of a retarding force. We see that when the retarding force is small, the oscillatory character of the motion is preserved but the amplitude decreases in time, with the result that the motion ultimately ceases. Any system that behaves in this way is known as a damped oscillator.

Damped motion varies with the fluid used. For example, if the fluid has a relatively low viscosity, the vibrating motion is preserved but the amplitude of vibration decreases in time and the motion ultimately ceases. This process is known as underdamped oscillation. The position versus time curve for an object undergoing such oscillation appears in Figure 50. Figure 51 compares three types of damped motion, with curve (a) representing underdamped oscillation. If the fluid viscosity is increased, the object returns rapidly to equilibrium after it's released and doesn't oscillate. In this case, the system is said to be critically damped, and is shown as curve (b) in Figure 51. The piston returns to the equilibrium position in the shortest time possible without once overshooting the equilibrium position. If the viscosity is made greater still, the system is said to be overdamped. In this case, the piston returns to equilibrium without ever passing through the equilibrium point, but the time required to reach equilibrium is greater than in critical damping, as illustrated by curve (c) in Figure 51.

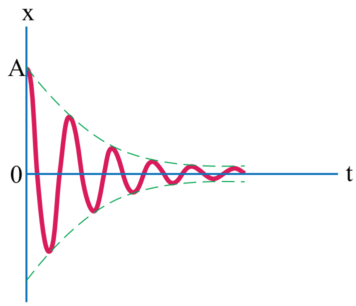


Figure 50. Graph of position versus time for a damped oscillator. Note the decrease in amplitude with time.

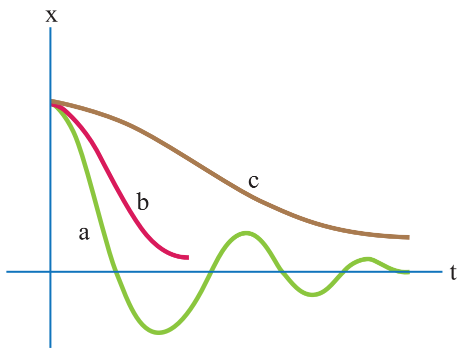


Figure 51. Plots of displacement versus time for (a) an underdamped oscillator, (b) a critically damped oscillator, and (c) an overdamped oscillator.

Did you know

To make automobiles more comfortable to ride in, shock absorbers Figure 52 are designed to be slightly under damped. This can be demonstrated by a sharp downward push on the hood of a car. After the applied force is removed, the body of the car oscillates a few times about the equilibrium position before returning to its fixed position.

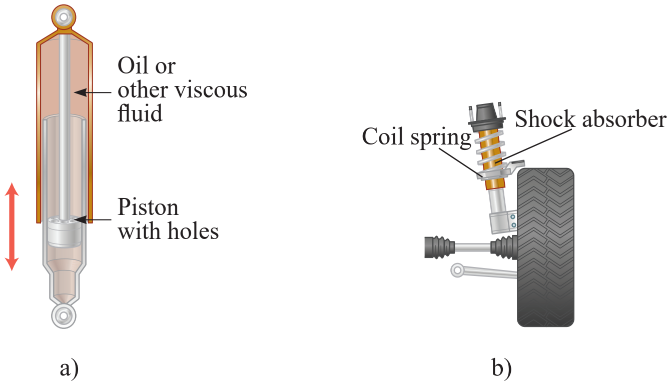


Figure 52. a) A shock absorber consists of a piston oscillating in a chamber filled with oil b) One type of automotive suspension system, in which a shock absorber is placed inside a coil spring at each wheel.

Key terms

- Damped oscillation: oscillation in which amplitude decreases in time.
- Under damped oscillation: with relatively low damping, the vibrating motion is preserved but the amplitude of vibration decreases in time
- Critically damped oscillation: the object returns rapidly to equilibrium after it's released and doesn't oscillate.
- Over damped oscillation: with relatively larger damping the piston returns to equilibrium without ever passing through the equilibrium point, but the time required to reach equilibrium is greater than in critical damping.

Forced Oscillations

We have seen that the mechanical energy of a damped oscillator decreases in time as a result of the resistive force. It is possible to compensate for this energy decrease by applying an external force that does positive work on the system. At any instant, energy can be transferred into the system by an applied force that acts in the direction of motion of the oscillator. For example, a child on a swing can be kept in motion by appropriately timed "pushes." The amplitude of motion remains constant if the energy input per cycle of motion exactly equals the decrease in mechanical energy in each cycle that results from resistive forces.

A person swinging in a swing without anyone pushing it is an example of free oscillation. However, if someone pushes the swing periodically, the swing has forced, or driven, oscillations. Two angular frequencies are associated with a system undergoing driven oscillations: (1) the natural angular frequency of the system, which is the angular frequency at which it would oscillate if it were suddenly disturbed and then left to oscillate freely, and (2) the angular frequency of the external driving force causing the driven oscillations.

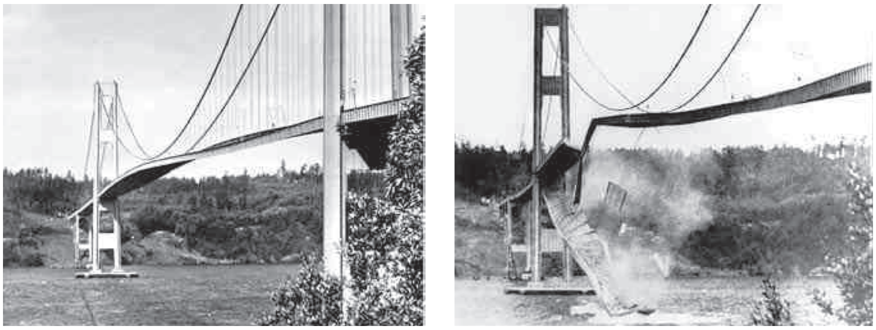
An oscillation in which external driving force is applied is known as forced oscillation. For small damping, the amplitude is large when the frequency of the driving force is near the natural frequency of oscillation. When the frequency of the driving force and the natural frequency of the oscillator much, a phenomenon known as Resonance takes place. At resonance the amplitude of oscillation increases. The frequency at which resonance happens is known as resonance frequency.

Did you know?

Bio Application Canine Resonance

Unlike humans, dogs have no sweet glands and so must pant in order to cool down. The frequency at which a dog pants is very close to the resonant frequency of its respiratory system. This cause the maximum amount of air inflow and outflow and so minimizes the effort that the dog must exert to cool itself.

All mechanical structures have one or more natural frequencies, and if a structure is subjected to a strong external driving force that matches one of these frequencies, the resulting oscillations of the structure may damage the structure, Figure 53.



a) b)

Figure 53. a) In 1940 turbulent winds set up torsional vibration in the Tacoma Narrows Bridge, causing it to oscillate at a frequency near one of the natural frequencies of the bridge structure. b) Once established, this resonance condition led to the bridge's collapse.

Did you know?

Resonance appears to be one reason buildings in Mexico City collapsed in September 1985 when a major earthquake (8.1 on the Richter scale) occurred on the western coast of Mexico, Figure 54.

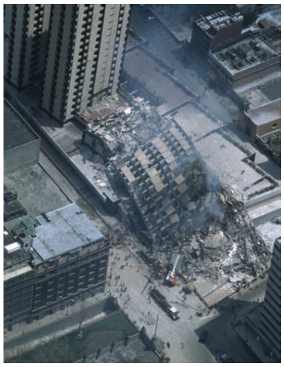


Figure 54. In 1985, buildings of intermediate height collapsed in Mexico City as a result of an earthquake far from the city. Taller and shorter buildings remained standing. (John T. Barr/Getty Images News and Sport Services)

Reducing Resonance Effects

KEY TERMS

- Forced oscillation: oscillation in which external driving force is applied.
- Resonance : a phenomenon that takes place when the natural frequency of oscillation matches with the frequency of driving force.
- Resonance frequency: the frequency at which resonance happens

To counter resonance effects on bridges and buildings, engineers build them in such a way as to reduce the amplitude of resonance. Bridge designers make bridges more streamlined so that the wind passes over without imparting much energy. Damping reduces the effect of resonance.

Skyscraper designers employ many strategies to lessen resonant vib-ration. One very effective approach is to use a large mass at the top of the building, called a "tuned mass damper," which is free to oscillate back and forth (Figure 55). Controlled by computers, it can be made to vibrate at the resonant frequency of the building.

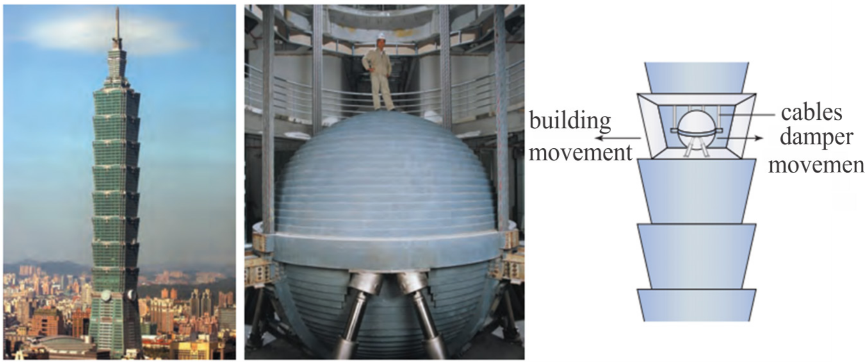


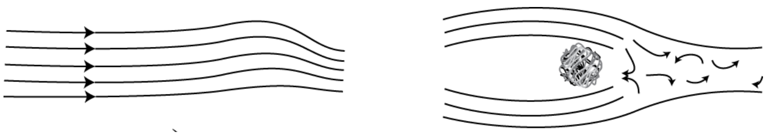
Figure 55. The Taipei 101 stories building. The inset shows a tuned mass damper in the building. It has a huge mass and vibrates opposite to the direction of the building, cancelling much of the amplitude of the resonant vibration.

Fluid in motion

Both liquids and gases are referred as fluids. The study of the mechanical motion of fluids is called hydro–dynamics or fluid dynamics. The basic descriptions of the flow of fluids are conventionality obtained by considering an ideal fluid. An ideal fluid is incompressible, non-viscous and steady.

1. An incompressible fluid is a fluid in which its density remains constant.
2. A non-viscous fluid is a fluid in which viscosity is neglected. Viscosity refers to a fluid's internal friction between different layers of fluids in relative motion. For non-viscous flow of fluids there is no loss of energy.
3. A fluid motion may be steady or unsteady. In steady flow, the velocity of the fluid particles at any point within it is constant. The velocity may be different at different points, but consecutive particles of fluid arriving at a given point will have a fixed velocity at that location.

When a fluid flows smoothly, such that neighboring layers of the fluid slide parallel to each other with no disruption between the layers, (Figure 56 (a)), the flow is said to be **streamline** or **laminar flow**. In this kind of flow, each particle of the fluid follows a smooth path, and these paths do not cross over one another. In contrast, at sufficiently high flow rates, or when boundary surfaces (or obstacles) cause abrupt changes in velocity, the flow can become irregular (unsteady) which is called **turbulent**, Figure 56 (b).



a) b)

Figure 56. (a) Streamline or laminar flow, (b) turbulent flow.

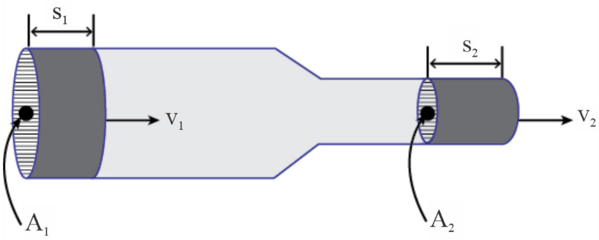
Factors Affecting Laminar Flow

There are four properties of fluid that affect the way it flows past an object. They are: density, compressibility, temperature and viscosity.

1. **A fluid is incompressible**, when its density is constant.
2. **A fluid is non-viscous**, when viscosity (fluid friction) is neglected.
3. **The motion of a fluid is steady**, when the particles of the fluid have the same velocity as they pass a given point.

Equation of continuity

The speed of a fluid changes as the cross-sectional area of the pipe through which it flow changes. The **equation of continuity** states that the volume of an ideal fluid flowing through a pipe is constant. This means that since liquids are incompressible the mass of fluid that flows through a pipe of different cross-section is also constant.



$$Q = \frac{V}{t} = \text{constant or } \frac{m}{t} = \text{constant}$$

This leads to the relation known as the **equation of continuity**.

$$A_1 v_1 = A_2 v_2$$

The product Av represents the volume of fluid per second that passes through the tube and is referred to as the **volume flow rate**. This means a fluid flows faster as the cross-sectional area of the pipe through which it flow decreases.

Examples

Water is flowing through a 6.0cm diameter hose at 25 cm/s. What will be the speed of the water, if the diameter of the pose is reduced to 1.0cm?

Solution

d_1 = 6.0cm, r_1 = 3.0cm, v_1= 25 cm/s, d_2 = 1.0cm, r_2 = 0.5cm.

$$A_1 v_1 = A_2 v_2$$

$$v_2 = \frac{A_1 v_1}{A_r} = \frac{\pi r_1^2 (v_1)}{\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2 v_1$$

$$= \left(\frac{3.0\text{cm}}{0.5\text{cm}}\right)^2 (25\,\text{cm/s}) = 9.0\,\text{m/s}$$

Examples

A pipe 5.0cm in diameter carries water (density = 1×10^3 kg/m^3) at a speed of 2.5 m/s. Calculate the mass – flow rate of the fluid.

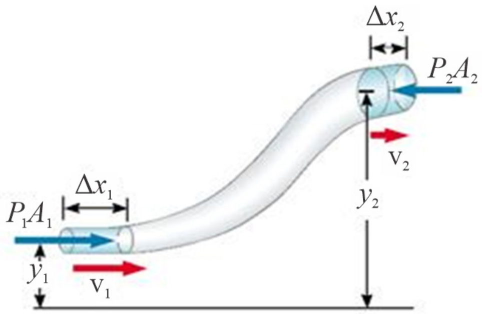
Solution

$d = 5.0cm,\ r = 2.5cm = 2.5 \times 10^{-2}\ m,\ \rho = 10^3 kg/m^3,\ v = 2.5m/s$

$$Qm = \rho Av = 10^3 kg/m^3 \left(3.14 \times (2.5 \times 10^{-2} m)^2\right)(2.5m/s)$$
$$= 4.9\ kg/s$$

Bernoulli's Equation

The Swiss physicist Daniel Bernoulli (1700–1783) developed the expression relating pressure, fluid speed and elevation. It arises from the principle that the work done on a fluid as it flows from one place to another is equal to the change in its mechanical energy (the work – energy theorem).



Bernoulli's equation can be written as:

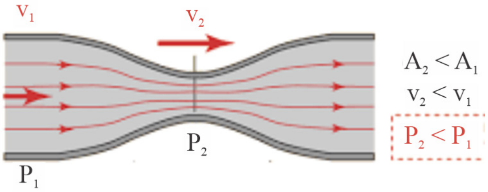
$$P + \frac{1}{2}\rho v^2 + \rho gy = constant$$

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gy_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gy_2$$

where

- points 1 and 2 lie on a streamline,
- the fluid has constant density,
- the flow is steady, and
- there is no friction.

When a liquid enters a **horizontal pipe** where the height of the two cross-sections of the pipe is the same ($y_1 = y_2$).



$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

Examples

Water flows through a horizontal pipe at a pressure of 85kpa with a speed of 2.6 m/s. The pipe widens, so that its area becomes larger by 35%. If the flow is to be at constant pressure, how far above the initial horizontal level should the pipe divert the water?

Solution

p1 = p2 = 85kpa, $v_1 = 2.6$m/s, $A_2 = A_1 + 35\% A_1 = 1.35A_1$, and $A_2 = 1.35A_1$

$$p_1 + \frac{1}{2}\rho v_1^2 = \rho gh_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2$$

Since $p_1 = p_2$

$$\frac{1}{2}\rho v_1^2 + \rho gh_1 = \frac{1}{2}\rho v_2^2 + \rho gh_2$$

$$h_2 - h_1 = \frac{\frac{1}{2}\rho\left(v_1^2 - v_2^2\right)}{\rho g} = \frac{v_1^2 - v_2^2}{2g}$$

From the equation of continuity

$$v_2 = \frac{A_1 v_1}{A_2} = \frac{A_1 v_1}{1.35A_1} = 0.74v_1$$

$$h_2 - h_1 = \frac{v_1^2 - (0.74v_1)^2}{29} = \frac{0.45v_1^2}{29} = \frac{0.45\left(2.6\,\mathrm{m/s^2}\right)}{20\,\mathrm{m/s^2}} = 0.15\mathrm{m}$$

Examples

During a storm, a 50 m/s wind blows horizontally across the flat level root of a house. If the roof has an area of 80m^2, what is the magnitude of the force that is likely to be exerted on it? Take density of air to be 1.0kg/m^3

Solution

$v_1 = 50$m/s (above the root), $v_2 = 0$ (below the root)

$A = 80m^2$, $\rho = 1.0kg/m^3$

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2$$

Since $h_1 = h_2$

$$p_2 - p_1 = \frac{1}{2}\rho\left(v_1^2 - v_2^2\right)$$

$$p_2 - p_1 = \frac{1}{2}\left(1.0\,kg/m^3\right)\left((50\,m/s)^2 - 0\right) = 1250\,N/m^2$$

$$F = \Delta pA = (p_2 - p_1)\,A = 1250\ N/m^2\ (80m^2) = 1.0 \times 10^5 N$$

$$F = \Delta pA = (p_2 - p_1)\,A = 1250\,N/m^2\left(80m^2\right) = 1.0$$

Exercises

1. Water is flowing smoothly through a pipe at a rate of $0.05m^3/s$. Two points in this pipe differ in height of 2.0m. The diameter of the pipe at point 1 is 20cm and at point 2 is 10cm. If the pressure at point 1 is 100kpa find (a) the velocity at points 1 and 2. (b) the pressure at point 2. Point 1 is at lower position than point 2.
2. A horizontal pipe 10cm in diameter is connected to another horizontal pipe with 5cm diameter. Water flows in the smaller pipe at 10m/s Find the difference in pressure between the two pipes.
3. Water is flowing through a pipe of cross – sectional area $8 \times 10^{-2}m^2$ with a speed of 8 m/s and has a pressure of 4×10^5 pa at this point. What will be the pressure at a point where the cross – sectional area of the pipe is $4 \times 10^{-2}m^2$ is risen by 12m from the for men place?

Applications of Bernoulli's Principle

Bernoulli's principle can be applied to many situations. Some of them are

(a) **Atomizer or paint sprayer:** A stream of air passing over an open tube reduces the pressure above the tube, causing the liquid to rise into the air stream. The liquid is then dispersed into a fine spray of droplets. This is used in atomizer, perfume bottles and paint sprayers.

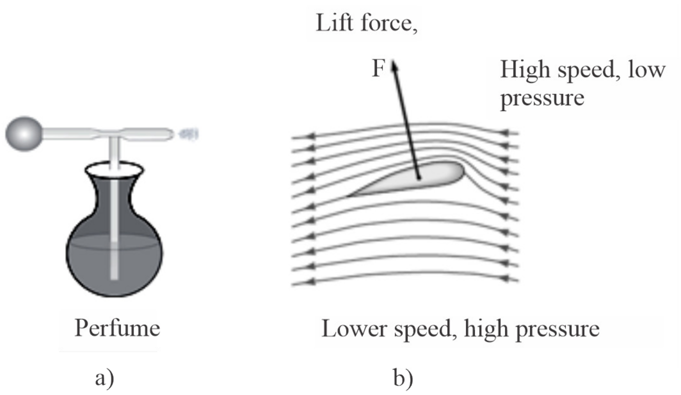


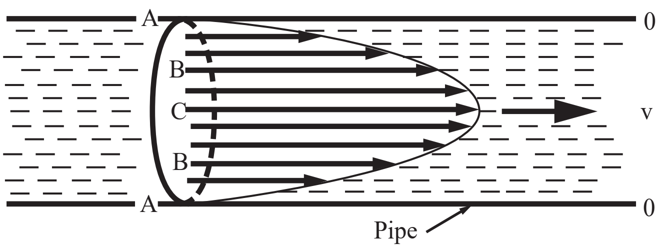
Figure 57.

(b) The forces on an airplane wings or airfoils can be explained by Bernoulli's principle. When a plane moves in horizontal or slightly upward flight, there is an increase in velocity of the air above the wing relative to that below. There is thus a reduction in pressure above the wing relative to that below. Such pressure difference results in a net upward force (dynamic lift) on the wing (Figure 57 (b)). The greater the speed difference, the greater the upward force.

Viscosity

Friction in fluids is called **viscosity**. Viscosity exists in both liquids and gases, and is essentially a frictional force between adjacent layers of fluid as the layers move past one another. Viscosity is "thickness" or "internal friction". Thus, water is "thin", having a lower viscosity, while honey is "thick", having a higher viscosity.

When water is flowing through a pipe, the layer of water touching the wall is at rest, the water in the center is moving fastest. The viscosity of different fluids can be expressed quantitatively by a *coefficient of viscosity*. The stationary layer of fluid retards the flow of the layer just above it, which in turn retards the flow of the next layer, and so on. Thus the velocity varies continuously from 0 to v, as shown in the figure.



The increase in velocity divided by the distance over which this change is made ($\frac{\Delta v}{\Delta y}$) is called the velocity gradient. For a given fluid, it is found that the force required, F, is proportional to the area of fluid in contact with each plate, A, and to the velocity gradient.

$$F \ \alpha \ A\frac{\Delta v}{\Delta y} \quad \Rightarrow F = \eta\, A\frac{\Delta v}{\Delta y}$$

The proportionality constant η is known as the coefficient of viscosity. Solving for η gives:

$$\eta = \frac{F\Delta y}{A\Delta v}$$

The SI unit of η is $N.s/m^2$ = Pa.s (Pascal.second).

Stokes' Law

For a spherical body of radius r, moving through a fluid of viscosity η, the magnitude of the drag force depends on:

1. The size (radius, r) of the body
2. The velocity v at which the sphere moves through the fluid
3. The viscosity η of the fluid

$\Rightarrow \ F = 6\pi\eta r v$

This equation was first deduced by Sir George Stokes in 1845 and is known as Stokes' law.

Examples

Find the force on a ball bearing of radius 4×10^{-3}m falling through a liquid of viscosity 0.985 at a velocity of 0.5 m/s.

Solution

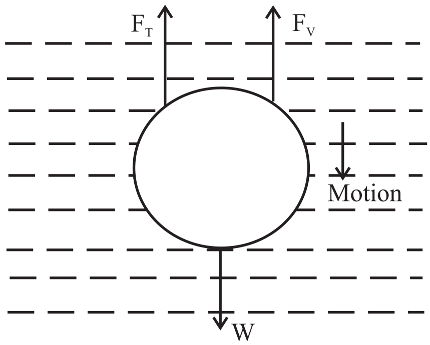
$r = 4 \times 10^{-3}$m, $\eta = 0.985$, v = 0.5 m/s.

$\Rightarrow F = 6\pi\,\eta\, rv = 6 \times 3.14 \times 0.985 \times 4 \times 10^{-3} \times 0.5$

$F = 3.7 \times 10^{-2}$N

Terminal velocity

The maximum constant velocity acquired by the body while falling freely in a viscous medium is called terminal velocity. When a small spherical body fall freely through a viscous medium three forces act on it.



(i) Weight of body = Volume × density × g, acting downward, $W = \frac{4}{3}\pi r^3 \rho g$ where ρ is density of material of spherical body.

(ii) Upward thrust due to buyonancy is equal to weight of medium displaced

F_T = Volume of medium displaced × density × g, $F_T = \frac{4}{3}\pi r^3 \sigma g$, where σ is density of medium

(iii) Viscous force, $F_v = 6\,\pi\eta r v$

where ρ is radius of spherical object falling with velocity v.

When the spherical body starts falling with a constant velocity called terminal velocity, weight of body is equal to the sum of viscous force and upward thrust due to buyonancy.

$$F_T + F_v = W$$

$$\frac{4}{3}\pi r^3 \sigma g + 6\pi\eta r v = \frac{4}{3}\pi r^3 \rho g$$

$$6\pi\eta r v = \frac{4}{3}\pi r^3 (\rho - \sigma) g$$

$$v = \frac{2r^2(\rho - \sigma)g}{9\eta}$$

If $\rho < \sigma$ then, velocity v is negative i.e. body will move up with a constant velocity. That is why gas bubbles rise up through soda water bottle.

Example : The terminal velocity of a copper ball of radius 2.0 mm falling through a tank of oil at 20°C is 6.5 cm s^{-1}. Compute the viscocity of the oil at 20°C. Density of oil is 1.5 x 10^3 kg m^{-3}, density of copper is 8.9 x 10^3 kg m^{-3}.

Solution : Here v = 6.5 × 10^{-2} ms^{-1}; r = 2.0 x 10^{-3} m; g = 9.8 m/s^2,

ρ = 8.9 x 10^3 kg m^{-3}, σ = 1.5 x 10^3 kg m^{-3}

$$\eta = \frac{2r^2(\rho-\sigma)g}{9\text{ v}} = \frac{2\times(2\cdot0\times10^{-3})^2\times(8\cdot9-1\cdot5)\times10^3\times9\cdot8}{2\times(6\cdot5\times10^{-2})} \approx 9\cdot9\times10^{-1}\text{Pa-s}$$

Summary

- Vectors have both magnitude (size) and direction while scalars have magnitude but no direction.
- A vector is represented geometrically by an arrow drawn to scale.
- Addition of vectors also known as composition of vectors involves both the magnitude and direction of the vector.
- Resolution of vector means breaking the vector into its component vectors.
- A unit vector is a dimensionless vector having a magnitude of exactly one or unity.
- In projectile motion with no air resistance, $a_x = 0$ and $a_y = -g$.
- It is useful to think of projectile motion as the superposition of two motions:
  1. constant-velocity motion in the x direction and
  2. free-fall motion in the vertical direction subject to a constant downward acceleration of magnitude g = 9.8 m/s^2.

     $x = (v_o \cos\theta)t,$

     $y = (v_o \sin\theta)t - \frac{1}{2} gt^2$

     $v_x = v_o \cos\theta$

     $v_y = v_o \sin\theta - gt$
- A particle in uniform circular motion travels around a circle or a circular arc at constant (uniform) speed.
- The acceleration associated with uniform circular motion that arises due to the change in direction of velocity is called centripetal a_c, where $a_c = \frac{v^2}{r}$
- In non-uniform circular motion there is a component of acceleration that is parallel to the instantaneous velocity known as tangential acceleration a_t.
- The total acceleration, a is the resultant of the centripetal acceleration (a_c) and tangential acceleration (a_t).

$$a = \sqrt{a_c^2 + a_t^2}$$

- The net force causing the centripetal acceleration is the centripetal force
$$F_c = \frac{mv^2}{r}$$
- Rotational motion is defined as the motion of a body about a fixed axis where all the particles constituting the body undergoes circular motion about a common axis.
- A rigid body is a body that can rotate with all its parts locked together and without any change in its shape.
- When a rigid object rotates about a fixed axis, every part of the object has the same angular speed and the same angular acceleration. The tangential values of velocity and acceleration depend on the distance of a point from the axis of rotation.
- Simple harmonic motion: type of oscillation in which acceleration is directly proportional to the displacement and directed toward the equilibrium position.
$$a = \frac{-k}{m}x, a = -\omega^2 x$$
- The position, velocity and acceleration of a SHM as a function of time are
- For x = A, at t=0, $x = A\cos(\omega t + \phi)$, $v = -\omega A\sin(\omega t + \phi)$, $a = -\omega^2 A\cos(\omega t + \phi)$,
- For x = 0, at t = 0, $x = A\sin(\omega t + \phi)$, $v = \omega A\cos(\omega t + \phi)$, $a = -\omega^2 A\sin(\omega t + \phi)$
- Velocity of SHM as a function of displacement is $v = \sqrt{\frac{k}{m}(A^2 - x^2)}$
- Period of SHM is $T = 2\pi\sqrt{\frac{m}{k}}$, and the frequency $f = \frac{1}{2\pi}\sqrt{\frac{m}{k}}$
- Period of simple pendulum of length is $T = 2\pi\sqrt{\frac{\ell}{g}}$, and the frequency
$$f = 2\pi\sqrt{\frac{g}{\ell}}$$
- Resonance occurs when the frequency of the driving force equals the natural frequency of the oscillator.

Review Exercises

1. Two vectors have magnitudes of 14 N and 8 N. What are the magnitudes of the maximum and the minimum values of the resultant of the vectors?
2. A commuter airplane starts from an airport and takes the route shown in Figure 58. The plane first flies to city A, located 175 km away in a direction 30° north of east. Next, it flies for 150 km 20° west of north, to city B. Finally, the plane flies 190 km due west, to city C. Find the location of city C relative to the location of the starting point.

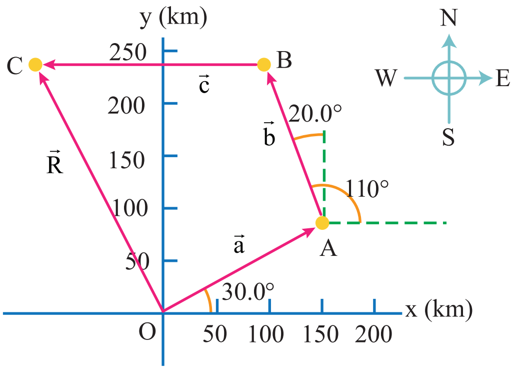


Figure 58.

3. An arrow is shot at an initial velocity of 25 m/s at an angle of 30° with respect to the horizontal.
   (a) What is the maximum height reached by the arrow?
   (b) How far does the arrow travel in the horizontal direction in returning to the level from which it was thrown?
   (c) With what velocity will it return to the level of throwing?
4. For what angle of projection will the range of a projectile be equal to the maximum height it attains? Would the answer be different on a different planet?
5. A basketball player who is 2 m tall is standing on the floor 10 m from the basket, as in Figure 59. If he shoots the ball at a 40° angle with the horizontal, at what initial speed must he throw so that it goes through the hoop without striking the backboard? The basket height is 3.05 m.

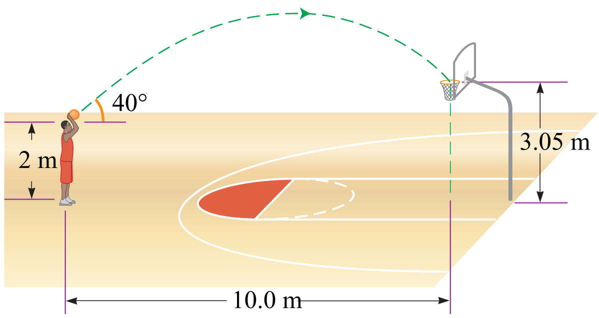


Figure 59.

In Figure 60, a car is driven at constant speed over a circular hill and then into a circular valley with the same radius. At the top of the hill, the normal force on the driver from the car seat is 0.The driver's mass is 70 kg. What is the magnitude of the normal force on the driver from the seat when the car passes through the bottom of the valley?

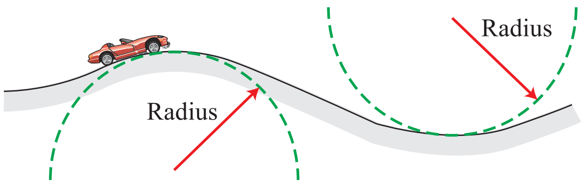


Figure 60.

A 2.40-kg ball is attached to an unknown spring and allowed to oscillate. Figure 61 shows a graph of the ball's position x as a function of time t. What are the oscillation's (a) period, (b) frequency, (c) angular frequency, and (d) amplitude? (e) What is the force constant of the spring?

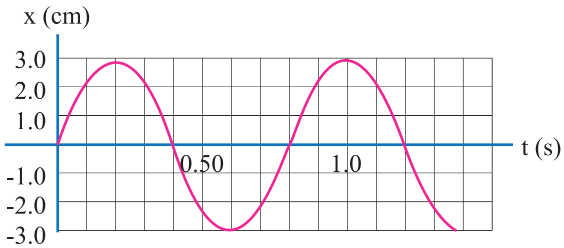


Figure 61.

P11CH02

CHAPTER 2

COMPOSITION AND RESOLUTION OF FORCES

Chapter Contents

2.1 Composition and Equilibrium of Forces
2.2 Composition and Resolution of Forces
2.3 Parallel Forces and Moments (torque), Center of Gravity, friction

- Summary
- Review Exercises

Chapter Outcomes

Learners will able to:

1. recognize the
   - various types of forces and the conditions that bring them to the state of equilibrium
   - advantages and disadvantages of friction
   - conditions for equilibrium of parallel forces
2. demonstrate the types of equilibrium.

Chapter Objectives

Upon completion of this chapter, learners will:

- combine force vectors to produce resultant force.
- use the cosine and sine laws in the resolution of force.
- distinguish between resultant and equilibrant force.
- demonstrate the two conditions for equilibrium.
- distinguish the types of Friction and calculate its coefficient.
- distinguish between center of gravity and center of mass.

2.1 Composition and Equilibrium of Forces

Introduction

It is our everyday experience that objects around us move from place to place. In kinematics motion is described in terms of position, velocity, and acceleration without considering what might cause bodies to move the way that they do. For example, why does a dropped feather fall more slowly than a dropped baseball? Why does a ball projected vertically upward come to rest at it reaches the maximum height? The answers to such questions take us into the subject of dynamics, the relationship of motion to the forces that cause it. Dynamics deals with the effects of forces on objects.

Before you can predict or explain the motion of an object, it is important to first understand what a force is and how to measure and calculate the sum of all forces acting on an object.

Key terms

- Kinematics: study of motion without regard to the cause of motion.
- Dynamics: deals with effects of a force on objects.

Force and interaction

In everyday language, a force is a push or a pull. A better definition is that a force is an interaction between two bodies or between a body and its environment. That's why we always refer to the force that one body exerts on a second body. When you push on a car that is stuck in the mud, you exert a force on the car; a steel cable exerts a force on the beam it is hoisting at a construction site; and so on.

Force is a Vector Quantity

You experience a force when you push or pull an object. A push or a pull can have different magnitudes and can be in different directions. For this reason, force is a vector quantity. In general, any force acting on an object can change the shape and/or velocity of the object (Figure 1). If you want to deform an object yet keep it stationary, at least two forces must be present.

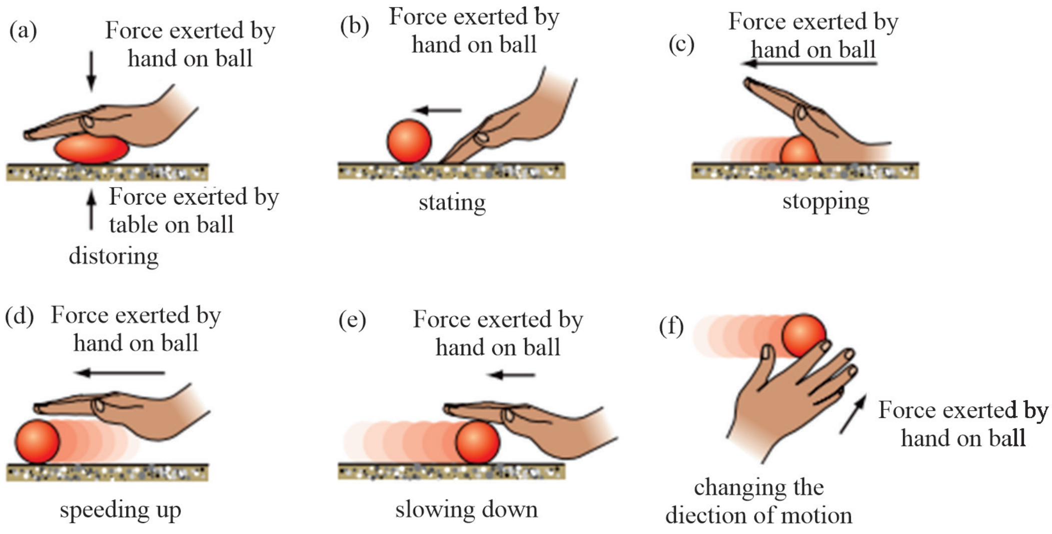


Figure 1. Effects of a force on a body

Some types of forces

When a force involves direct contact between two bodies, such as a push or pull that you exert on an object with your hand, we call it a contact force, Figure 2.

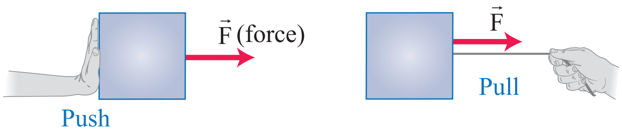


Figure 2. Force is a push or a pull exerted on an object

Figures 3a, 3b, and 3c show three common types of contact forces. The normal force (Figure 3a) is exerted on an object by any surface with which it is in contact. The adjective normal means that the force always acts perpendicular to the surface of contact, no matter what the angle of that surface. By contrast, the friction force (Figure 3b) exerted on an object by a surface acts parallel to the surface, in the direction that opposes sliding. The pulling force exerted by a stretched rope or cord on an object to which it's attached is called a tension force (Figure. 3c).

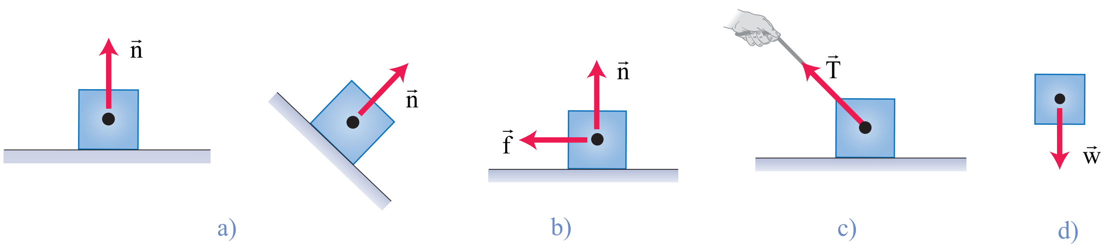


Figure 3. Some types of forces

In addition to contact forces, there are long-range forces that act even when the bodies are separated by empty space. The force between two magnets is an example of a long-range force, as is the force of gravity (Fig. 3d); the earth pulls a dropped object toward it even though there is no direct contact between the object and the earth. The gravitational force that the earth exerts on your body is called your weight.

Representing forces using free-body diagrams

A free-body diagram is a powerful tool that can be used to analyze situations involving forces. This diagram is a sketch that shows the object by itself, isolated from all others with which it may be interacting, Figure 4.

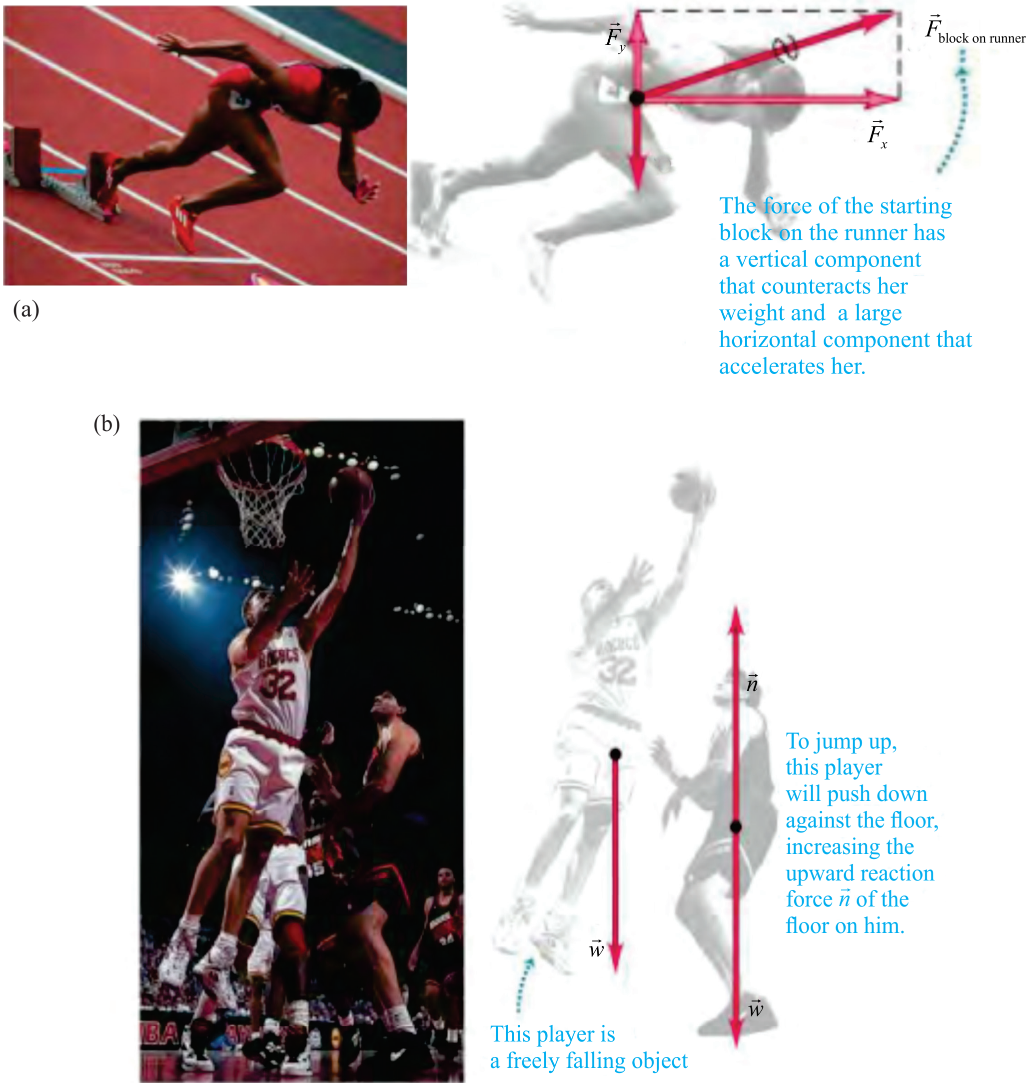


Figure 4. Examples of free-body diagrams

When you throw a ball, at least two forces act on it: the push of your hand and the downward pull of gravity. A package dropped from a certain height experiences gravitational pull of the earth and the drag fore (air resistance). When you pull a box on the floor, the force you apply and friction between the box and the floor act on the box. Suppose a number of forces F_1 and F_2, F_3, etc., act at the same time at the same point on a body. These forces could be replaced by a single force R which has the same effect as the number of forces acting on the body. R is known as the resultant force or net force. According to the principle of superposition of forces resultant is the vector sum of the forces written as

$R = F_1 + F_2 + F_3 + \ldots$

In Figure 5, two forces of F_1 and F_2 are acting on a body at point o. The resultant R has the same effect as the two forces. $R = F_1 + F_2$

KEY TERMS

- Force: a push or a pull exerted on a body
- Normal force n: When an object rests or pushes on a surface, the surface exerts a push on it that is directed perpendicular to the surface.
- Friction force f: In addition to the normal force, a surface may exert a friction force on an object, directed parallel to the surface.
- Tension force T: A pulling force exerted on an object by a rope, cord, etc.
- Weight w: The pull of gravity on an object is a long-range force (a force that acts over a distance).

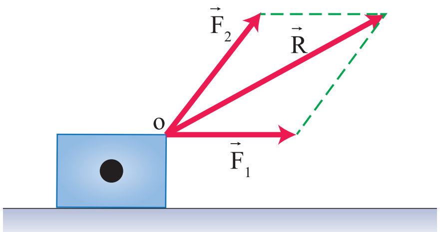


Figure 5. A resultant is the vector sum of forces

Since forces are vector quantities and add like vectors, we can use all of the rules of vector mathematics that we learned in unit 1 to solve problems that involve vectors.

We discussed that it's easiest to add vectors by using components. We resolve the vectors into their components so that the x component of the resultant is the sum of the x components of the vectors and the y component of the resultant is the sum of the y components of the vectors.

KEY TERMS

- Net force: vector sum of all the forces acting simultaneously on an object
- Superposition of forces: finding the vector sum of forces

Note that the x- and y-coordinate axes do not have to be horizontal and vertical, respectively. As an example, Fig. 6 shows a crate being pulled up a ramp by a force F. In this situation it's most convenient to choose one axis to be parallel to the

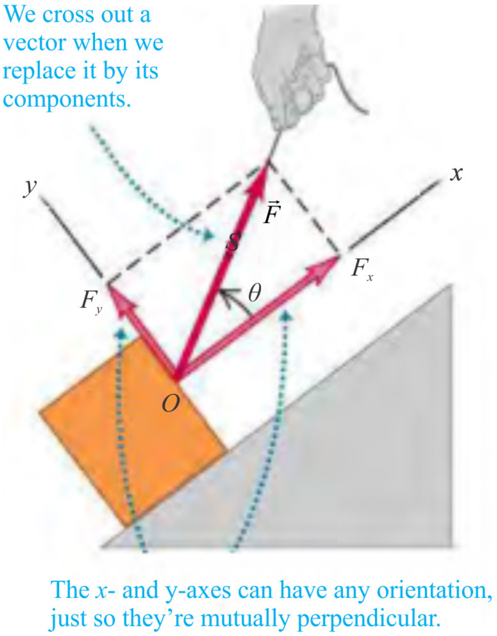


Figure 6. The vector F can be replaced by its components $F_x = F\cos\theta$ and $F_y = F\sin\theta$

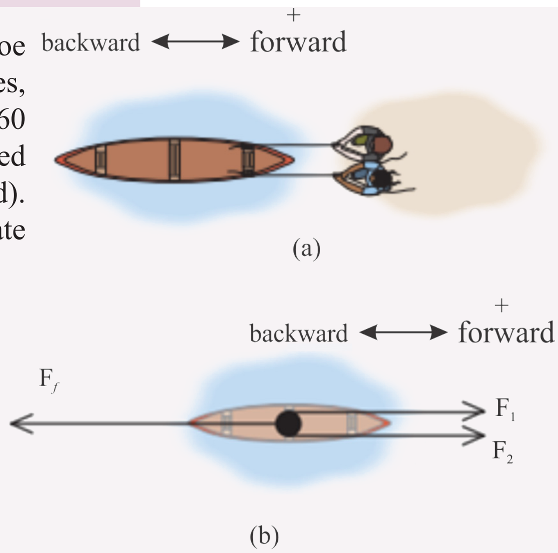
ramp and the other to be perpendicular to the ramp. Depending on your choice of axes and the orientation of the force F, either F_x or F_y may be negative or zero.

Adding Collinear Forces

Vectors that are parallel are collinear, even if they have opposite directions. Figure 7 shows two collinear forces acting on a canoe. The canoe is dragged using two ropes. The magnitude of the force exerted by a rope on an object at the point where the rope is attached to the object is called the tension in the rope.

Examples

Two people, A and B, are dragging a canoe out of a lake onto a beach using light ropes, Figure 7 (a). Each person applies a force of 60 N (forward) on the rope. The friction exerted by the beach on the canoe is 85 N (backward). Starting with a free-body diagram, calculate the net force on the canoe.



Solution

Given $F_1 = F_2 = 60$ N, $F_f = 85$ N

The free-body diagram can be drawn as Figure 7 (b)

Figure 7 shows graphical method of addition of the three vectors. F_1 and F_2 are in the forward direction while F_f is in a backward direction.

The algebraic method of addition can be done as

$F_{net} = F_1 + F_2 + F_f$

The magnitude of the net force is

$F_{net} = F_1 + F_2 + F_f = 60\text{ N} + 60\text{ N} + (-85\text{ N})$
$= 35\text{ N}$

Therefore, $F_{net} = 35$ N, in the forward direction

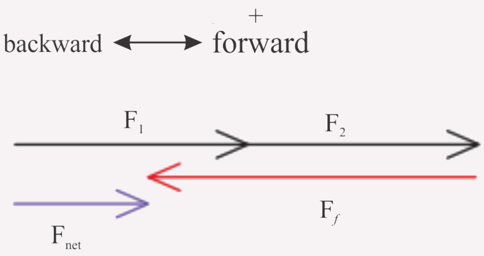


Figure 7. A simplified diagram where the forces are shown on separate lines. F_1, F_2 and F_f must be connected tip to tail to give the resultant F_{net}.

Examples

A child pulls on a rope attached to a sled with a force of 60 N. The rope makes an angle of 40° to the ground. (a) Compute the effective value of the pull tending to move the sled along the ground. (b) Compute the force tending to lift the sled vertically.

Solution

As shown in Figure 8, the x and y components of the force on the sled are $F_x = (60\text{ N})\cos 40^o = 46$ N, and $F_y = (60\text{ N})\sin 40^o = 39$ N, respectively.

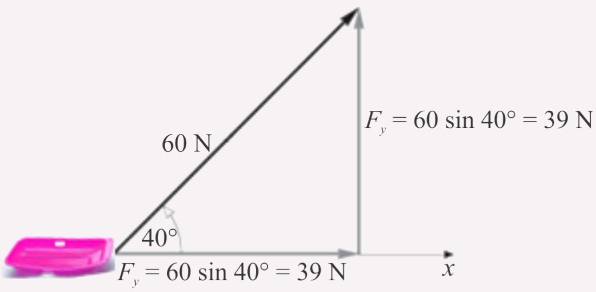


Figure 8.

(a) The force that tends to move the sled along the horizontal is the component of the force along the horizontal, $F_x = 46$ N.

(b) The force that tends to lift the sled vertically upward is the y component of the force $F_y = 39$ N

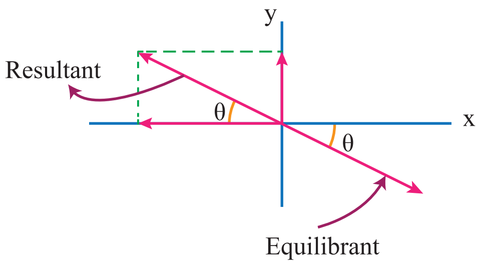


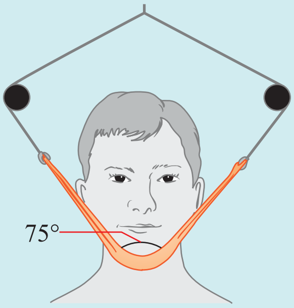
Figure 9. Equilibrant and resultant are equal and opposite

Equilibrant

We have discussed that the vector sum of two or more forces is the resultant and that the resultant can, in effect, replace the individual vectors. The equilibrant of a set of forces is the force needed to keep the system in equilibrium. It is equal and opposite to the resultant of the set of forces, Figure 9.

The x and y components of the equilibrant will be equal to the negative of the respective values of the resultant. $\Sigma F_x = 95.49$ N and $\Sigma F_y = -70.35$ N

ACTIVITY 1



Bio: Jaw Injury.

Due to a jaw injury, a patient must wear a strap Figure 10 that produces a net upward force of 5 N on his chin. The tension is the same throughout the strap. To what tension must the strap be adjusted to provide the necessary upward force?

Figure 10. Treatment of Jaw Injury where the concept of vectors apply

Experiment to determine Equilibrant

Title

Demonstration (gravity and equilibrium) experiment using Force Table

Aim

The objective of this experiment is to find the equilibrant of one or more known forces using a force table and compare the results to that obtained by analytical method.

Equipment

- Force table
- Ruler
- Strings
- Weight hangers
- Assorted weights
- Bubble level

Procedure

Given two force vectors you will determine the third force that will produce equilibrium in the system. This third force is known as the equilibrant and it will be equal and opposite to the resultant of the two known forces.

You will use a force table as shown in Figure 11, and work with force vectors. The force table is a circular platform mounted on a tripod stand. The three legs of the tripod have adjustable screws that can be used to level the circular platform.

The circular platform has angle markings, in degrees, on its surface. Two or more pulleys can be clamped at any location along the edge of the platform. In this lab we will use three pulleys. Three strings are attached to a central ring and then each string is passed over a pulley. Masses are added to the other end of the strings.

The hanging masses will produce a tension force in each string. The masses are directly proportional to the gravitational force (which you will learn about later in the course). The tension force in each string is equal to the gravitational force. For example, doubling the mass doubles the force, etc. When the forces are balanced, the ring will be positioned at the exact center of the table. When the forces are not balanced, the ring will rest against one side of the central post.

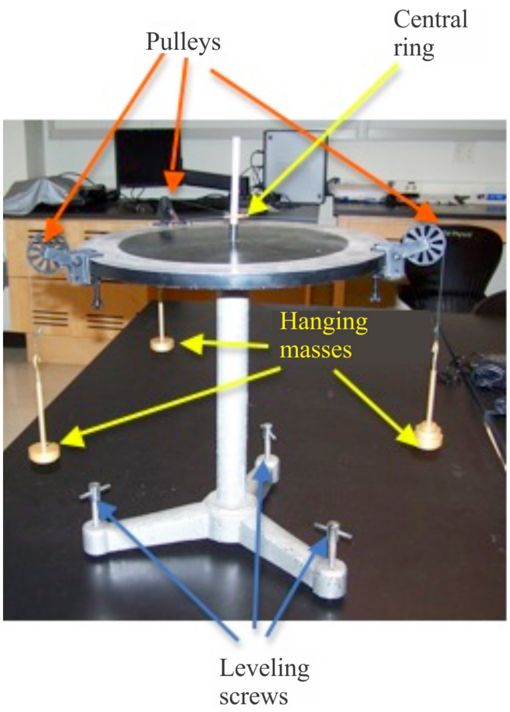


Figure 11. Force table

Note: The force due to each hanging mass will be mg where g is the acceleration due to gravity.

To make it easier to read the angles, assume the x-axis to be from the 180° mark to the 0° mark, with 0° being the positive x direction, and the y-axis to be from the 270° mark to the 90° mark with 90° being the positive y direction. See Figure 12.

Figure 12. Force table with axes

Procedure

1. Use the bubble level to check if the circular platform is horizontal. Use the leveling screws, if necessary, to make the necessary adjustments.
2. You are given two equal masses that are to be placed at 60° and 300°.

Remember that the weight hangers have their own masses and this needs to be included as part of the hanging mass.

You will determine the magnitude (in newton) and angle of the third force needed to balance the forces due to these two masses.

3. Represent these forces as vectors on the diagram in the worksheet. Be sure to include the axes.

   Each vector in the diagram should be drawn so that the larger the vector the bigger the force it represents.

4. Calculate the x and y components (to the nearest thousandth of a newton) and enter these values in a Data Table on the worksheet.
5. Find the x and y components of the resultant of the two vectors and enter these values in the Data Table 1 on the worksheet.
6. Now calculate the x and y components of the equilibrant of these two vectors and enter these values on the worksheet.
7. Calculate the magnitude and angle of the equilibrant. Enter these values on the worksheet. These are the calculated value of the third force.

   Use the equation A = for the magnitude of the vector

   $\theta = \cos^{-1}(Ax/A)$, $\theta = \sin^{-1}(Ay/A)$, $\theta = \tan^{-1}(Ay/Ax)$ for the direction of the vector.
8. Add this vector to your diagram to represent the third force.
9. Position the third string at the angle you determined in step 7 and hang the mass (including the hanger mass) corresponding to the calculated third force to represent the third force.
   Make adjustments (if needed) to the mass and the angle until the ring is at the center. Record this value on the worksheet.
10. Compare the calculated and experimental values for the third force by computing the percent difference between the two values.
11. Compare the calculated and experimental values of the angle for the third force by computing the percent difference between the two angle values.

Newton's first Laws of motion

In the absence of external forces, when viewed from an inertial reference frame, an object at rest remains at rest and an object in motion continues in motion with a constant velocity (that is, with a constant speed in a straight line). In simpler terms, we can say that when no force acts on an object, the acceleration of the object is zero. If nothing acts to change the object's motion, then its velocity does not change. From the first law, we conclude that any isolated object (one that does not interact with its environment) is either at rest or moving with constant velocity.

The tendency of an object to resist any attempt to change its velocity is called inertia. Mass is that property of an object that specifies how much resistance an object exhibits to changes in its velocity. Newton's first law of motion is also known as the law of inertia.

Newton's second law of motion

The acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass. Thus, we can relate mass, acceleration, and force through the following mathematical statement of Newton's second law:

$\Sigma F = ma$

The above equation is a vector expression and hence is equivalent to three component equations:

$\Sigma \mathbf{F}_x = m\mathbf{a}_x \qquad \Sigma \mathbf{F}_y = m\mathbf{a}_y \qquad \Sigma F_z = ma_z$

Newton's third law of motion

If two objects interact, the force F_{12} exerted by object 1 on object 2 is equal in magnitude and opposite in direction to the force F_{21} exerted by object 2 on object 1:

$\mathbf{F}_{12} = - \mathbf{F}_{21}$

The third law is equivalent to stating that forces always occur in pairs, or that a single isolated force cannot exist. The force that object 1 exerts on object 2 may be called the action force and the force of object 2 on object 1 the reaction force. In reality, either force can be labeled the action or reaction force. The action force is equal in magnitude to the reaction force and opposite in direction. In all cases, the action and reaction forces act on different objects and must be of the same type. Action and reaction forces do not cancel each other.

Exercises

1. A girl walked through the following displacements. Find the magnitude and direction of the resultant displacement: 20 m, due East, 30 m, due North-East, and 10 m, due West.
2. What is the result of applying an equilibrant force on the motion of a body that is acted up on by a number of forces?

2.2 Composition and Resolution of Forces

Adding Non-Collinear Forces

Worked Example 3 demonstrates how to find the net force on an object if the forces acting on it are neither parallel nor perpendicular. By observing the relationship between the components of the force vectors, you can greatly simplify the calculations.

We will often need to find the vector sum (resultant) of all forces acting on a body. We will use the Greek letter Σ (capital sigma, equivalent to the Roman S) as a shorthand notation for a sum. If the forces are labeled $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, and so on, we write $\mathbf{R} = \Sigma\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \ldots$

Component wise we write,

$\mathbf{R}_x = \Sigma F_x = F_{1x} + F_{2x} + F_{3x} + \ldots$

$\mathbf{R}_y = \Sigma F_y = F_{1y} + F_{2y} + F_{3y} + \ldots$

Each component may be positive or negative, so be careful with signs when you evaluate these sums.

Activity 2

Can a net force on an object ever equal zero? Explain using an example and a free-body diagram.

Once we obtain $\mathbf{R}_x$ and $\mathbf{R}_y$ we can find the magnitude and direction of the net force $\mathbf{R} = \Sigma\mathbf{F}$ acting on the body. The magnitude of $R = \sqrt{R_x^2 + R_y^2}$, and the angle θ between **R** and the x axis can be found from the relationship $\tan\theta = \frac{R_y}{R_x}$. The components $\mathbf{R}_x$ and $\mathbf{R}_y$ may be positive, negative, or zero, and the angle θ may be in any of the four quadrants.

In three-dimensional problems, forces may also have z-components; then we add the equation $\mathbf{R}_z = \Sigma\mathbf{F}_z = \mathbf{F}_{1z} + \mathbf{F}_{2z} + \mathbf{F}_{3z} + \ldots$

The magnitude of the net force is then

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

Isaac Newton English Physicist and Mathematician (1642–1727)

Newton was one of the most brilliant scientists in history. Before he was 30, he formulated the basic concepts and laws of mechanics, discovered the law of universal gravitation, and invented the mathematical methods of calculus. As a consequence of his theories, Newton was able to explain the motions of the planets, the ebb and flow of the tides, and many special features of the motions of the Moon and Earth. He also interpreted many fundamental observations concerning the nature of light. His contributions to physical theories dominated scientific thought for two centuries and remain important today.

Examples

As an introduction to dealing with force vectors, consider the four coplanar forces acting on a body at point O as shown in Figure 13a. (a) Find their resultant graphically, and (b) algebraically.

Solution

(a) Composition of vectors graphically involves joining the vectors tip to tail. Starting from O, the four vectors are plotted in turn as drawn in Figure 13b. Place the tail end of each vector at the tip end of the preceding one. The arrow from O to the tip of the last vector represents the resultant of the vectors.

Measure R from the scale drawing in Fig. 13b and find it to be 119 N. Angle α is measured by protractor and is found to be 37°. Hence, the resultant makes an angle θ = 180° – 37° = 143° with the positive x-axis. The resultant is 119 N at 143°.

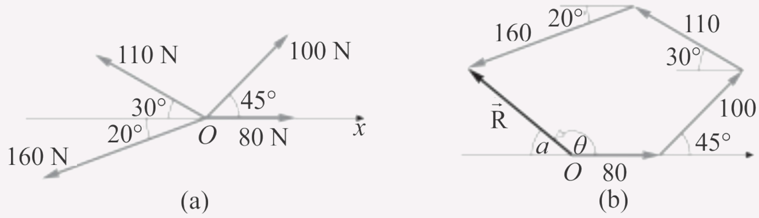


Figure 13.

(b) Component method involves finding the x and y components of each of the vectors (Table 2.1)

Table 1 The vectors are resolved into their components

	x component	y component
F_1 = 80 N	80 N	0
F_2 = 100 N	(100 N) cos45° = 70.07 N	(100 N) sin45° = 70.07 N
F_3 = 110 N	-(110 N) cos30° = -95.26 N	(110 N) sin30° = 55.00 N
F_4 = 160 N	-(160 N) cos20° = -150.3 N	-(160 N) sin20° = -54.72 N
	ΣF_x = -95.49 N	ΣF_y = 70.35 N

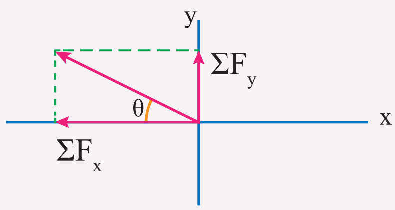


Figure 14. The x and y components are place on the coordinate axes

Place the components on the x and y axes, as in Figure 14.

$R_x = \Sigma F_x = -95.49$ N and $R_y = \Sigma F_y = 70.35$ N

The magnitude of

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(95.49\text{N})^2 + (70.35\text{N})^2} = 119\text{ N}$$

The angle between the vector and the negative x axis is $\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}(0.74) = 37^o$, above the negative x axis, hence 143^o with the positive x axis.

Examples

The following example demonstrates how free-body diagrams and the concept of net force apply to a stationary object. Stationary objects are examples of objects at equilibrium because the net force acting on them equals zero. A store sign that experiences a downward gravitational force of 245 N is suspended as shown in Figure 15a. Calculate the forces F_1 and F_2 exerted at the point at which the sign is suspended. $\theta_1 = 55^o$, $\theta_2 = 90^o$

Solution

Draw a free-body diagram for the sign as in Fig 15b. The forces are resolved into their components as in Table 2.2

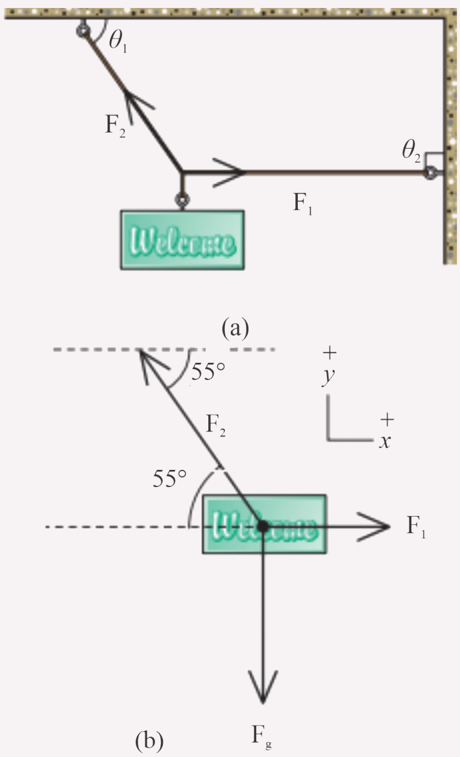


Figure 15.

Table 2

	x component	y component
F_1	$F_{1x} = F_1$	$F_{2x} = 0$
F_2	$F_{2x} = -F_2 \cos 55^o$	$F_{2y} = F_2 \sin 55^o$
F_g	$F_{gx} = 0$	$F_{gy} = F_g = -245$ N

Since the sign is in equilibrium, the net force in both x and y direction is zero.

$(F_{net})_x = (F_{net})_y = 0$

$(F_{net})_x = 0$ gives $F_1 - F_2 \cos 55^o = 0$

$(F_{net})_y = 0$ gives $0 + F_2 \sin 55^o - 245 \text{ N} = 0$

$$F_2 \sin 55^o = 245 \text{ N}$$

$$F_2 = 245 \frac{N}{F^2} \sin 55^o = 299 \text{ N}$$

Substituting $F_2 = 299$ N in $F_1 - F_2 \cos 55^o = 0$ and solving for F_1 we obtain $F_1 = F_2 \cos 55^o = 299 \times \cos 55^o = 172$ N

Activity 3

Refer to Figure 15 of worked example 4 above.

(a) As θ_1 decreases, what happens to F_1 and F_2?

(b) Explain why θ_1 can never be zero.

Exercises

1. If two forces act on an object, state the angle between these forces that will result in the net force given below. Explain using sketches.
   (a) maximum net force
   (b) minimum net force
2.
   (a) Explain what a force is, and state the SI unit of force.
   (b) Why is force a dynamics quantity and not a kinematics quantity?
3. When three force vectors are added graphically by joining tip to tail they form a triangle. What can you say about their resultant?
4. Any two forces that are equal in magnitude and opposite in direction are action and reaction. Is this statement correct? If not why?
5. Forces F_1 and F_2 act at a point. The magnitude of F_1 is 9 N, and its direction is 60° above the x-axis in the second quadrant. The magnitude of F_2 is 6 N, and its direction is 53° below the x-axis in the third quadrant.

(a) What are the x- and y-components of the resultant force?
(b) What is the magnitude of the resultant force?

2.3 Parallel Forces and Moments (Torque), Center of Gravity, Friction

Figure 16. A mechanic uses torque to tighten up or loosen the nut

Why are a door's hinges and its doorknob placed near opposite edges of the door? Imagine trying to rotate a door by applying a force of magnitude F perpendicular to the door surface but at various distances from the hinges. You will achieve a more rapid rate of rotation for the door by applying the force near the doorknob than by applying it near the hinges.

If you cannot loosen a stubborn bolt with a socket wrench, what would you do in an effort to loosen the bolt? You may intuitively try using a wrench with a longer handle or slip a pipe over the existing wrench to make it longer. This is similar to the situation with the door. You are more successful at causing a change in rotational motion (of the door or the bolt) by applying the force farther away from the rotation axis.

In this chapter we'll define a new physical quantity, torque, that describes the twisting or turning effort of a force. We'll find that the net torque acting on a rigid body determines its angular acceleration, in the same way that the net force on a body determines its linear acceleration. In order to describe the effects of a force on a body the magnitude and direction of the force are important, but so is the point on the body where the force is applied.

Key terms

- Torque: a measure of tuning effect of a force
- Moment arm (or lever arm): the perpendicular distance between the line of action of the force and the pivot.

Torque

When a force is exerted on a rigid object pivoted about an axis, the object tends to rotate about that axis. The tendency of a force to rotate an object about some axis is measured by a vector quantity called torque τ (Greek tau). Torque is a vector, and it is defined by the cross product

$$\tau = \mathbf{r} \times \mathbf{F}$$

$\tau = (rF\sin\theta)\,\hat{n}$, where $rF\sin\theta$ is the magnitude of τ and $\hat{n}$ is a unit vector along the direction of torque τ.

Consider the wrench pivoted on the axis through O in Figure 17.

The applied force F acts at an angle ϕ to the horizontal. We define the magnitude of the torque associated with the force F by the expression

$$\tau = Fr \sin\phi$$

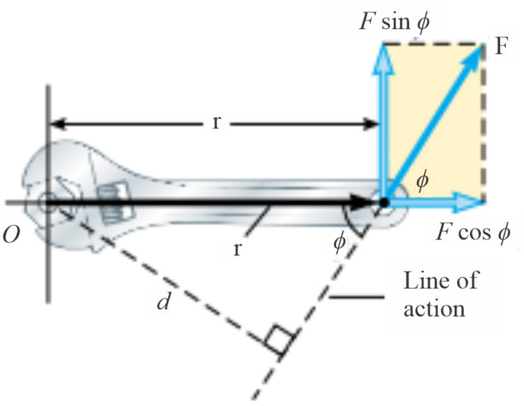


Figure 17.

From the right triangle in Figure 17 that has the wrench as its hypotenuse, we see that $d = r \sin \phi$, and we have $\tau = F d$, where r is the distance between the pivot point and the point of application of **F** and d is the perpendicular distance from the pivot point to the line of action of **F**. (The line of action of a force is an imaginary line extending out both ends of the vector representing the force. The quantity d is called the moment arm (or lever arm) of **F**.

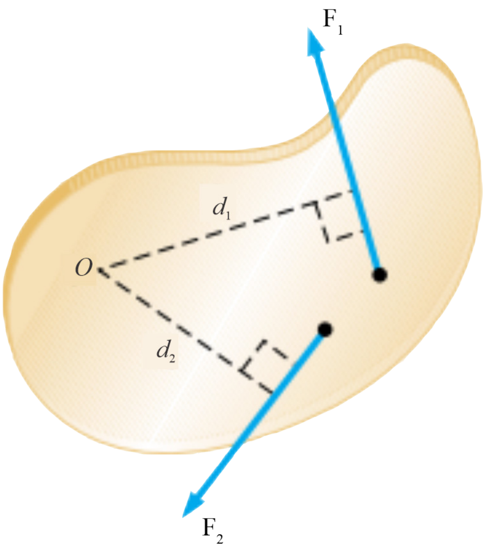


Figure 18.

The SI unit of torque is Newton-meter (Nm).

The torque can also be expressed in terms of the component of the force perpendicular to the line drawn from the pivot to the point of application of the force. In Figure 17, the only component of F that tends to cause rotation is F $\sin\phi$, the component perpendicular to a line drawn from the rotation axis to the point of application of the force. The horizontal component F $\cos\phi$, because its line of action passes through O, has no tendency to produce rotation about an axis passing through O. From the definition of torque, we see that the rotating tendency increases as F increases and as d increases.

ACTIVITY 4

Based on the expression used to calculate torque, $\tau = Fr \sin\phi$, explain the observation that

(a) it is easier to rotate a door if we push at the doorknob rather than at a point close to the hinge.

(b) We also want to apply our push as closely perpendicular to the door as we can. Pushing sideways on the doorknob will not cause the door to rotate.

If two or more forces are acting on a rigid object, as in Figure 18, each tends to produce rotation about the axis at O. In this example, $\mathbf{F}_2$ tends to rotate the object clockwise and $\mathbf{F}_1$ tends to rotate it counterclockwise.

We use the convention that the sign of the torque resulting from a force is negative if the turning tendency of the force is clockwise and is positive if the turning tendency is countErclockwise. For example, in Figure 18, the torque resulting from $\mathbf{F}_1$, which has a moment arm d_1, is positive and equal to $+F_1d_1$; the torque from F_2 is negative and equal to $-F_2d_2$. Hence, the net torque about O is

$\Sigma\tau = \tau + \tau = F_1d_1$, counterclockwise + F_2d_2, clockwise

$\Sigma\tau = F_1d_1 - F_2d_2$

Torque should not be confused with force. Forces can cause a change in linear motion, as described by Newton's second law. Forces can also cause a change in rotational motion, but the effectiveness of the forces in causing this change depends on both the forces and the moment arms of the forces, in the combination that we call torque. Torque has units of force times length, newton-meter in SI units, and should be reported in these units. Do not confuse torque and work, where work is given in joule while torque is given in newton-meter.

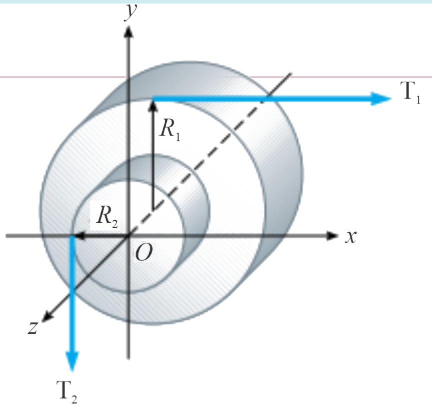
Activity 5

In small groups discuss the following

If you are trying to loosen a stubborn screw from a piece of wood with a screwdriver and fail, should you find a screwdriver for which the handle is (a) longer or (b) fatter? If you are trying to loosen a stubborn bolt from a piece of metal with a wrench and fail, should you find a wrench for which the handle is (a) longer (b) fatter?

Did you know?

Zero Torque: Zero net torque does not mean an absence of rotational motion. An object which is rotating at a constant angular speed can be under the influence of net torque of zero. This is analogous to the translational situation— zero net force does not mean an absence of translational motion.



a

Figure 19. A solid cylinder pivoted about the z axis through O. The moment arm of T_1 is R_1, and the moment arm of T_2 is R_2.

Examples

A one-piece cylinder is shaped as shown in Figure 2.19, with a core section protruding from the larger drum. The cylinder is free to rotate about the central axis shown in the drawing. A rope wrapped around the drum, which has radius R_1 = 1 m exerts a force $\mathbf{T}_1$ = 5 N to the right on the cylinder. A rope wrapped around the core, which has radius R_2 = 0.5 m exerts a force $\mathbf{T}_2$ = 19 N downward on the cylinder. What is the net torque about the rotation axis, and which way does the cylinder rotate starting from rest?

Solution

$\Sigma\tau$ = (19 N) (0.5 m), Counterclockwise + (5 N)(1 m), clockwise = 4.5 Nm, counterclockwise

Because this torque is positive, the cylinder will begin to rotate in the counterclockwise direction.

Activity 6

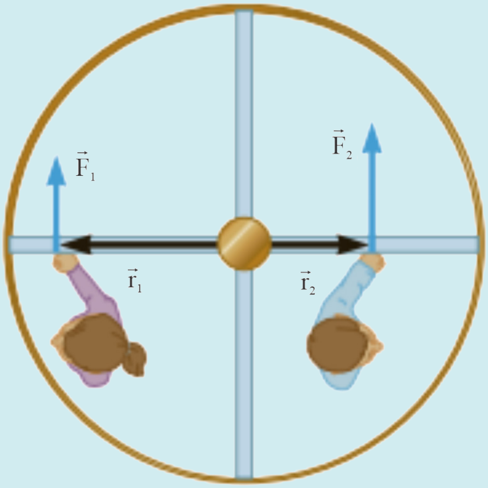


Figure 20. Battle of the Revolving Door

Apply the basic definition of torque.

Two disgruntled businesspeople are trying to use a revolving door, which is initially at rest (see Figure. 20.) The woman on the left exerts a force of 625 N perpendicular to the door and 1.2 m from the hub's center, while the man on the right exerts a force of 850 N perpendicular to the door and 0.8 m from the hub's center. Find the net torque on the revolving door.

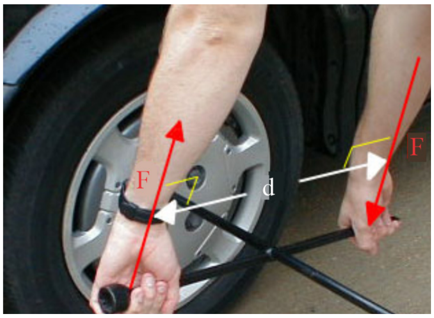


Figure 21. A mechanic applies equal and opposite forces at the opposite end of a 4-way wrench.

What are couples?

Two forces equal in magnitude and opposite in direction, acting on an object at two different points, along differnt lines of action form what is called a couple, Figure 21.

Application of Couple in Physics

There are different applications of couples. Some of them are:

(a) Steering wheel applied by the car driver
(b) Opening and closing of a water tap
(c) Winding the spring of an alarm clock
(d) Unlocking the locker by using a key
(e) Opening and closing of a cap of a water bottle, or jug.
(f) Turning of a screwdriver

Characteristics of couples

The couple does not induce translational motion because the two forces that make up the couple are equal and opposing. When it is applied to a body, the net resultant force is zero. Because the algebraic sum of the moments of the two forces around any point in their plane is not zero, a net torque results and it causes pure rotational motion in the body. The size and direction of a couple's moment about any point on its plane are both constant. The net torque is equal to the product of one of the forces and the distance between the lines of action of the forces.

$\tau_{net} = Fd$

Relationship between Torque and Angular Acceleration

Newton's second law of motion explains that a net force on an object causes an acceleration of the object and that the acceleration is proportional to the net force. In this section we see the rotational analog of Newton's second law—the angular acceleration of a rigid object rotating about a fixed axis is proportional to the net torque acting about that axis. Before discussing the more complex case of rigid-object rotation, however, it is instructive first to discuss the case of a particle moving in a circular path about some fixed point under the influence of an external force, Figure 22.

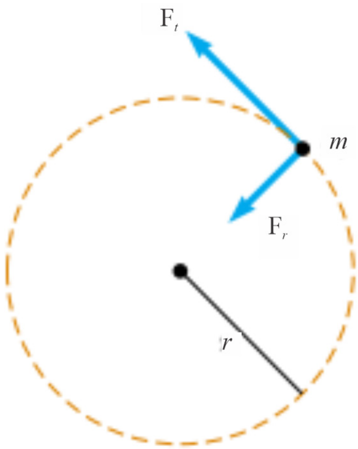


Figure 22. A particle rotating in circle under the influence of a tangential force F_t.

Consider a particle of mass m rotating in a circle of radius r under the influence of a tangential force $\mathbf{F}_t$ and a radial force $\mathbf{F}_r$, as shown in Figure 22. A force $\mathbf{F}_r$ in the radial direction must be present to maintain the circular motion.

The tangential force provides a tangential acceleration $\mathbf{a}_t$, and

$\mathbf{F}_t = m\mathbf{a}_t$

The magnitude of the torque about the center of the circle due to $\mathbf{F}_t$ is

$\tau = F_t r = (ma_t)r$

Because the tangential acceleration is related to the angular acceleration through the relationship $a_t = \alpha r$, the torque can be expressed as

$\tau = (m\alpha r)r = (mr^2)\alpha$

The quantity represented by mr^2 is the moment of inertia of the particle about the z axis passing through the origin, so that

$\tau = I\alpha$

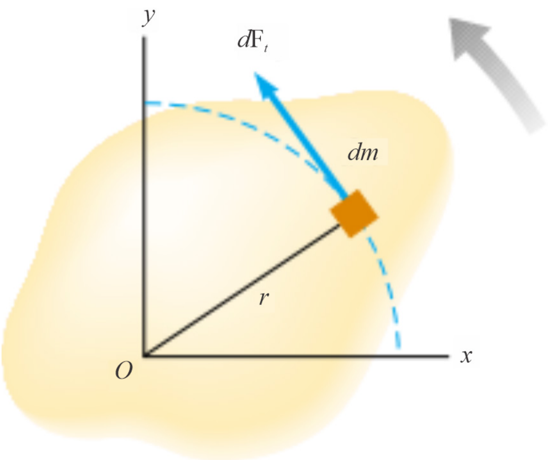


Figure 23. A rigid object rotating about an axis through O.

That is, the torque acting on the particle is proportional to its angular acceleration, and the proportionality constant is the moment of inertia. Note that $\tau = I\alpha$ is the rotational analog of Newton's second law of motion, F = ma.

Moment of inertia is a measure of inertia of a body in rotational motion. It depends not only on the mass but also on the distribution of the mass from the axis of rotation.

When the above discussion is extended to the rotation of a rigid body, Figure 23, the body can be regarded as an infinite number of mass elements dm of infinitesimal size. Each mass element rotates in a circle about the origin, and each has a tangential acceleration a_t produced by an external tangential force dF_t. And also each mass element rotates about O with the same angular acceleration α, and the net torque on the object is proportional to α.

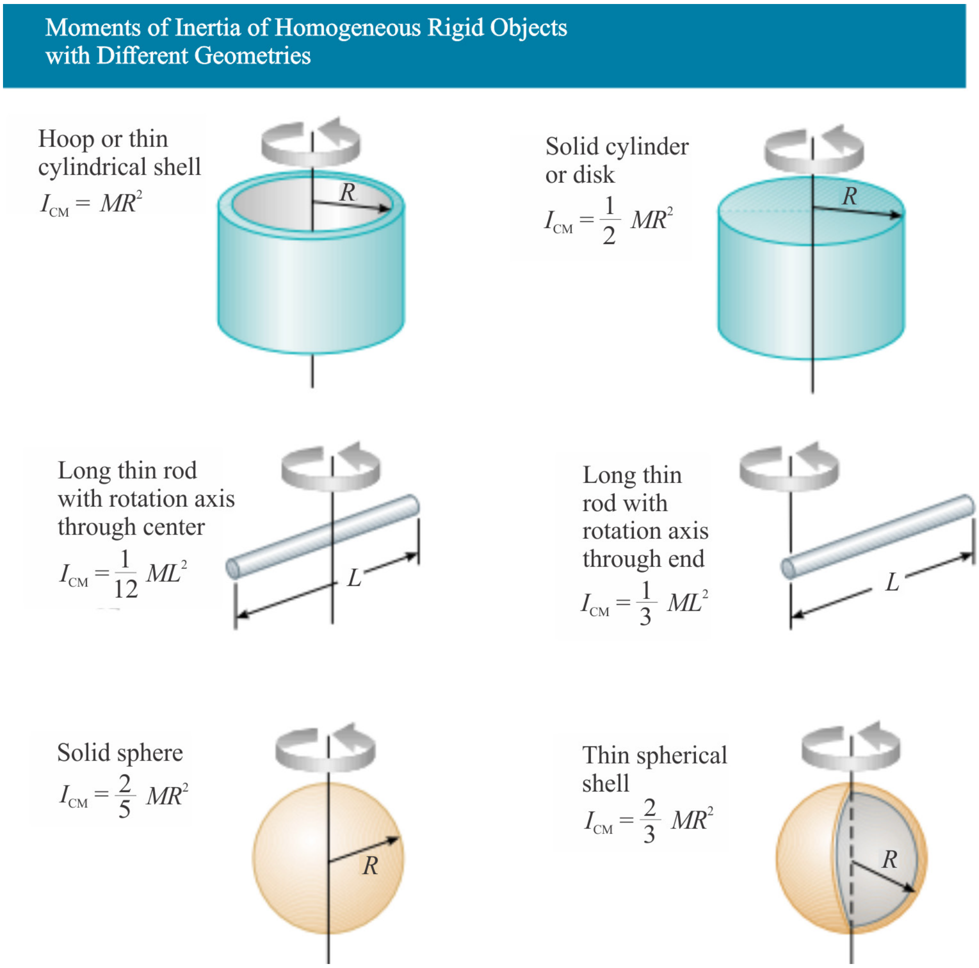
Key terms

- Angular acceleration: the rate of change of angular velocity
- Moment of inertia: Inertia of a body in rotational motion.

Although each mass element of the rigid object may have a different linear acceleration a_t, they all have the same angular acceleration α. The net torque Στ about O due to the external forces can be written as

$$\Sigma\tau = I\alpha$$

Table 3 **Moments of inertia of objects about specific axes.**



Examples

A flywheel of moment of inertia 4.5 kgm^2 is rotating with an angular velocity of 4.8 rad/s. If it accelerates uniformly to 10.5 rad/s over 3 s,

(a) what is the angular acceleration of the flywheel?

(b) what is the unbalanced torque causing the acceleration?

Solution

Given

Initial angular velocity ω_i = 4.8 rad/s, final angular velocity ω_f = 10.5 rad/s, time t = 3 s, moment of inertia I = 4.5 kgm^2

(a) Angular acceleration $\alpha = \frac{\omega_f - \omega_i}{t} = \frac{10.5 \text{ rad/s-4.8 rad/s}}{3s} = \frac{1.9 \text{ rad}}{s^2}$

(b) Unbalanced torque $\tau = I\,\alpha = 4.5 \text{ kgm}^2 \text{ x } \frac{1.9 \text{ rad}}{s^2} = 8.6 \text{ Nm}$

Examples

A 1 kg solid sphere of radius 50 cm rotates about an axis through its center. If it changes is speed from rest to 25 rev/min in 10 s, find the torque applied on it. Take moment of inertia of a solid sphere to be $I = \frac{2}{5} mR^2$.

Solution

Given

Mass m = 1 kg, Initial angular velocity $\omega_i = 0$, final angular velocity $\omega_f = 25$ rev/min $= 2.6$ rad/s, time t = 10 s, moment of inertia $I = \frac{2}{5} mR^2 = \frac{2}{5}(1 \text{ kg})\,(0.5 \text{ m})^2 = 0.1 \text{ kgm}^2$

The angular acceleration $\alpha = \frac{\omega_f - \omega_i}{t} = \frac{26 \text{ rad/s-orad/s}}{10s} = 2.6 \text{ rad/s}^2$

The torque $\tau = I\,\alpha = 0.1 \text{ kgm}^2 \text{ x } 2.6 \text{ rad/s}^2 = 2.6 \text{ Nm}$

Equilibrium

A body is said to be in equilibrium if it does not change the state of rest or motion, even under the influence of external forces.

Types of equilibrium

1. Stable equilibrium
2. Unstable equilibrium
3. Neutral equilibrium

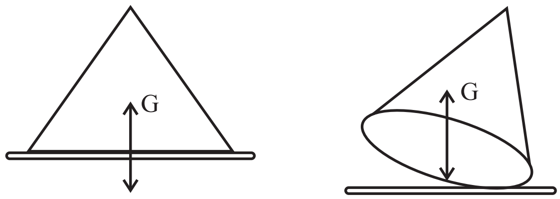
1. Stable equilibrium

A body is said to be in stable equilibrium if it regains its original position after removal of external force after being slightly disturbed by it. In stable equilibrium, the line joining the point where entire weight of the body supposed to be concentrated i.e. centre of gravity and centre of earth must fall within the base of body.

Examples

1. A cone resting on its base

2. A book lying on a flat surface



From above figure, line joining the centre of gravity and centre of earth falls within the base of body, even after being lightly distubred by it. So the cone is stable equilibrium.

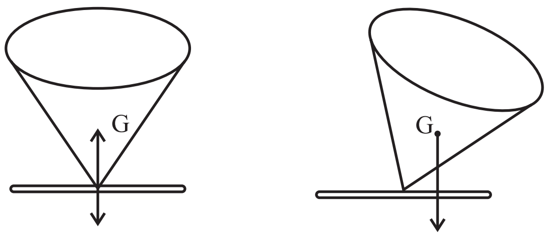
2. Unstable equilibrium

A body is said to be in unstable equilibrium if it does not regains its original position after being slightly distubed by an enternal force. The line joining the centre of gravity and centre of earth falls outside the base, after being slightly disturbed by external force.

Examples

1. A cone resting on its apex.

2. A bottle standing on the edge of its mouth.



Here the base of body is small and the top portion is heavier as it raises the height of the centre of gravity from its base.

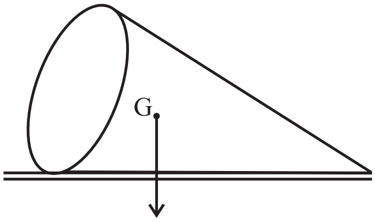
3. Neutral equilibrium

A body is said to be in neutral equilibrium, when it moves to a new place on the application of an external force and on the removal of external force the body may or may not come back to its original position but the height of its centre of gravity from a reference surface remains the same.

Examples

1. A rolling ball

2. A cone resting on its side.



Here the line joining the centre of gravity and the centre of earth falls within the base of the body on application of an external force and height of centre of gravity does not change on the removal of external force.

Condition of equilibrium of a body acted upon by a number of parallel forces is,

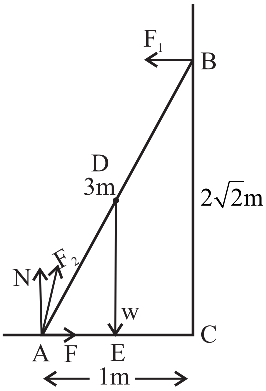
1. The Algebric sum of the forces must be zero i.e., sum of the forces acting in the upward direction must be equal to the sum of the forces acting in the downward direction.

$$\Sigma F = 0$$

2. The algebric sum of the moments of the forces about any point in the plane must be zero i.e. sum of moments of the forces about a point in the anticlockwise direction is equal to the sum of moments of the forces about the same point in the clockwise direction.

Examples

A 3m long ladder weighing 20kg leaves on a frictionless wall. Its feet rest on the floor 1m from the wall as shown in the figure. Find the reaction forces of the wall and the floor. ($g = 9.8\ m/s^2$)



Solution

From the figure, $BC = 2\sqrt{2}$ m

Forces on the ladder are:

1. Weight W acting at centre of gravity D.

2. Reaction forces F_1 and F_2 of the wall and floor. F_1 is perpendicular to the wall and F_2 can be resolved into the components N and F.

F prevents the ladder from sliding away from the wall & developed towards wall.

For equilibrium, taking the forces in vertical directions,

$N - W = 0$ (1)

horizontal direction,

$F - F_1 = 0$ (2)

Taking the moments,

$2\sqrt{2}\ F_1 - \left(\frac{1}{2}\right) w = o$ (3)

Now W = 20 kg = 20 × 9.8 = 196.0 N.

From (1) N = 196 N.

From (3) $F_1 = \frac{W}{4\sqrt{2}} = \frac{196}{4\sqrt{2}} = 34.6$ N

From (2) F = F1 = 34.6 N

$F_2 = \sqrt{F^2 + N^2} = 199$ N

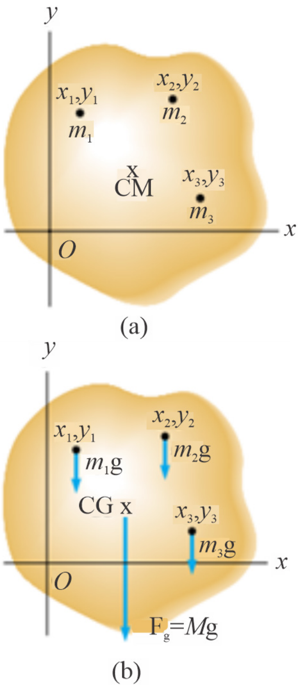
The force makes F_2 moves an angle α with the horizontal.

$\tan\alpha = \frac{N}{F} = 4\sqrt{2}$

Center of gravity

In most equilibrium problems, one of the forces acting on the body is its weight. We need to be able to calculate the torque of this force. The weight doesn't act at a single point; it is distributed over the entire body. But we can always calculate the torque due to the body's weight by assuming that the entire force of gravity (weight) is concentrated at a point called the center of gravity (abbreviated "cg"), Figure 24a. The acceleration due to gravity decreases with altitude; but if we can ignore this variation over the vertical dimension of the body, then the body's center of gravity is identical to its center of mass (abbreviated "cm"), Fig 24b.

Finding the center of mass of a system of masses m_1, m_2, m_3, … at points with (x_1, y_1, z_1), (x_1, y_1, z_1), (x_1, y_1, z_1), …, the coordinates of the center of mass x_{cm}, y_{cm}, and z_{cm} of the center of mass of the collection on masses are, Figure 24a.



$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \ldots}{m_1 + m_2 + m_3 + \ldots} = \frac{\sum m_i x_i}{\sum m_i}$$

$$y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + \ldots}{m_1 + m_2 + m_3 + \ldots} = \frac{\sum m_i y_i}{\sum m_i}$$

$$z_{cm} = \frac{m_1z_1 + m_2z_2 + m_3z_3 + \ldots}{m_1 + m_2 + m_3 + \ldots} = \frac{\sum m_i z_i}{\sum m_i}$$

The center of gravity, or the center of mass for homogeneous, symmetric objects such as sphere, rod of uniform thickness, coincides with their geometric center.

Also, x_{cm}, y_{cm}, and z_{cm} are the components of the position vector r_{cm} of the center of mass of the system of masses are given by vector equation

$$\boldsymbol{r}_{cm} = \frac{m_1\boldsymbol{r}_1 + m_2\boldsymbol{r}_2 + m_3\boldsymbol{r}_3 + \ldots}{m_1 + m_2 + m_3 + \ldots} = \frac{\sum m_i \boldsymbol{r}_i}{\sum m_i}$$

Figure 24. a) An object can be divided into many small particles each having a specific mass and specific coordinates. These particles can be used to locate the center of mass. b) The center of gravity of an object is located at the center of mass if g is constant over the object.

Did you know?

The Earth and the Moon have a center of mass that lies on a straight line connecting them. It is between them but closer to the Earth than the Moon.

Examples

Consider the following mass distribution: 5 kg at (0, 0) m, 3 kg at (0, 4) m, and 4 kg at (3, 0) m.

Where is the center of mass of the system of masses?

Solution

Given three masses of 5 kg at (0, 0) m, 3 kg at (0, 4) m, and 4 kg at (3, 0) m, the x and y coordinates of the center of mass are

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{(5kg)(0m) + (3kg)(0m) + (4kg)(3m)}{5kg + 3kg + 4kg} = 1m$$

$$y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} = \frac{(5kg)(0m) + (3kg)(4m) + (4kg)(0m)}{5kg + 3kg + 4kg} = 1m$$

The center of mass is at (1,1)m

Center of Gravity Experiment

Aim: Finding the center of mass of a cardboard (Plumb line method)

Theory: The weight of an object is concentrated at the center of gravity. The term center of gravity is used interchangeably with center of mass. For a symmetrical object the center of mass is located at the geometric center of the object. If the object is not symmetrical we can determine the center of mass using the method below.

Materials:

Cardboard

Weight (washer, bolt, or fishing weight)

Hole punch

Nail

Pencil

Procedure:

Cut the cardboard into a strange shape. Do not use a circle, square, rectangle, or any other common geometric shape.

Punch a hole near the edge of the cut-out cardboard piece and hang it from a nail.

Place the weight, a washer or bolt, on a thread and tie it off.

Hang the thread and weight from the nail in front of the cardboard.

Use a pencil to draw a plumb line down the cardboard where the thread touches. This marks your center line from that hanging point.

Repeat step 2 and 5 in two other places on the cardboard.

When a body in rotational equilibrium and acted on by gravity is supported or suspended at a single point, the center of gravity is always at or directly above or below the point of suspension. If it were anywhere else, the weight would have a torque with respect to the point of suspension, and the body could not be in rotational equilibrium. Figure 25 shows an application of this idea.

A body supported at several points must have its center of gravity somewhere within the area bounded by the supports. This explains why a car can drive on a straight but slanted road if the slant angle is relatively small (Fig. 25a) but will tip over if the angle is too steep (Fig. 25b). The truck in Fig. 25c has a higher center of gravity than the car and will tip over on a shallower incline.

ACTIVITY 7

Discuss in groups

Based on the concept of equilibrium and how it is affected by the position of center of gravity and the way an object is supported what will happen to the vehicles shown in Figure 25?

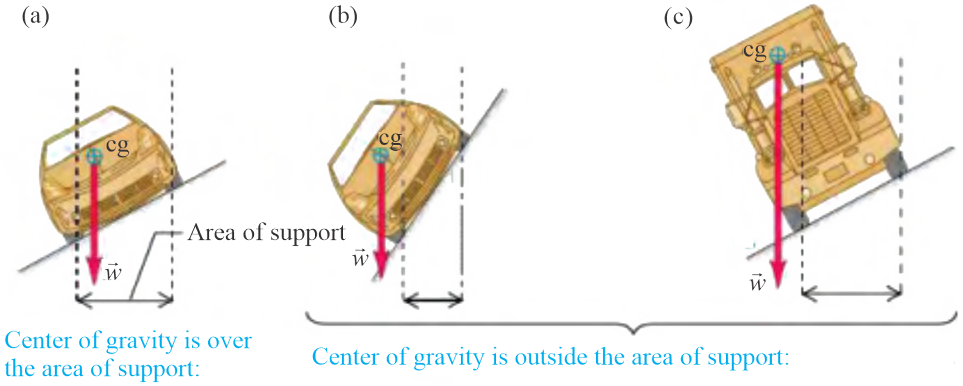


Figure 25.

Stand with your back and feet against a wall. Have someone place a book on the floor at your feet. Try to pick it up. Can you do it? Discuss your observation with a friend.

ACTIVITY 8

Stand against a wall sideways with your arm and leg touching the wall, with nothing to hold onto. Try to lift your other leg straight out away from the wall. Will you be able to do it?

Forces of Friction

When an object is in motion either on a surface or in a viscous medium such as air or water, there is resistance to the motion because the object interacts with its surroundings. We call such resistance a force of friction. Forces of friction are very important in our everyday lives. They allow us to walk or run and are necessary for the motion of wheeled vehicles.

Imagine that you are working in your garden and have filled a trash can with yard clippings. You then try to drag the trash can across the surface of your concrete patio, as in Figure 26. This is a real surface, not an idealized, frictionless surface. If we apply an external horizontal force F to the trash can, acting to the right, the trash can remains stationary if F is small. The force that counteracts F and keeps the trash can from moving acts to the left and is called the force of static friction f_s. As long as the trash can is not moving, $f_s = F$. Thus, if F is increased, f_s also increases. Likewise, if F decreases, f_s also decreases.

Figure 26. When a person falls in midair, the air resistance that acts on a parachute slows the fall. In this case, friction allows a parachutist to land without injury

Experiments show that the friction force arises from the nature of the two surfaces: because of their roughness, contact is made only at a few locations where peaks of the material touch, as shown in the magnified view of the surface in Figure 27 (a).

At these locations, the friction force arises in part because one peak physically blocks the motion of a peak from the opposing surface. Although the details of friction are quite complex at the atomic level, this force ultimately involves an electrical interaction between atoms or molecules.

If we increase the magnitude of F, as in Figure 27 (b), the trash can eventually slips. When the trash can is on the verge of slipping, f_s has its maximum value $f_{s,max}$, as shown in Figure 27 (c).

When F exceeds $f_{s,max}$, the trash can moves and accelerates to the right. When the trash can is in motion, the friction force is less than $f_{s,max}$ (Figure. 27 (c). We call the friction force for an object in motion the force of kinetic friction f_k.

Experimentally, we find that, to a good approximation, both $f_{s,max}$ and f_k are proportional to the magnitude of the normal force, F_N.

The following empirical laws of friction summarize the experimental observations:

- The magnitude of the force of static friction between any two surfaces in contact can have the values

  $f_s \leq, \mu_s F_N$,

where the dimensionless constant μ_s is called the coefficient of static friction and F_N is the magnitude of the normal force exerted by one surface on the other.

- The magnitude of the force of kinetic friction acting between two surfaces is

  $F_f = \mu_k F_N$

  where μ_k is the coefficient of kinetic friction. Although the coefficient of kinetic friction can vary with speed, we shall usually neglect any such variations for low speeds.

The values of μ_k and f_s depend on the nature of the surfaces, but μ_k is generally less than μ_s. Typical values range from around 0.03 to 1.

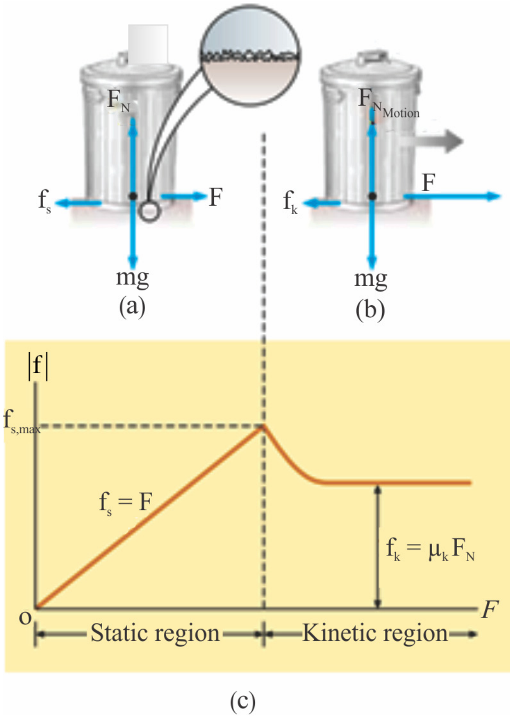


Figure 27. The direction of the force of friction f between a trash can and a rough surface is opposite the direction of the applied force F.

- The direction of the friction force on an object is parallel to the surface with which the object is in contact and opposite to the actual motion (kinetic friction) or the impending motion (static friction) of the object relative to the surface.
- The coefficients of friction are nearly independent of the area of contact between the surfaces.

ACTIVITY 9

Discuss in groups

The equality in $f_s \leq \mu_s F_N$ holds when the surfaces are on the verge of slipping, that is, when $f_s = f_{s,max} = f_s F_N$. This situation is called impending motion. The inequality holds when the surfaces are not on the verge of slipping.

Experimental Determination of μ_s and μ_k

The following is a simple method of measuring coefficients of friction: Suppose a block is placed on a rough surface inclined relative to the horizontal, as shown in Figure 28. The incline angle is increased until the block starts to move. Show that by measuring the critical angle θ_c at which this slipping just occurs, we can obtain μ_s.

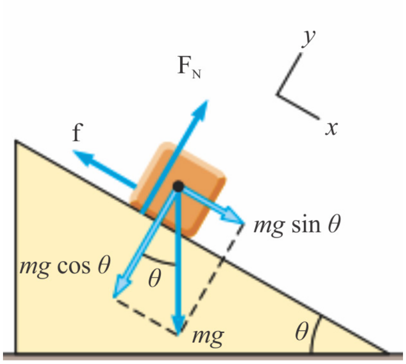


Figure 28.

Note that the only forces acting on the block are the gravitational force mg, the normal force F_N, and the force of static friction f_s. These forces balance when the block is not moving. When we choose x to be parallel to the plane and y perpendicular to it, Newton's second law applied to the block for this balanced situation gives

$\Sigma F_x = mg \sin\theta - f = ma_x = 0$, from which $mg \sin\theta = f$

Static friction force $f_s = \mu_s F_N$, we write $mg \sin\theta = \mu_s F_N$

From $\Sigma F_y = F_N - mg \cos\theta = ma_y = 0$ we get $F_N = \mu_s mg \cos\theta$

Using $F_N = \mu_s mg \cos\theta$ in $mg \sin\theta = \mu_s F_N$

$$mg \sin\theta = \mu_s mg \cos\theta$$

Rearranging this we get $\mu_s = \tan\theta$

To finalize the problem, note that once the block starts to move at $\theta \geq \theta_c$, it accelerates down the incline and the force of friction is $f_k = \mu_k F_N$.

ACTIVITY 10

In the arrangement of the experimental determination of μ_s (Figure 28) the critical angle θ_c is found to be 20^o. What is (a) the coefficient of static friction between the block and the inclined surface? (b) the normal force? (c) the force that tends to move the block down the plane? (d) the static friction force just before the block starts to slide?

Advantages and disadvantages of friction

Advantages

- Friction helps us to walk, turn and stop. Without friction, it is difficult to walk.
- Friction helps to wear cap, ring, and belt. Without friction, our hats, ring on our fingers, etc., will slide off.
- Friction helps in the transfer of energy. In mechanical engineering, the friction between parts in a machine helps in the transfer of energy.
- Friction helps in making fire. The rubbing of two bodies produces heat

due to friction.

- Friction helps to hold onto objects. The friction between our hands and the object that we are holding onto helps us to grip the object.

Disadvantages

- Produces unwanted heat.
- Reduces efficiency of machines.
- Eco-hazard.
- Friction causes a reduction in efficiency and makes us consume more fuel. This additional fuel consumption causes environmental hazards.
- Produces unwanted noise. Friction is always characterized by heat and noise.
- Causes wear and tear. Friction always causes wear and tear.

Methods of reducing friction

Wear and tear due to friction depends on two factors the roughness of two surfaces its content and teh amount of time the two surfaces rub against each other. Frictional force can be reduced in the following ways :

1. **Use of Lubricants:** Friction can be reduced by applying lubricatns between the contact surfaces to fill the fine pores or depressions in the surfaces and make them smooth thereby reducing friction.
2. **Polishing:** Unevenness of the surfaces can be reduced by polishing, thereby reducing the friction.
3. **Use of ball bearings:** In rotating machines, shafts are mounted on ball bearings. By doing so, rolling friction occurs lesser than sliding friction, thereby reducing the friction.
4. **By streamlining:** Air friction is reduced by designing streamline bodies of cars and aeroplanes. Similarly, if the bodies of boats and ships are streamlines, friction of water can be reduced.

Did you know?

Rollers are extremely simple and a basic part of many complex machines. They were used by the ancient Egyptians in making their pyramids.

Examples

A block lies on a horizontal floor. (a) What is the magnitude of the frictional force on it from the floor? (b) If a horizontal force of 5 N is now applied to the block, but the block does not move, what is the magnitude of the frictional force on it? (c) If the maximum value $f_{s,max}$ of the static frictional force on the block is 10 N, will the block move if the magnitude of the horizontally applied force is 8 N? (d) If it is 12 N? (e) What is the magnitude of the frictional force in part (c)?

Solution

(a) No force is acting on the block and it does not slide or try to slide over the surface. There is no friction.

(b) If a 5 N force acting in the block does not move the block, the static friction must be 5 N.

(c) Since the maximum static friction force if 10 N, an 8 N force cannot move the block.

(d) A 12 N force exceeds the $f_{s,max}$ and the block will slide.

(e) For an applied force less than $f_{s,max}$ =10 N, the block remains at rest and the F_f will be equal to the applied force, F_f = 8 N

Exercises

1. The center of gravity of an object may be located outside the object. Give a few examples for which this is the case.
2. Why is it easier to hold a 10-kg dumbbell in your hand at your side than it is to hold it with your arm extended horizontally?
3. Consider the following mass distribution: 5.00 kg at (0, 0) m, 3 kg at (0, 4) m, and 4 kg at (3, 0) m. Where should a fourth object of mass 8 kg be placed so that the center of gravity of the four-object arrangement will be at (0, 0)?
4. A 5 kg block is placed on a horizontal rough surface. The block just begins to slide when a horizontal force of 7.5 N applied on it. What is the coefficient of static friction between the block and the surface?

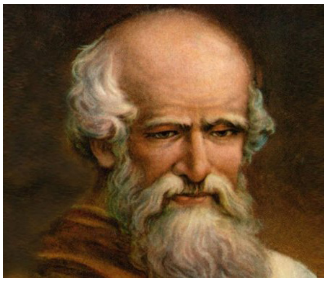
Equilibrium of bodies in liquids

Fluids play a vital role in many aspects of everyday life. We drink them, breathe them, swim in them. They circulate through our bodies and control our weather. The physics of fluids is therefore crucial to our understanding of both nature and technology. In this section we will study fluid statics, the study of fluids at rest in equilibrium situations. Like other equilibrium situations, it is based on Newton’s first and third laws.

A fluid is any substance that can flow and change the shape of the volume that it occupies. (By contrast, a solid tends to maintain its shape.) We use the term “fluid” for both gases and liquids. The key difference between them is that a liquid has cohesion, while a gas does not. The molecules in a liquid are close to one another, so they can exert attractive forces on each other and thus tend to stay together (that is, to cohere). The molecules of a gas, by contrast, are separated on average by distances far larger than the size of a molecule. Hence the forces between molecules are weak, there is little or no cohesion, and a gas can easily change in volume.

Buoyant Forces and Archimedes’ Principle

A fundamental principle affecting objects submerged in fluids was discovered by Greek mathematician and natural philosopher Archimedes. Archimedes’ principle can be stated as follows:



Archimedes

(287 BC - 212 BC)

Any object completely or partially submerged in a fluid is buoyed up by a force with magnitude equal to the weight of the fluid displaced by the object.

The up thrust is known as Buoyant force (F_B).

F_B = Weight of the displaced fluid

$F_B = M_f \times g$

$F_B = \rho_f V_f g$

Did you know?

Archimedes was an ancient Greek mathematician, physicist, astronomer, philosopher and engineer. He is considered the first and greatest scientist of the ancient world. He laid the foundations of hydrostatics and mechanics. According to legend, King Hieron asked him to determine whether the king’s crown was pure gold or a gold alloy. Archimedes allegedly arrived at a solution when bathing, noticing a partial loss of weight on lowering himself into the water. He was so excited that he reportedly ran naked through the streets of Syracuse shouting “Eureka!” which is Greek for “I have found it!” Everyone has experienced Archimedes’ principle. It’s relatively easy, for example, to lift someone if you’re both standing in a swimming pool, whereas lifting that same individual on dry land may be a difficult task.

Buoyant force (F_B) is equal to the difference between the weight of the object in air (W_a) and its weight in the fluid (W_f).

$$FB = Wa - Wf$$

Activity 11

Discuss the following:

The buoyant force on an object is exerted by the fluid and is the same, regardless of the density of the object.

It's instructive to compare the forces on a totally submerged object with those on a floating object.

Case I: A Totally Submerged object. When an object is totally submerged in a fluid of density ρ_f, the upward buoyant force acting on the object has a magnitude of $F_B = \rho_f V_o g$, where V_o is the volume of the object. If the object has density ρ_o, the downward gravitational force acting on the object has a magnitude equal to $w = mg = \rho_o V_o g$, and the net force on it is $F_B - w = (\rho_f - \rho_o)V_o g$. Therefore, if the density of the object is less than the density of the fluid, the net force exerted on the object is positive (upward) and the object accelerates upward, as in Figures 29 a. If the density of the object is greater than the density of the fluid, as in Figure 29 b, the net force is negative and the object accelerates downward.

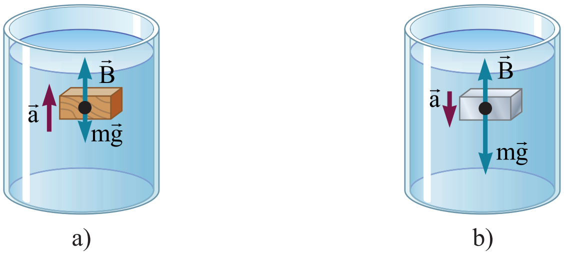
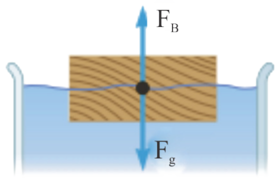


Figure 29. a) A totally submerged object that is less dense than the fluid in which it is submerged is acted upon by a net upward force. b) A totally submerged object that is denser than the fluid sinks.

Case II: A Floating object. Now consider a partially submerged object in static equilibrium floating in a fluid, as in Figure 30a. In this case, the upward buoyant force is balanced by the downward force of gravity acting on the object (Figure 30 b). If V_f is the volume of the fluid displaced by the object (which corresponds to the volume of the part of the object beneath the fluid level), then the magnitude of the buoyant force is given by $F_B = \rho_f V_f g$. Because the weight of the object is $w = mg = \rho_o V_o g$, and because $w = F_B$, it follows that

$$\rho_f V_f g = \rho_o V_o g, \text{ or } \frac{\rho_o}{\rho_f} = \frac{V_f}{V_o}$$

a)



b)

Figure 30. a) An iceberg floating in water. Most of the volume of the iceberg is beneath the water. b) An object floating on the surface of a fluid is acted upon by two forces: the gravitational force F_g and the buoyant force F_B.

Activity 12

Can you determine what fraction of the total volume of an iceberg of figure 30a is under sea water? Density of ice is 917 kg/m^3 and sea water has density 1 025 kg/m^3.

Examples

An object weighs 20 N in air and 12 N when completely immersed in water of density 1000 kg/m^3. Find (a) the volume of the body, and (b) its density.

Solution

Given: $W_a = 20$ N, $W_w = 12$ N, $\rho_w = 1000$ kg/m^3

(a) According to Archimedes' $F_B = \rho_w V_w g = 1000 \text{ kg/m}^3 \text{ x } V_w \text{ x } 10 \text{ m/s}^2$

And also $F_B = W_a - W_w = 20 \text{ N} - 12 \text{ N} = 8\text{N}$

$1000 \text{ kg/m}^3 \times V_w \text{ x } 10 \text{ m/s}^2 = 8 \text{ N}$

$V_w = 8 \text{ x } 10^{-4} \text{ m}^3$

As the object is completely submerged, we have $V_o = V_w = 8 \text{ x } 10^{-4} \text{ m}^3$

(b) $\rho_o \dfrac{M_o}{V_o}$, $Mo = \dfrac{W_a}{g} = \dfrac{20\text{N}}{10\text{m/s}^2} = 2\text{kg}$

$\rho_o = \dfrac{2\text{kg}}{8\times10^{-4}\text{m}^3} = 2500 \text{ kg/m}^3$

Examples

A certain block floats on water with only one-fourth of its volume above the surface of water. What is (a) the buoyant force on the block, and (b) density of the block?

Solution

Applying the Archimedes' principle for the case of flotation, we write

$$\rho_f V_f\, g = \rho_o V_o g,$$

$$\frac{\rho_o}{\rho_f} = \frac{V_f}{V_o}$$

As one fourth of the volume is above we see that three fourth of the volume is below the surface. $V_w = \frac{3}{4} V_o$.

$$\rho_o =_w \frac{V_w}{V_o} = (1000\ \text{kg/m}^3)\frac{(3/4)V_o}{V_o} = 750\ \text{kg/m}^3$$

Summary

- A body is in equilibrium when the vector sum of all the forces (resultant force) acting on it is zero.
- Equilibrant has the same magnitude but opposite direction to the resultant.
- A force is a push or a pull exerted on a body.
- Resultant force is a single force that can replace the number of forces acting on a body.
- When a body under the action of forces is in equilibrium, the net force acting on it is zero.
- Equilibrant force has the same magnitude as the resultant but opposite in direction to the resultant.
- Torque is a measure of the turning effect of a force.
- A pair of forces of the same magnitude, opposite in direction, and acting along different lines of action constitutes a couple.
- A couple results in a net torque while the net force is zero.
- The net torque resulting from a couple is equal to the product of one of the forces and the distance between the lines of action of the forces.
- An unbalanced torque produces angular acceleration given by $\tau = I\alpha$, where I is moment of inertia.
- The moment of inertia of a body depends on both mass and distribution of the mass about the axis of rotation.
- Center of mass is a point where the total mass of a body is assumed to be concentrated.

Review Exercises

1. If only one force acts on an object, can it be in equilibrium? Explain.
2. If a fly collides with the windshield of a fast-moving bus, which object experiences an impact force with a larger magnitude? (a) the fly (b) the bus (c) the same force is experienced by both.
3. If a fly collides with the windshield of a fast-moving bus, which object experiences the greater acceleration: (a) the fly (b) the bus (c) the same acceleration is experienced by both.
4. A man is dragging a trunk up the loading ramp of a mover's truck. The ramp has a slope angle of 20°, and the man pulls upward with a force F whose direction makes an angle of 30° with the ramp, Figure. 31. (a) How large a force F is necessary for the component F_x parallel to the ramp to be 90 N? (b) How large will the component F_y perpendicular to the ramp be then?

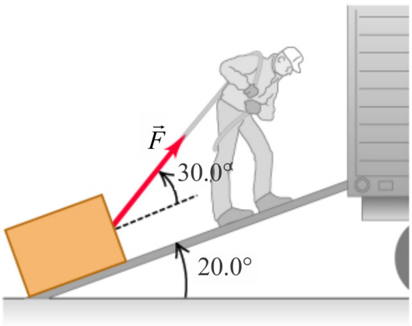


Figure 31.

5. To extricate an SUV stuck in the mud, workmen use three horizontal ropes, producing the force vectors shown in Figure 32. (a) Find the x- and y-components of each of the three pulls. (b) Use the components to find the magnitude and direction of the resultant of the three pulls.

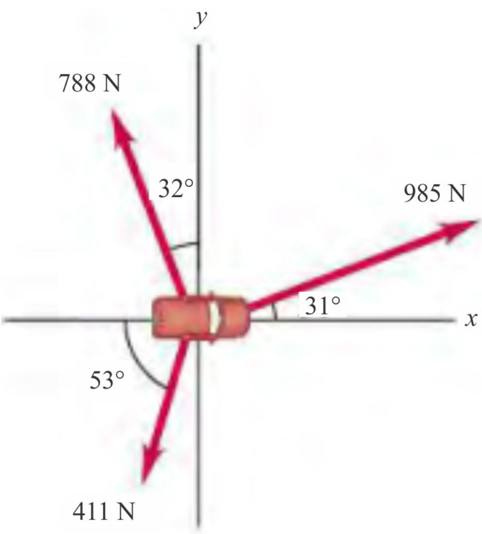


Figure 32.

6. During pregnancy, women often develop back pains from leaning backward while walking. Why do they have to walk this way?
7. Figure 33 shows three uniform objects: a rod, a right triangle, and a square. Their masses and their coordinates in meters are given. Determine the center of gravity for the three-object system.

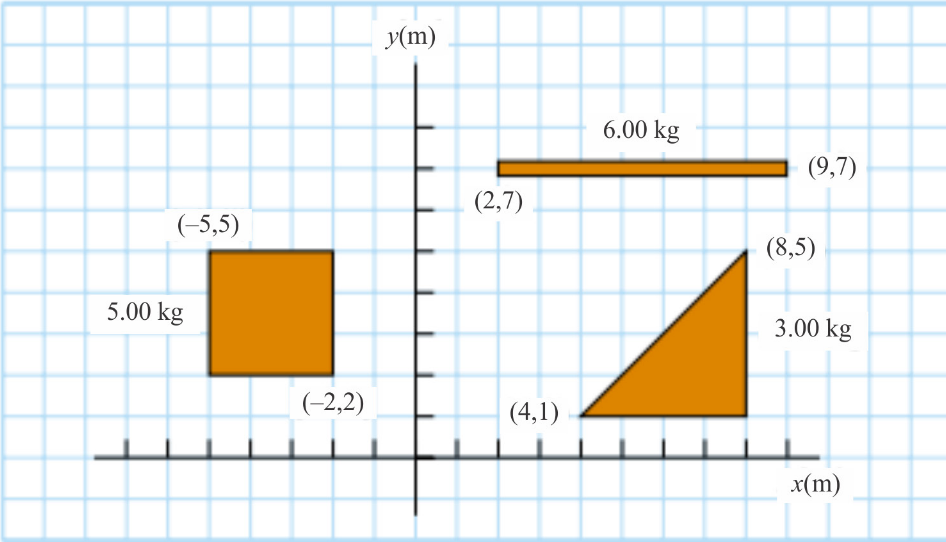


Figure 33.

8. You press your physics textbook flat against a vertical wall with your hand. What is the direction of the friction force exerted by the wall on the book?
9. A crate is located in the center of a flatbed truck. The truck accelerates to the east, and the crate moves with it, not sliding at all. What is the direction of the friction force exerted by the truck on the crate?
10. A mechanic has two spanners, one of 15 cm long and the other 20 cm long. She applies a force of 40 N with the first spanner and 30 N with the second, with which spanner does she produce the larger torque?
11. A 25.0-kg block is initially at rest on a horizontal surface. A horizontal force of 75 N is required to set the block in motion. After it is in motion, a horizontal force of 60 N is required to keep the block moving with constant speed. Find the coefficients of static and kinetic friction from this information.
12. In a laboratory experiment on friction, a 135 N block resting on a rough horizontal table is pulled by a horizontal wire. The pull gradually increases until the block begins to move and continues to increase thereafter. Figure 34 shows a graph of the friction force on this block as a function of the pull. (a) Identify the regions of the graph where static friction and kinetic friction occur. (b) Find the coefficients of static friction and kinetic friction between the block and the table. (c) Why does the graph slant upward at first but then level out?

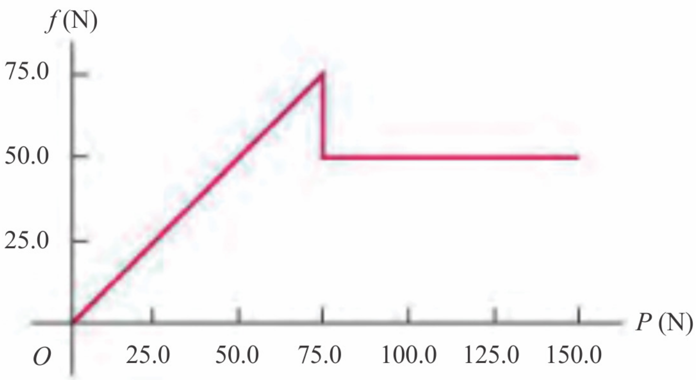


Figure 34.

13. A stockroom worker pushes a box with mass 16.8 kg on a horizontal surface with a constant speed. The coefficient of kinetic friction between the box and the surface is 0.20. (a) What horizontal force must the worker apply to maintain the motion?
14. A box of bananas weighing 40.0 N rests on a horizontal surface. The coefficient of static friction between the box and the surface is 0.40, and the coefficient of kinetic friction is 0.20. (a) If no horizontal force is applied to the box and the box is at rest, how large is the friction force exerted on it? (b) What is the magnitude of the friction force if a monkey applies a horizontal force of 6.0 N to the box and the box is initially at rest? (c) What minimum horizontal force must the monkey apply to start the box in motion? (d) What minimum horizontal force must the monkey apply to keep the box moving at constant velocity once it has been started? (e) If the monkey applies a horizontal force of 18.0 N, what is the magnitude of the friction force?

P11CH03

CHAPTER 3

MOMENTUM AND ITS CONSERVATION

Chapter Outcome

Learners will able to:

- recognize the dangers in the collision of moving objects and take measure in avoiding it.

Chapter Objectives

Upon completion of this chapter, learners will:

- recognize the dangers the collision of moving objects and take measure in avoiding it;
- analyze the nature and effect of momentum;
- differentiate between impulse and momentum;
- distinguish between elastic and inelastic momentum;
- distinguish between the laws of conservation of linear and angular momentum and their application.

Introduction

Many questions involving forces can't be answered by directly applying Newton's second law, $\Sigma F = ma$. For example, when a truck collides head-on with a compact car, what determines which way the wreckage moves after the collision? In playing pool, how do you decide how to aim the cue ball in order to knock the eight ball into the pocket? And when a meteorite collides with the earth, how much of the meteorite's kinetic energy is released in the impact? All of these questions involve forces about which we know very little: the forces between the car and the truck, between the two pool balls, or between the meteorite and the earth. Remarkably, we will find in this unit that we don't have to know anything about these forces to answer questions of this kind! Our approach uses two new concepts, momentum and impulse, and a new conservation law, conservation of momentum.

3.1 Nature of Linear Momentum and Impulse

Momentum is a commonly used term in sports. When a sports announcer says that a team has the momentum they mean that the team is really on the move and is going to be hard to stop. The term momentum is a physics concept. Any object with momentum is going to be hard to stop. To stop such an object, it is necessary to apply a force against its motion for a given period of time. The more momentum that an object has, the harder it is to stop. Thus, it would require a greater amount of force or a longer amount of time or both to bring such an object to a halt. As the force acts upon the object for a given amount of time, the object's velocity is changed; and hence, the object's momentum is changed.

Key terms

- Momentum: the product of mass and velocity of a body. It is a vector quantity which takes the direction of velocity. It describes the effort required to set a body in motion or to stop it.
- Impulse: The product of the force applied and the time for which the force is acting. It is equal to the change of momentum.

Linear momentum is the product of a system's mass and its velocity. In equation form, linear momentum **p** is

$$p = mv \qquad \text{Eq. 3.1}$$

You can see from the equation that momentum is directly proportional to the object's mass (m) and velocity (**v**). Therefore, the greater an object's mass or the greater its velocity, the greater its momentum. A large, fast-moving object has greater momentum than a smaller, slower object.

Momentum is a vector and has the same direction as velocity **v**. The SI unit for momentum is kg m/s.

KEY TERMS

- Newton's second law
- A non-zero net force applied on a body accelerates the body. The acceleration is proportional to the force and inversely proportional to the mass of the object.

Momentum is so important for understanding motion that it was called the quantity of motion by physicists such as Newton. Force influences momentum, and we can rearrange Newton's second law of motion to show the relationship between force and momentum.

Newton's second law of motion in equation form is $\mathbf{F}_{net} = m\mathbf{a}$. Newton actually stated his second law of motion in terms of momentum the time rate of change of the linear momentum of a particle is equal to the net force acting on the particle.

$$\mathbf{F}_{net} = \frac{\Delta \mathbf{p}}{\Delta t} \qquad \text{Eq. 3.2}$$

where $\mathbf{F}_{net}$ is the net external force, $\Delta\mathbf{p}$ is the change in momentum, and Δt is the change in time.

ACTIVITY 1

Assuming mass m to be constant use the equations $\mathbf{F}_{net} = m\mathbf{a}$ and $\mathbf{a} = \frac{\Delta \mathbf{V}}{\Delta t}$ to show that

$$\mathbf{F}_{net} = \frac{\Delta \mathbf{p}}{\Delta t}$$

We can solve for $\Delta\mathbf{p}$ by rearranging the equation $F_{net} = \frac{\Delta \mathbf{p}}{\Delta t}$ as

$\Delta\mathbf{p} = \mathbf{F}_{net}\,\Delta t$.

The product $\mathbf{F}_{net}\,\Delta t$ is known as impulse (I) and this equation is known as the impulse-momentum theorem.

$$\mathbf{J} = \Delta\mathbf{p} = \mathbf{F}_{net}\,\Delta t \qquad \text{Eq. 3.3}$$

From the equation, we see that the impulse equals the average net external force multiplied by the time this force acts. It is equal to the change in momentum. The effect of a force on an object depends on how long it acts, as well as the strength of the force. Impulse is a useful concept because it quantifies the effect of a force. Very large force acting for a short time can have a great effect on the momentum of an object, such as the force of a racket hitting a tennis ball. A small force could cause the same change in momentum, but it would have to act for a much longer time.

Activity 2

Discuss the following practical application of the concept of impulse-momentum relationship Airbags, (Figure 1), in automobiles have saved countless lives in accidents. The airbag increases the time interval during which the passenger is brought to rest, thereby decreasing the force on (and resultant injury to) the passenger.

Figure 1. An airbag in use

Examples

A 180 kg bumper car carrying a 70 kg driver has a constant velocity of 4 m/s, toward East. What is the momentum of the car-driver system?

Solution

Given mass of bumper m_b = 180 kg, mass of driver m_d = 70 kg and velocity v = 3 m/s, E, the momentum of the car-driver system will be

$$\mathbf{P} = mv = (m_b + m_d)\,\mathbf{v} = (180 \text{ kg} + 70 \text{ kg})(3 \text{ m/s, E}) = 750 \text{ kgm/s, E}$$

Activity 3

Group discussion

Legendary stunt person Dar Robinson broke a world record of a cable jump from the CN Tower in 1980 for the film The World’s Most Spectacular Stuntman. While tied to a 3-mm steel cable, Robinson jumped more than 366 m and stopped only a short distance above the ground. The thick mattress on the ground provides a protective cushion for the stunt person when he lands (Figure 2) Why do you think the hardness of a surface affects the extent of injury upon impact?

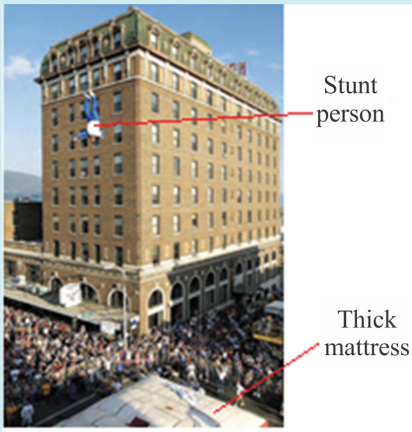
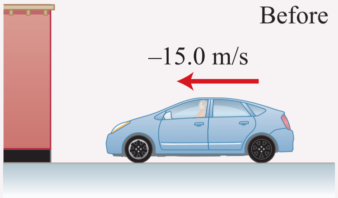


Figure 2.

Examples



In a particular crash test, a car of mass 1400 kg collides with a wall, as shown in Figure 3. The initial and final velocities of the car are v_i = -(15 m/s) $\hat{i}$ and v_f = (2.5 m/s) $\hat{i}$, respectively. If the collision lasts for 0.15 s, find the impulse caused by the collision and the average force exerted on the car.

Solution

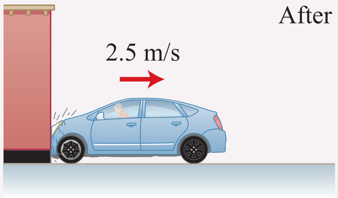


Figure 3.

Given mass m = 1400 kg, initial velocity = -(15 m/s) $\hat{i}$, final velocity = (2.5 m/s) $\hat{i}$, the impulse can be calculated as **J** = Δ**P**

$\Delta \mathbf{P} = mv_f - mv_i = m(v_f - v_i) = 1400\ kg((2.5\ m/s)\ \hat{i} - (-15\ m/s)\ \hat{i}) = (2.45 \times 10^4\ kgm/s)\ \hat{i}$

Therefore, the impulse **J** = $(2.45 \times 10^4$ kgm/s$)\ \hat{i}$

Activity 4

Many modern rifles use bullets that have less mass and reach higher speeds than bullets for older rifles, resulting in increased accuracy over longer distances. The momentum of a bullet is initially 8.25 kgm/s, West. What is the momentum if the speed of the bullet increases by a factor of 3/2 and its mass decreases by a factor of 3/4?

Did you know?

Figure 4. A Woodpecker

BIO: Application Woodpecker Impulse The pileated woodpecker (Dryocopus pileatus) has been known to strike its beak against a tree up to 20 times a second and up to 12,000 times a day. The impact force can be as much as 1200 times the weight of the bird's head. Because the impact lasts such a short time, the impulse—the product of the net force during the impact multiplied by the duration of the impact—is relatively small. (The woodpecker has a thick skull of spongy bone as well as shock-absorbing cartilage at the base of the lower jaw, and so avoids injury.), Figure 4.

Examples

An object has a momentum of 2.5×10^2 kg m/s, North. Determine the momentum of the object if its mass decreases to one-third its original value and an applied force causes the speed to increase by exactly five times. The direction of the velocity remains constant.

Solution

The initial momentum is $\mathbf{P}_1 = m_1\mathbf{v}_1 = 2.5 \times 10^2$ kg m/s, N and the new momentum is $\mathbf{P}_2 = m_2\mathbf{v}_2$.

Using the new values of mass and velocity as $m_2 = \frac{1}{3} m_1$ and $v_2 = 5v_1$, we write

$$P_2 = (\frac{1}{3} m_1)(5v_1) = \frac{1}{3} m_1 v_1 = \frac{5}{3} P_1 = \frac{5}{3} (2.5 \times 10^2 \text{ kg m/s, N}) = 4.17 \times 10^2 \text{ kg m/s, N}$$

Activity 5

Using the concept of impulse, explain how a karate expert can break a board.

Activity 6

What is the effect of varying either the net force or the interaction time on the momentum of an object?

Did you know?

Some early cars were built with spring bumpers that tended to bounce off whatever they hit. These bumpers were used at a time when people generally traveled at much slower speeds. For safety reasons, cars today are built to crumple upon impact, not bounce. This results in a smaller change in momentum and a reduced average net force on motorists. The crushing also increases the time interval during the impulse, further decreasing the net force on motorists

Impulse Can Be Calculated Using a Net Force-Time Graph

One way to calculate the impulse provided to an object is to graph the net force acting on the object as a function of the interaction time. Suppose a net force of magnitude 30 N acts on a model rocket for 0.6 s during liftoff (Figure 5). From the net force-time graph in Figure 5, the product $F_{net}\Delta t$ is equal to the magnitude of the impulse. But this product is also the area under the graph.

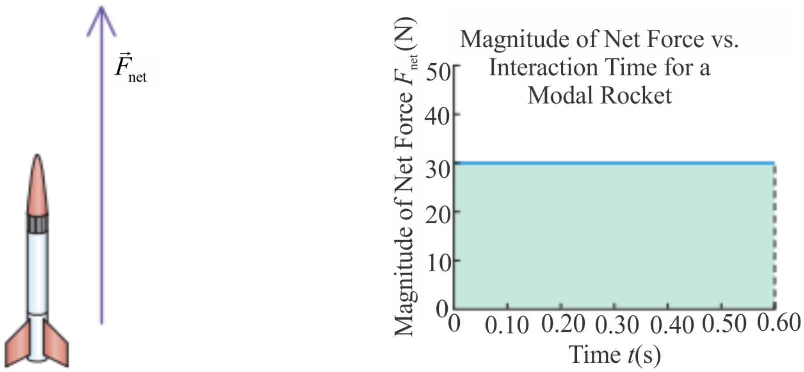


Figure 5. a) A model rocket during liftoff b) Magnitude of net force as a function of interaction time for a model rocket. The area under the graph is equal to the magnitude of the impulse provided to the rocket.

When F_{net} is not constant, you can still calculate the impulse by finding the area under a net force-time graph. Figure 6 shows the magnitude of the net force exerted by a bow on an arrow during the first part of its release. The magnitude of the net force is greatest at the beginning and decreases linearly with time.

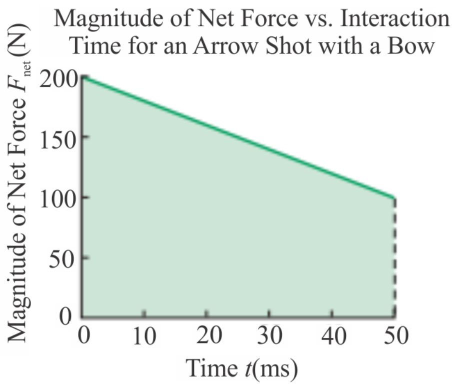


Figure 6. Magnitude of net force as a function of interaction time for an arrow shot with a bow.

Examples

A golfer hits a long drive sending a 45.9 g golf ball due east. Figure 7 shows an approximation of the net force as a function of time for the collision between the golf club and the ball. What is the impulse provided to the ball?

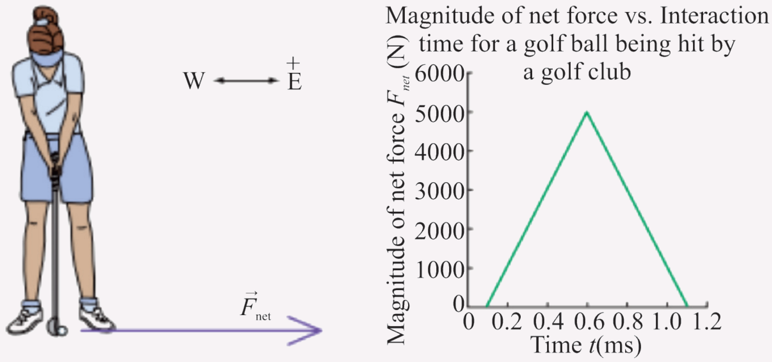


Figure 7.

Solution

Given m = 45.9 g = 0.0459 kg

From time t_1 = 0.1m s, and t_2 = 1.1 ms, $\Delta t = t_2 - t_1$ = 1.1m s – 0.1 ms = 1 ms, F_{max} = 5000 N

Magnitude of impulse = Area under net force-time graph

Area of the triangle = $\frac{1}{2}$ $(\Delta t)(F_{max})$ = $\frac{1}{2}$ $(1 \times 10^{-3} s)(5000\ N)$ = 2.5 kgm/s

What is the velocity of the ball at the moment the golf club and ball separate?

Activity 7

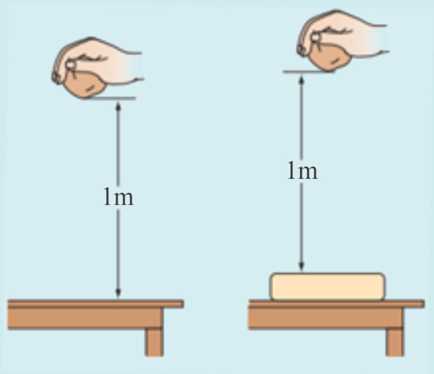
Boxers in the nineteenth century used their bare fists. In modern boxing, fighters wear padded gloves. How do gloves protect the brain of the boxer from injury?

Activity 8

Softening the hint

Problems

How is the change in the shape of a putty ball upon impact related lo the structure of the landing surface?



Materials

Putty-type material meter-stick closed cell foam or fell pad urethane loam pad or pillow waxed paper or plastic wrap

Procedure

1. Choose three surfaces of varying softness onto which to drop a putty ball. One of the surfaces should be either a lab bench or the floor. Cover each surface with some waxed paper or plastic wrap to protect it.

2. Knead or work, the putty unlit you can form three pliable baits of equal size.
3. Measure a height of I m above the top or each surface. Then drop the balls. o0ne for each surface

Questions

1. Describe any different in the shape of the putty balls after impact
2. How does the amount of cushioning affect the deformation or the putty?
3. Discuss how the softness of the landing surface might be related to the time required for the putty ball to come to a stop. Justify your answer with an analysis involving the kinematics equations.

Exercises

1. Does a large force always produce a larger impulse on an object than a smaller force does? Explain.
2. If the speed of a particle is doubled, by what factor is its momentum changed? By what factor is its kinetic energy changed?
3. If two particles have equal kinetic energies, are their momenta necessarily equal? Explain.
4. A 2 kg ball traveling toward North with a speed of 1.8 m/s hits a rigid wall and rebounds backwards with the same speed. Find
   (a) the change of momentum of the ball, and
   (b) if the interaction with the wall lased for 0.01 s, what is the force exerted by the wall on the ball?
5. A loaded transport truck with a mass of 38000 kg is travelling at 1.20 m/s, West. What will be the velocity of a 1400-kg car if it has the same momentum?
6. In the overhead view of Figure 8, a 300 g ball with a speed v of 6 m/s strikes a wall at an angle θ of 30° and then rebounds with the same speed and angle. It is in contact with the wall for 10 ms. In unit vector notation, what are
   (a) the impulse on the ball from the wall and
   (b) the average force on the wall from the ball?

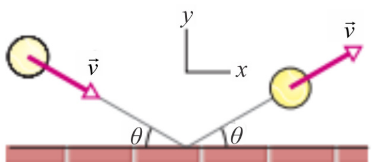


Figure 8.

3.2 Elastic and Inelastic Collision

We use the term collision to represent an event during which two particles come close to each other and interact by means of forces. The time interval during which the velocities of the particles change from initial to final values is assumed to be short. The interaction forces are assumed to be much greater than any external forces present, so we can use the impulse approximation.

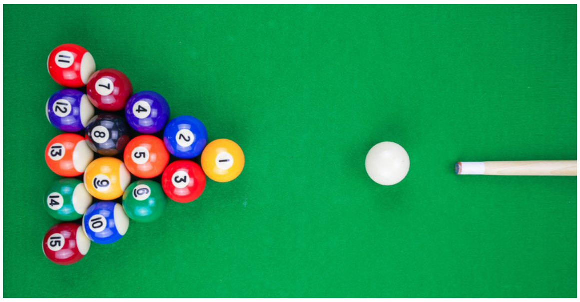


Figure 9. Many collisions take place during a game of pool. What evidence suggests that momentum is conserved during the collision shown in the photo?

Elastic Collisions

Suppose you hit a stationary pool ball dead center (head-on) with another pool ball so that the collision is collinear and the balls move without spinning immediately after impact. What will be the resulting motion of both balls (Figure 10)?

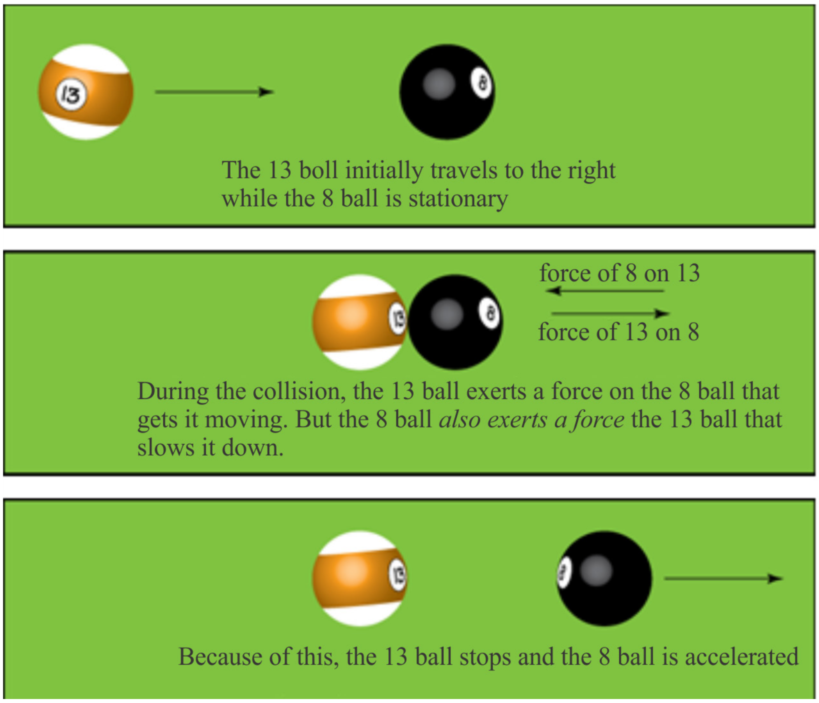


Figure 10. Elastic collision between two identical pool balls

> **KEY TERMS**
> - Kinetic energy: energy possessed by a body due to its motion.

The ball that was initially moving will become stationary upon impact, while the other ball will start moving in the same direction as the incoming ball. If you measure the speed of both balls just before and just after collision, you will find that the speed of the incoming ball is almost the same as that of the outgoing ball. Using the equation of kinetic energy of a body as the product $E_k = \frac{1}{2}mv^2$, it can be shown that the final kinetic energy of the system is almost the same as the initial kinetic energy of the system.

A collision in which the total kinetic energy (as well as total momentum) of the system is the same before and after the collision is known as Elastic collision.

$E_{ki} = E_{kf}$

Figure 11 shows a model for an elastic collision. When the gliders collide, their springs are momentarily compressed and some of the original kinetic energy is momentarily converted to elastic potential energy. Then the gliders bounce apart, the springs expand, and this potential energy is converted back to kinetic energy.

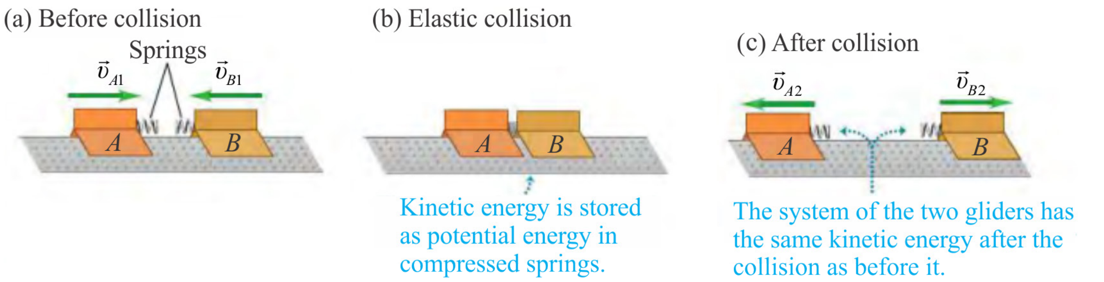
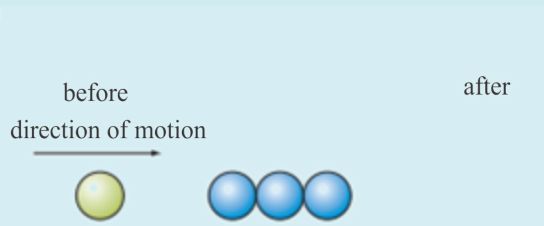


Figure 11. Two gliders undergoing an elastic collision on a frictionless surface. Each glider has a steel spring bumper that exerts a conservative force on the other glider.

ACTIVITY 9

Observing Collinear Collisions



Problem

What happens when spheres collide in one dimension?

Materials

One set of four identical ball bearings or marbles (set A) a second set or four identical ball bearings or marbles of double the mass (set B) a third set of four identical ball bearings or marbles of half the mass (set C) 1-m length of an I-beam curtain rod or two meter-sticks with smooth edges masking tape.

Procedure

1. Lay the curtain rod flat on a be1lch to provide a horizontal track for the spheres. Tape the ends of the rod securely. If you are using meter-sticks. Tape them 5 mm apart to form a uniform straight horizontal track.
2. Using set A. place three of the spheres tightly together at the center of the track.
3. Predict what will happen when one sphere of set A moves along the track and collides with the three stationary spheres.
4. Test your prediction. Ensure that the spheres remain on the track after collision. Record your observations
5. Repeat steps 2 to 4, but this time use set B. spheres of greater mass.
6. Repeat steps 2 to 4. but this time use set C. spheres of lesser mass.
7. Repeat steps 2 to 4 using different numbers of stationary spheres. The stationary spheres should all be the same mass. but the moving sphere should be of a different mass in some of the trials.

Questions

1. Describe the motion of the spheres in steps 4 to 6.
2. Explain what happened when
   (a) a sphere of lesser mass collided with a number or spheres of greater mass and
   (b) a sphere of greater mass collided with a number of spheres of lesser mass.

In activity 9 above we see that, for each set of spheres A to C, when one sphere hit a row of three stationary ones from the same set, the last sphere in the row moved outward at about the same speed as the incoming sphere. But when one sphere from set A hit a row of spheres from set B, the last sphere in the row moved outward at a much slower speed than the incoming sphere, and the incoming sphere may even have rebounded. When one sphere from set A hit a row of spheres from set C, the last sphere in the row moved outward at a greater speed than the incoming sphere, and the incoming sphere continued moving forward.

Inelastic collision

Most macroscopic interactions in the real world involve some of the initial kinetic energy of the system being converted to sound, light, or deformation (Figure 12). When deformation occurs, some of the initial kinetic energy of the system is converted to heat because friction acts on objects in almost all situations.

An inelastic collision is one in which the total kinetic energy of the system is greater before than after the collision (even though the momentum of the system is conserved).

Inelastic collisions are of two types. When the colliding objects stick together after the collision, as happens when a meteorite collides with the Earth, the collision is called perfectly inelastic. When the colliding objects do not stick together, but some kinetic energy is lost, as in the case of a rubber ball colliding with a hard surface, the collision is called inelastic (with no modifying adverb).

Figure 12. During impact the side of the ball momentarily flattens.

When the rubber ball collides with the hard surface, some of the kinetic energy of the ball is lost when the ball is deformed while it is in contact with the surface. In most collisions, the kinetic energy of the system is not conserved because some of the energy is converted to internal energy and some of it is transferred away by means of sound. Elastic and perfectly inelastic collisions are limiting cases; most collisions fall somewhere between them.

$E_{ki} \neq E_{kf}$

Figure 13 shows a model for an inelastic collision. The spring bumpers in Figure 11 are replaced with Velcro, which sticks the two gliders together.

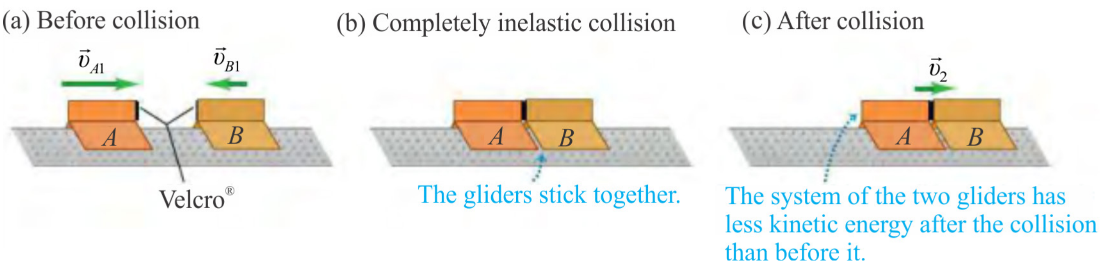


Figure 13. Two gliders undergoing a perfectly inelastic collision. The spring bumpers on the gliders in Figure 10 are replaced by Velcro (for fastening the gliders), so the gliders stick together after collision.

The important distinction between elastic and inelastic collisions is that momentum of the system is conserved in all collisions, but kinetic energy of the system is conserved only in elastic collisions.

Did you know?

Bio: Application of collision - Glaucoma testing

As a practical application, an inelastic collision is used to detect glaucoma, a disease in which the pressure inside the eye builds up and leads to blindness by damaging the cells of the retina. In this application, medical professionals use a device called a tonometer to measure the pressure inside the eye. This device releases a puff of air against the outer surface of the eye and measures the speed of the air after reflection from the eye. At normal pressure, the eye is slightly spongy, and the pulse is reflected at low speed. As the pressure inside the eye increases, the outer surface becomes more rigid, and the speed of the reflected pulse increases. In this way, the speed of the reflected puff of air can measure the internal pressure of the eye.

Key terms

- Collision: represents an event during which two particles come close to each other and interact by means of forces.
- Elastic collision: Collision in which total kinetic energy is conserved.
- Inelastic collision: Collision in which the total kinetic energy of the system is not conserved.

Did you know?

The law of conservation of energy states that the total energy of an isolated system remains constant. The energy may change into several different forms. This law has no known exceptions.

Activity 10

An executive stress reliever

An ingenious device that illustrates conservation of momentum and kinetic energy is shown in Figure 14. It consists of five identical hard balls supported by strings of equal lengths. When ball 1 is pulled out and released, after the almost-elastic collision between it and ball 2, ball 5 moves out, as shown in Figure 14 b. If balls 1 and 2 are pulled out and released, balls 4 and 5 swing out, and so forth. Is it ever possible that when ball 1 is released, balls 4 and 5 will swing out on the opposite side and travel with half the speed of ball 1, as in Figure 13 c?

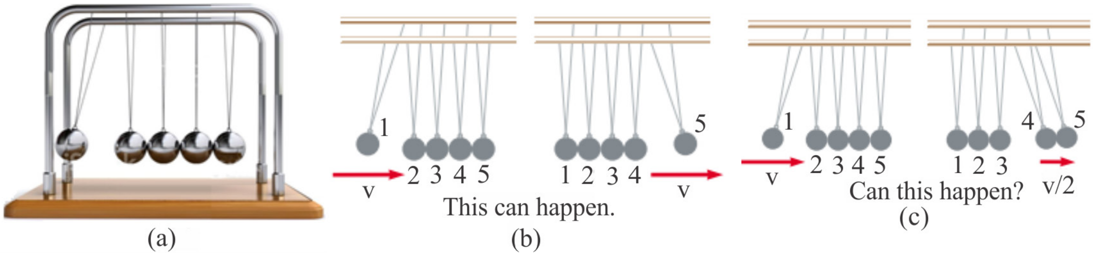


Figure 14. An executive stress reliever (Newton's cradle)

Activity 11

Group discussion

A steel sphere will bounce as high on a steel anvil as a rubber ball will on concrete. However, when a steel sphere is dropped on linoleum or hardwood the sphere hardly bounces at all. Explain why.

Examples

An argon atom with a mass of 6.63×10^{-26} kg travels at 17 m/s toward the right and strikes another identical argon atom dead center travelling at 20 m/s toward the left. The first atom rebounds at 20 m/s toward left, while the second atom moves at 17 m/s toward right. Determine if the collision is elastic.

Solution

Given: m1 = 6.63×10^{-26} kg, initial velocity of m_1 is v_{1i} = 17 m/s, right, and final velocity of m1 is v_{1f} = 20 m/s, left m_2 = 6.63×10^{-26} kg, initial velocity of m_2 is v_{2i} = 20 m/s, left, and final velocity of m2 is v_{2f} = 17 m/s, right

In order to decide whether or not the collision is elastic we calculate the total kinetic energy before collision with that after collision.

The total kinetic energy before collision is $E_{ki} = E_{k1i} + E_{k2i}$

$$E_{ki} = \frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 = \frac{1}{2}\left(6.63\times10^{-26}\,kg\right)\left(17\,m/s\right)^2 + \frac{1}{2}\left(6.63\times10^{-26}\,kg\right)\left(20\,m/s\right)^2$$

$E_{ki} = 2.28 \text{ x } 10^{-23}$ J

$$E_{kf} = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2 = \frac{1}{2}\left(6.63\times10^{-26}\,kg\right)\left(20\,m/s\right)^2 + \frac{1}{2}\left(6.63\times10^{-26}\,kg\right)\left(17m/s\right)^2$$

$E_{kf} = 2.28 \text{ x } 10^{-23}$ J

Since $E_{ki} = E_{kf}$, the collision is elastic.

Exercises

1. Explain the difference between elastic and inelastic collision.
2. Give examples of elastic collisions and inelastic collisions.
3. What do we mean by ‘kinetic energy is conserved during collision’?
4. Why can’t collisions be always elastic?
5. If the collision between two objects is completely inelastic, what can you say about the velocity of the objects after collision?
6. Show that the kinetic energy of a particle of mass m is related to the magnitude of the momentum p of that particle by $Ek = p^2/2m$.
7. Two masses m_1 and m_2, with $m_1 = m_2$, have equal kinetic energy. How do the magnitudes of their momenta compare?

3.3 Conservation of Momentum

When a collision occurs in an isolated system, the total momentum of the system doesn’t change with the passage of time. Instead, it remains constant both in magnitude and in direction. The momenta of the individual objects in the system may change, but the vector sum of all the momenta will not change. The total momentum is therefore said to be conserved. In this section, we will see how the laws of motion lead us to this important conservation law.

The concept of momentum is particularly important in situations in which we have two or more bodies that interact. To see why, let’s consider first an idealized system of two bodies that interact with each other but not with anything else— for example, two astronauts who touch each other as they float freely in the zero-gravity environment of outer space (Figure 15). Think of the astronauts as particles.

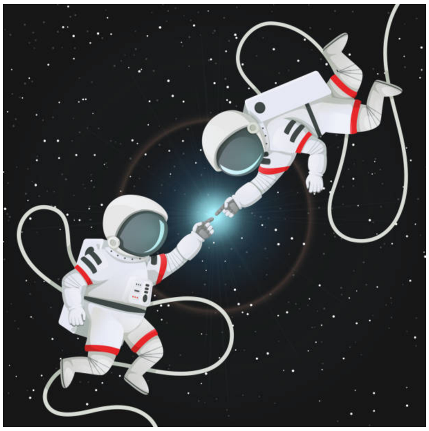


Figure 15. Two astronauts push each other as they float freely in the zero-gravity environment of space.

Each particle exerts a force on the other; according to Newton’s third law, the two forces are always equal in magnitude and opposite in direction. Hence, the impulses that act on the two particles are equal in magnitude and opposite in direction, as are the changes in momentum of the two particles.

Let’s go over that again with some new terminology. For any system, the forces that the particles of the system exert on each other are called internal forces. Forces exerted on any part of the system by some object outside it are called external forces. For the system shown in Figure 14, the internal forces are $\mathbf{F}_{BA}$, exerted by particle B on particle A, and $\mathbf{F}_{AB}$, exerted by particle A on particle B. There are no external forces; when this is the case, we have an isolated system.

The net force on particle A is $\mathbf{F}_{BA}$ and the net force on particle B is $\mathbf{F}_{AB}$, so from the equation $\mathbf{F}_{net} = \frac{\Delta \mathbf{p}}{\Delta t}$, the rates of change of the momenta of the two particles are

$$\mathbf{F}_{BA} = \frac{\Delta P_A}{\Delta t} \qquad \mathbf{F}_{AB} = \frac{\Delta P_B}{\Delta t}$$

The momentum of each particle changes, but these changes are related to each other by Newton's third law: Forces $\mathbf{F}_{BA}$ and $\mathbf{F}_{AB}$ are always equal in magnitude and opposite in direction. That is, $\mathbf{F}_{BA} = -\mathbf{F}_{AB}$, so $\mathbf{F}_{BA} + \mathbf{F}_{AB} = 0$.

Adding $\mathbf{F}_{BA}$ and $\mathbf{F}_{AB}$ we have $\mathbf{F}_{BA} + \mathbf{F}_{AB} = \frac{\Delta \mathbf{P}_A}{\Delta t} + \frac{\Delta \mathbf{P}_B}{\Delta t} = \frac{\Delta(\mathbf{P}_A + \mathbf{P}_B)}{\Delta t} = 0$

The rates of change of the two momenta are equal and opposite, so the rate of change of the vector sum $\mathbf{P}_A + \mathbf{P}_B$ is zero. We define the total momentum **P** of the system of two particles as the vector sum of the momenta of the individual particles; that is,

$$\mathbf{P} = \mathbf{P}_A + \mathbf{P}_B \qquad \text{Eq. 3.4}$$

From $\mathbf{F}_{BA} + \mathbf{F}_{AB} = 0$ we write

$$\frac{\Delta \mathbf{P}}{\Delta t} = 0$$

Did you know?

Newton's Third Law: Matter interacts with matter—forces come in pairs. For each force exerted on one body, there is an equal, but oppositely directed, force on some other body interacting with it. This is often called the Law of Action and Reaction. Notice that the action and reaction forces act on the two different interacting objects.

The time rate of change of the total momentum **P** is zero. Hence the total momentum of the system is constant, even though the individual momenta of the particles that make up the system can change. Thus we have the following general result:

If the vector sum of the external forces on a system is zero, the total momentum of the system is constant.

This is the simplest form of the principle of conservation of momentum. This principle is a direct consequence of Newton's third law.

In equation form the law of conservation of linear momentum can be written as

$$\frac{\Delta \mathbf{P}_A}{\Delta t} + \frac{\Delta \mathbf{P}_B}{\Delta t} = 0$$

$$(m_1\mathbf{v}_{1f} - m_1\mathbf{v}_{1i}) + (m_2\mathbf{v}_{2f} - m_2\mathbf{v}_{2i}) = 0$$

This equation can be rearranged to give the following important result:

$$m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = m_1\mathbf{v}_{1f} + m_2\mathbf{v}_{2f} \qquad \text{Eq. 3.5}$$

This result is a special case of the law of conservation of momentum and is true of isolated systems containing any number of interacting objects **Note that**: Momentum Conservation applies to a System! The momentum of an isolated system is conserved, but not necessarily the momentum of one particle within that system, because other particles in the system may be interacting with it. Apply conservation of momentum to an isolated system only.

Examples

A75 kg hunter in a stationary boat throws a 0.72 kg harpoon at 12 m/s, toward right. The mass of the kayak is 10 kg. What will be the velocity of the kayak and hunter immediately after the harpoon is released, Figure 16.

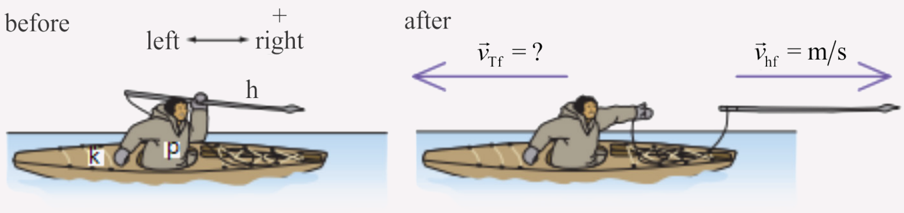


Figure 16.

Solution

Given

$m_p = 75$ kg	$m_k = 10$ kg	$m_h = 0.72$ m/s
$\mathbf{v}_{pi} = 0$ m/s	$\mathbf{v}_{ki} = 0$m/s	$\mathbf{v}_{hi} = 0$ m/s
		$\mathbf{v}_{hf} = 12$ m/s, right

Initially, the person (p), kayak (k) and harpoon (h) are all at rest. The system has initial total momentum of

$$\mathbf{P}_{Ti} = m_p\mathbf{v}_{pi} + m_k\mathbf{v}_{ki} + m_h\mathbf{v}_{hi} = 0$$

Just after the harpoon is thrown, the total momentum of the system will be

$\mathbf{P}_{Tf} = m_p\mathbf{v}_{pf} + m_k\mathbf{v}_{kf} + m_h\mathbf{v}_{hf}$

The kayak and the person are together when the harpoon is thrown, so they have the same velocity.

We write $\mathbf{v}_{pf} = \mathbf{v}_{kf} = \mathbf{v}$

$\mathbf{P}_{Tf} = (m_p + m_k)\mathbf{v} + m_h\mathbf{v}_{hf}$ = (75 kg + 10 kg) **v** + (0.72 kg)(12 m/s, right = (85 kg)**v** + 8.64 kgm/s, right

Applying the law of conservation of linear momentum,

Total momentum before = Total momentum after

$\mathbf{P}_{Ti} = \mathbf{P}_{Tf}$

0 = (85 kg)**v** + 8.64 kgm/s, right

Solving for v we obtain **v** = -0.1m/s, right = 0.1 m/s, left

Examples

Apply conservation of momentum to a one-dimensional inelastic collision.

A pickup truck with mass 1.8×10^3 kg is traveling eastbound at +15 m/s, while a compact car with mass 9×10^2 kg is traveling westbound at -15 m/s, Figure 17 a. The vehicles collide head-on, becoming entangled, Figure 17 b (a) Find the speed of the entangled vehicles after the collision. (b) Find the change in the velocity of each vehicle. (c) Find the change in the kinetic energy of the system consisting of both vehicles.(d) Is the collision elastic or inelastic?

Figure 17.

Solution

Given

$m_T = 1.8 \times 10^3$ kg, $m_c = 9 \times 10^2$ kg

$v_{Ti} = +15$ m/s $v_{ci} = -15$ m/s

Since the vehicles stick together after collision, $\mathbf{v}_{Tf} = \mathbf{v}_{cf} = \mathbf{v}$

(a) Applying the law of conservation of linear momentum,

$m_T\mathbf{v}_{Ti} + m_c\mathbf{v}_{ci} = m_T\mathbf{v}_{Tf} + m_c\mathbf{v}_{cf}$

$m_T\mathbf{v}_{Ti} + m_c\mathbf{v}_{ci} = (m_T + m_c)\mathbf{v}$

$(1.8 \times 10^3 \text{ kg}) \times (15 \text{ m/s}) + (9 \times 10^2 \text{ kg}) \times (-15 \text{ m/s}) = (1.8 \times 10^3 \text{ kg} + 9 \times 10^2 \text{ kg})\mathbf{v}$

Solving for v we get v = + 5 m/s = 5 m/s, toward right

Therefore, the speed of the entangled vehicles after the collision $\mathbf{v}_{Tf} = \mathbf{v}_{cf} = 5$ m/s, right

(b) Change in velocity of the truck = $v_{Tf} - v_{Ti} = 5 \text{ m/s} - 15 \text{ m/s} = -10 \text{ m/s}$

Change in velocity of the car = $v_{cf} - v_{ci} = 5 \text{ m/s} - (-15 \text{ m/s}) = 20 \text{ m/s}$

(c) The total initial kinetic energy of the system,

$$E_{ki} = \frac{1}{2}m_T v_{Ti}^2 + \frac{1}{2}m_c v_{ci}^2 = \frac{1}{2} = \frac{1}{2} \ (1.8 \times 10^3 \text{ kg})(15 \text{ m/s})^2 + \frac{1}{2} \ (9 \times 10^2)(-15 \text{ m/s})^2$$

$$= 3.04 \times 10^5 \text{ J}$$

The total final kinetic energy of the system,

$$E_{kf} = \frac{1}{2} \ (m_T + m_c)v^2 = \frac{1}{2} \ (1.8 \times 10^3 \text{ kg} + 9 \times 10^2)(5 \text{ m/s})^2 = 3.38 \times 10^4 \text{ J}$$

The change of kinetic energy of the system

$$\Delta E_k = E_{kf} - E_{ki} = 3.38 \times 10^4 \text{ J} - 3.04 \times 10^5 \text{ J} = -2.7 \times 10^5 \text{ J}$$

During the collision, the system lost almost 90% of its kinetic energy, $\left(\frac{E_{ki}}{E_{ki}} \times 100\%\right)$.

The change in velocity of the pickup truck was only 10 m/s, compared to twice that for the compact car. This example underscores perhaps the most important safety feature of any car: its mass. Injury is caused by a change in velocity, and the more massive vehicle undergoes a smaller velocity change in a typical accident.

(d) The collision is perfectly inelastic. Momentum is conserved while kinetic energy is not.

For an elastic collision between two masses where both momentum and kinetic energy are conserved, we write

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2 \qquad \text{Eq 3.6}$$

An alternate approach simplifies the quadratic equation to another linear equation, facilitating solution. Canceling the factor $\frac{1}{2}$ in Equation 3.6, we rewrite the equation as

$$m_1(v_{1i}^2 - v_{1f}^2) = - m_2(v_{2i}^2 - v_{2f}^2)$$

Here we have moved the terms containing m_1 to one side of the equation and those containing m_2 to the other. Next, we factor both sides of the equation:

$$m_1(v_{1i} - v_{1f})(v_{1i} + v_{1f}) = m_2(v_{2f} - v_{2i})(v_{2f} + v_{2i}) \qquad \text{Eq. 3.7}$$

Separating the terms containing m_1 and m_2 in equation for the conservation of momentum (Eq 3.5) we get

$$m_1(v_{1i} - v_{1f}) = m_2(v_{2f} - v_{2i}) \qquad \text{Eq 3.8}$$

Dividing Eq __ by Eq ___, produces

$$v_{1i} + v_{1f} = v_{2f} + v_{2i}$$

$$v_{1i} - v_{2i} = -(v_{1f} - v_{2f}) \qquad \text{Eq 3.9}$$

Examples

A basketball player and her wheelchair (player A) have combined mass of 58 kg. She moves at 0.6 m/s due East and pushes off a stationary player B whiling jockeying for a position near the basket. Player A ends up moving at 0.2 m/s due West. The combined mass of player B and her wheelchair is 85 kg. What will be player B's velocity immediately after the interaction, Figure 18?

Solution

Given

$m_A = 58$ kg $m_B = 85$ kg

$\mathbf{v}_{Ai} = 0.6$ m/s, E, $= +0.6$ m/s $\mathbf{v}_{Bi} = 0$ m/s

$\mathbf{v}_{Af} = 0.2$ m/s, W, $= -0.2$ m/s

We need to determine final velocity of player B, $\mathbf{v}_{Bf}$.

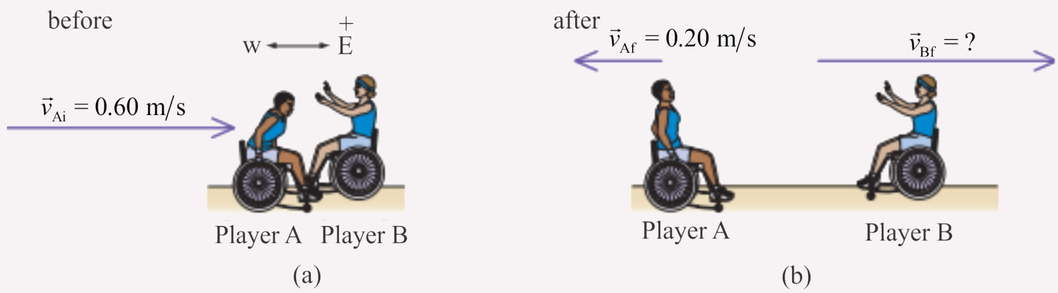


Figure 18.

Assuming players A and B as an isolated system, we apply the law of conservation of linear momentum as

$$m_A v_{Ai} + m_B v_{Bi} = m_A v_{Af} + m_B v_{Bf}$$

$$(58\text{ kg})(0.6\text{ m/s}) + (85\text{ kg})(0\text{ m/s}) = (58\text{ kg})(-0.2\text{ m/s}) + (85\text{ kg})\mathbf{v}_{Bf}$$

Solving for $v_{Bf} = +0.55$ m/s $= 0.55$ m/s, East

Note that: Conservation of Momentum When students were given a problem about conservation of momentum, more than 29% gave an incorrect response.

Common errors:

- Forgetting that momentum **P** is a vector.

Its components can be positive or negative depending on the direction of **P**

- Adding momenta incorrectly. If two momentum vectors point in different directions, you cannot find the total momentum by simply adding the magnitudes of the two momenta.

Examples

Two billiard balls of identical mass move toward each other as in Figure 19, with the positive x-axis to the right. Assume that the collision between them is perfectly elastic. If the initial velocities of the balls are 0.3 m/s and -0.2 m/s, what are the velocities of the balls after the collision? Assume friction and rotation are unimportant.

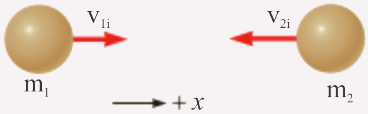


Figure 19.

Solution

Write the conservation of momentum equation.

$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$

Because $m_1 = m_2$, we can cancel the masses, then substitute $v_{1i} = 0.3$m/s and $v_{2i} = -0.2$m/s $0.3 \text{ m/s} + (-0.2\text{m/s}) = v_{1f} + v_{2f}$.

$0.1\text{m/s} = v_{1f} + v_{2f}$ Eq. 3.10

Next, apply conservation of energy in the form of

Equation 3.9,

$v_{1i} - v_{2i} = -(v_{1f} - v_{2f})$ Eq. 3.11

$0.3 \text{ m/s} - (-0.2\text{m/s}) = v_{2f} - v_{1f}$

$0.5\text{m/s} = v_{2f} - v_{1f}$ Eq. 3.12

Now solve Equations 3.10 and 3.12 simultaneously by adding them together

$0.1\text{m/s} + 0.5\text{m/s} = (v_{1f} + v_{2f}) - (v_{2f} - v_{1f})$

$0.6\text{m/s} = 2v_{2f}$

$v_{2f} = 0.3\text{m/s}$

Substitute the answer for v_{2f} into Equation 3.10:

$0.1\text{m/s} = v_{1f} + 0.3\text{m/s}$

$v_{1f} = -0.2\text{m/s}$

Notice the balls exchanged velocities—almost as if they'd passed through each other. This is always the case when two objects of equal mass undergo an elastic head-on collision.

Did you know?

Figure 20. Squid swimming in water

BIO: Animal Propulsion.

Squids and octopuses propel themselves by expelling water.

They do this by keeping water in a cavity and then suddenly contracting the cavity to force out the water through an opening, Figure 20.

Examples

Consider a large mass m_1 of 5 kg and a small mass m_2 of 2 kg that are in contact (not attached) are placed on a frictionless horizontal surface as shown in Figure 21. The smaller mass suddenly moves to the right at 13 m/s. What is the recoil velocity of the larger mass?

Solution

Given

$m_1 = 5$ kg　　　　$m_2 = 2$ kg

$v_{1i} = 0$ m/s　　　　$v_{2i} = 0$ m/s

$v_{2f} = 13$ m/s, right $= +\,13$ m/s

Required is the final velocity of m_1, v_{1f}

Applying the law of conservation of linear momentum for the isolated system we write

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$$

$$(5\text{ kg})(0\text{ m/s}) + (2\text{ kg})(0\text{ m/s}) = (5\text{ kg})(v_{1f}) + (3\text{ kg})(13\text{ m/s})$$

Solving for v_{1f} gives $v_{1f} = -\,7.8$ m/s, toward left as in Figure 21b.

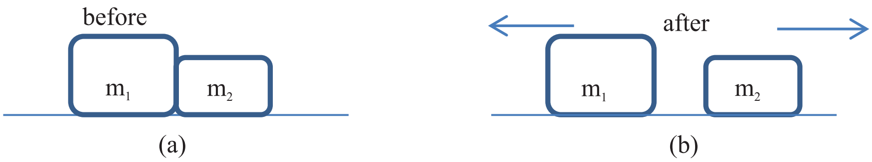


Figure 21.

Did you know?

The horns of a bighorn ram can account for more than 10% of its mass, which is about 125 kg. Rams collide at about 9 m/s, and average about 5 collisions per hour. Mating contests between any two rams may last for more than 24 h in total, Figure 22

Figure 22.

Two-dimensional Collisions

In the last section, we showed that the momentum of a system of two particles is conserved when the system is isolated. For any collision of two particles, this result implies that the momentum in each of the directions x, y, and z is conserved.

An important subset of collisions takes place in a plane. The game of billiards is a familiar example involving multiple collisions of objects moving on a two-dimensional surface. For such two dimensional collisions, we obtain two component equations for conservation of momentum:

$$m_1v_{1ix} + m_2v_{2ix} = m_1v_{1fx} + m_2v_{2fx} \qquad \text{Eq. 3.13}$$

$$m_1v_{1iy} + m_2v_{2iy} = m_1v_{1fy} + m_2v_{2fy} \qquad \text{Eq. 3.14}$$

Let us consider a two-dimensional problem in which particle 1 of mass m_1 collides with particle 2 of mass m_2 , where particle 2 is initially at rest, as in Figure 23.

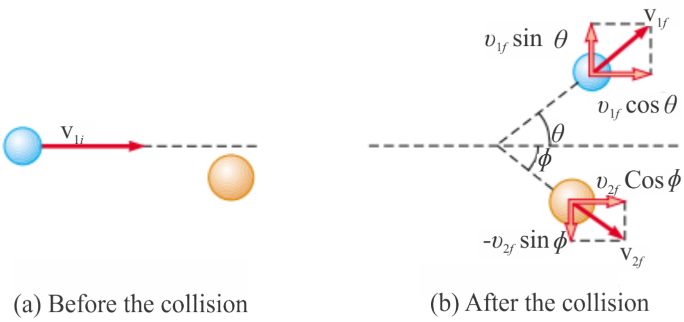


Figure 23. An elastic glancing collision between two particles

After the collision, particle 1 moves at an angle θ with respect to the horizontal and particle 2 moves at an angle ϕ with respect to the horizontal. This is called a glancing collision. Applying the law of conservation of momentum in component form and noting that the initial y component of the momentum of the two-particle system is zero, we obtain

$$m_1 v_{1i} = m_1 m_{1f} \cos\theta + m_2 v_{2f} \cos\phi$$

$$0 = m_1 m_{1f} \sin\theta - m_2 v_{2f} \sin\phi$$

where the minus sign in the equations comes from the fact that after the collision, particle 2 has a y component of velocity that is downward.

If the collision is elastic, we can also use the equation of conservation of kinetic energy with $v_{2i} = 0$ to give

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

If the collision is inelastic, kinetic energy is not conserved so that the kinetic energy equation does not apply.

Examples

View from above the robot velocities.

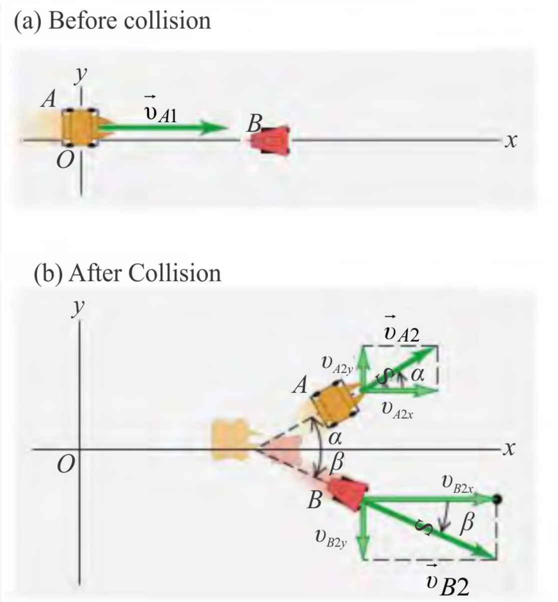


Figure 24. View from above the robot velocity

Figure 24 a shows two battling robots on a frictionless surface. Robot A, with mass 20 kg, initially moves at 2 m/s parallel to the x-axis. It collides with robot B, which has mass 12 kg and is initially at rest. After the collision, robot A moves at 1 m/s in a direction that makes an angle α = 30° with its initial direction (Figure 24b). What is the final velocity of robot B?

Solution

The momentum-conservation equations and their solutions for v_{B2x} and v_{B2y} are

$$m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$$

$$v_{B2x} = \frac{m_A v_{A1x} + m_B v_{B1x} - m_A v_{A2x}}{m_B}$$

$$= \frac{(20kg)(2m/s) + (12kg)(0) - (20kg)(1m/s)(\cos 30°)}{12kg} = 1.89m/s$$

$$m_A v_{A1y} + m_B v_{B1y} = m_A v_{A2y} + m_B v_{B2y}$$

$$v_{B2y} = \frac{m_A v_{A1y} + m_B v_{B1y} - m_A v_{A2y}}{m_B}$$

$$= \frac{(20kg)(0) + (12kg)(0) - (20kg)(1m/s)(\sin 30°)}{12kg} = -0.83 m/s$$

Figure 23 b shows the motion of robot *B* after the collision. The magnitude of $\mathbf{v}_{B2}$ is

$$v_{B2} = \sqrt{(1.89m/s)^2 + (-0.83m/s)^2} = 2.1m/s$$

and the angle of its direction from the positive x-axis is

$$\beta = \tan\text{-}1\left(\frac{-0.83m/s}{1.89m/s}\right) = -24°$$

Exercises

1. What is an isolated system?
2. State the law of conservation of momentum and the conditions in which it applies.
3. Consider two objects moving with velocities of v_{1i} and v_{2i}, respectively before collision and that their respective velocities just after interaction are v_{1f} and v_{2f}. Assuming a perfectly elastic collision, show that $(v_{1f} - v_{2f}) = -(v_{1i} - v_{2i})$.
4. A 0.25 kg volleyball is flying west at 2 m/s when it strikes a stationary 0.58 kg basketball dead center (head-on). The volleyball rebounds east at 0.79 m/s. What will be the velocity of the basketball immediately after impact?
5. A 9500 kg rail flatcar moving forward at 0.7 m/s strikes a stationary 18000 kg boxcar, causing it to move forward at 0.42 m/s. What will be the velocity of the flatcar immediately after collision if they fail to connect?

3.4 Weightlessness: Apparent Weight of a Body in a Lift/Elevator

When a body of mass m is placed on a weighing machine which is placed in a lift, then actual weight of the body is mg, this acts on a weighing machine which offers a reaction R given by the reading of weighing machine. The reaction exerted by the surface of contact on the body is the apparent weight of the body.

Cases: 1. When lift is at rest then velocity $v = 0$ and acceleration $a = 0$

so, $R - mg = 0 \Rightarrow R = mg$

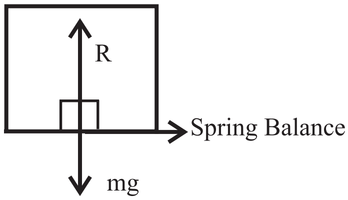
Therefore, appreant weight = actual weight

2. Lift moving upward with constant velocity

v = constant and a = 0

so R - mg = 0 ⇒ R = mg

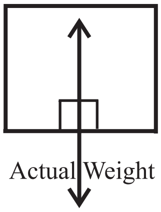
Again appreant wieght = actual weight



3. Lift accelerating upward with acceleration 'a'

R- mg = ma ⇒ R = m (g+a)

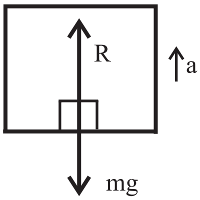
Apparent weight > Actual Weight



4. Lift is accelerating downward with acceleration 'a' :

mg - R = ma ⇒ R = m (g-a)

Apparent weight < actual weight



5. Lift accelerating downward at the rate of 'g'

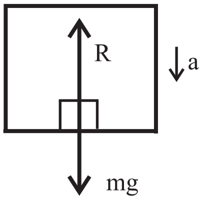
if a = g

R = m (g-g)

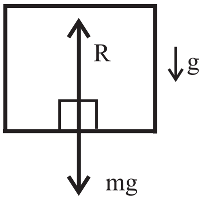
R= o

Apparent weight = 0

= Weight less ness



Thus when an object is in free fall it is weightless and this phenomena is called weightlessness.



3.5 Angular Momentum and its Conservation

Gyroscope is a device containing a rapidly spinning wheel that is used to detect the deviation of an object from its desired orientation. Gyroscopes are used in compasses and automatic pilots on ships and aircraft, in the steering mechanisms of torpedoes, and in the inertial guidance systems installed in space launch vehicles, ballistic missiles, and orbiting satellites.

Figure 25. Gyroscope: makes use of the concept of angular momentum

Figure 26. A skater moving in a straight line has angular momentum about any axis displaced from the path of the skater.

The central topic of this chapter is angular momentum, a quantity that plays a key role in rotational dynamics. Every rotational quantity that we have encountered is the analog of some quantity in the translational motion of a particle. The analog of momentum of a particle is angular momentum, a vector quantity denoted as **L**. In analogy to the principle of conservation of linear momentum, we find that the angular momentum of a system is conserved if no external torque acts on the system. Like the law of conservation of linear momentum, the law of conservation of angular momentum is a fundamental law of physics.

Imagine a rigid pole sticking up through the ice on a frozen pond Figure 26. A skater glides rapidly toward the pole, aiming a little to the side so that she does not hit it. As she approaches the pole, she reaches out and grabs it, an action that causes her to move in a circular path around the pole. Just as the idea of linear momentum helps us analyze translational motion, a rotational angular momentum helps us analyze the motion of this skater and other objects undergoing rotational motion.

Every rotational quantity that we have encountered in rotational motion is the analog of some quantity in the translational motion of a particle. The analog of momentum of a particle is angular momentum, a vector quantity denoted as **L**.

Its relationship to momentum **P** (which we will often call linear momentum for clarity) is exactly the same as the relationship of torque to force, τ = **r** x **F**. For a particle with constant mass m and velocity **v**, the angular momentum is

L = **r** x **P** = **r** x mv   Eq. 15

The value of **L** depends on the choice of origin O, since it involves the particle's position vector **r** relative to O. The units of angular momentum are kg m²/s.

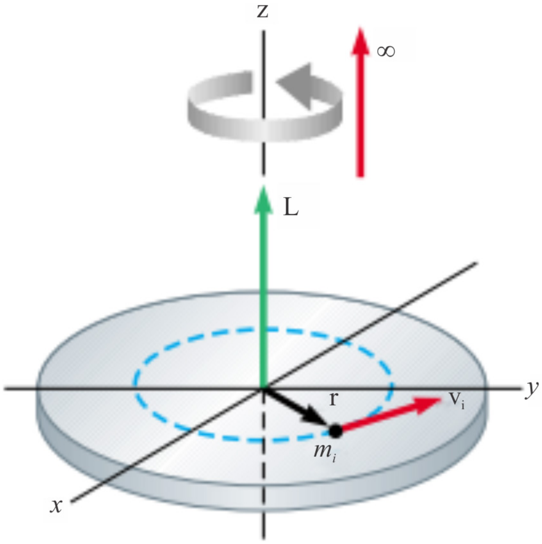


Figure 27. Angular momentum L is perpendicular to the plane of motion

KEY TERMS

- Angular momentum: momentum of a body due to its angular velocity.

The direction of **L** is determined by the right-hand rule. If a particle moves on the xy plane, its position vector **r** and linear momentum **P** = mv are both on the xy plane and, therefore, the angular momentum is along the z axis (perpendicular to the xy plane), Figure 27.

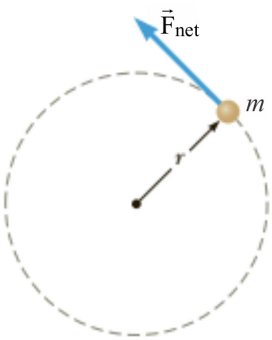


Figure 28. Mass m rotates in a circular path of radius r, acted on by a net force, F_{net}

Consider an object of mass m rotates in a circular path of radius r, acted on by a net force, F_{net} (Figure 28). The resulting net torque on the object increases its angular speed from the value ω_o to the value ω in a time interval Δt. Therefore, we can write

$$\Sigma\boldsymbol{\tau} = \mathbf{I}\boldsymbol{\alpha} = I\frac{\Delta\omega}{\Delta t} = I\left(\frac{\omega_f - \omega_i}{\Delta t}\right) = \frac{I\omega_f - I\omega_i}{\Delta t}$$

The product Iω is defined as the angular momentum (L) of the object.

$L = I\omega$ Eq. 3.16

Did you know?

Is Rotation Necessary for Angular Momentum?

Notice that we can define angular momentum even if the particle is not moving in a circular path. Even a particle moving in a straight line has angular momentum about any axis displaced from the path of the particle.

Activity 12

Recall the skater described at the beginning of this section, Figure 25. Let her mass be m. What would be her angular momentum relative to the pole at the instant she is a distance d from the pole if she were skating directly toward it at speed v? (a) zero (b) mvd (c) impossible to determine.

Activity 13

Consider again the skater in Figure 26. What would be her angular momentum relative to the pole at the instant she is a distance d from the pole if she were skating at speed v along a straight line that would pass within a distance a from the pole? (a) zero (b) mvd (c) mva (d) impossible to determine

Examples

A uniform solid sphere of moment of inertia 1.5 kgm^2 turns counterclockwise about a vertical axis through its center. Find its vector angular momentum when its angular speed is 3 rad/s. Assume the axis along the vertical is the z-axis.

Solution

Given

$I = 1.5\ kgm^2$, and $\omega = 3$ rad/s, we use $L = I\omega = 1.5\ kgm^2 \times 3\ rad/s = 4.5\ kgm^2/s$

Since the sphere rotates counterclockwise about the vertical axis, the angular momentum vector is directed upward in the +z direction.

The rotational analog of Newton's second law, which can be written in the form $\Sigma F = \Delta p/\Delta t$ is

$$\sum \tau = \frac{\Delta L}{\Delta t} = \frac{\textit{Change of angular momentum}}{\textit{time interval}}$$ Eq. 3.17

It states that the net torque acting on an object is equal to the time rate of change of the object's angular momentum. Recall that this equation also parallels the impulse–momentum theorem.

When the net external torque ($\Sigma\tau$) acting on a system is zero,

$$\Sigma\tau = \frac{\Delta L}{\Delta t} = 0,$$

which says that the time rate of change of the system's angular momentum is zero. We then have the following important result:

Let L_i and L_f be the angular momenta of a system at two different times, and suppose there is no net external torque, so $\Sigma\tau = 0$. Then

$$L_i = L_f \qquad \text{Eq. 3.18}$$

This gives us another conservation law to add to our list: conservation of angular momentum. We can now state that linear momentum and angular momentum of an isolated system all remain constant.

Activity 14

Group discussion.

What is a gyroscope and how does it work?

If the moment of inertia of an isolated rotating system changes, the system's angular speed will change.

Conservation of angular momentum then requires that

$$I_i\omega_i = I_f\omega_f \qquad \text{Eq. 3.19}$$

Note that conservation of angular momentum applies to macroscopic objects such as planets and people, as well as to atoms and molecules. There are many examples of conservation of angular momentum; one of the most dramatic is that of a figure skater spinning in the finale of his act. In Figure 29a, the skater has pulled his arms and legs close to his body, reducing their distance from his axis of rotation and hence also reducing his moment of inertia. By conservation of angular momentum, a reduction in his moment of inertia

(a) (b)

Figure 29. A skater varies his moment of inertia to change his angular speed. (a) By pulling in his arms and legs, he reduces his moment of inertia and increases his angular speed (rate of spin). (b) Upon landing, extending his arms and legs increases his moment of inertia and helps slow his spin.

must increase his angular speed. Coming out of the spin in Figure 28 b, he needs to reduce his angular speed, so he extends his arms and legs again, increasing his moment of inertia and thereby slowing his rotation.

Similarly, when a diver or an acrobat wishes to make several somersaults (Figure 30), she pulls her hands and feet close to the trunk of her body to rotate at a greater angular speed. In this case, the external force due to gravity acts through her center of gravity and hence exerts no torque about her axis of rotation, so the angular momentum about her center of gravity is conserved.

Figure 30. Tightly curling her body, a diver decreases her moment of inertia, increasing her angular speed.

For example, when a diver wishes to double her angular speed, she must reduce her moment of inertia to half its initial value.

Did you know?

An interesting astrophysical example of conservation of angular momentum occurs when a massive star, at the end of its lifetime, uses up all its fuel and collapses under the influence of gravitational forces, causing a gigantic outburst of energy called a supernova.

Activity 15

If global warming continues, it's likely that some ice from the polar ice caps of the Earth will melt and the water will be distributed closer to the equator. If this occurs, would the length of the day (one rotation) (a) increase, (b) decrease, or (c) remain the same?

Activity 16

A cat usually lands on its feet regardless of the position from which it is dropped. A slow-motion film of a cat falling shows that the upper half of its body twists in one direction while the lower half twists in the opposite direction. (See Figure 31.) Why does this type of rotation occur?

Figure 31. Slow-motion picture shows how cats survive falls from heights.

Examples

A student sits on a pivoted stool while holding a pair of weights, (See Figure 32). The stool is free to rotate about a vertical axis with negligible friction. The moment of inertia of student, weights, and stool is 2.25 kg m2. The student is set in rotation with arms outstretched, making one complete turn every 1.25 s, arms outstretched. (a) What is the initial angular speed of the system? (b) As he rotates, he pulls the weights inward so that the new moment of inertia of the system (student, objects, and stool) becomes 1.8 kg m^2. What is the new angular speed of the system?

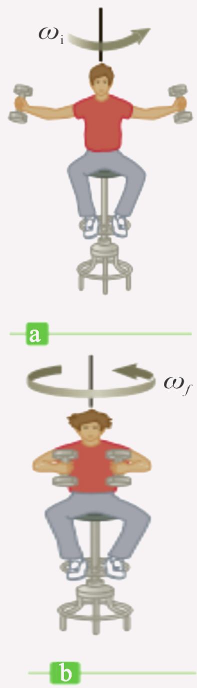


Figure 32.

Solution

Given

Time taken for one complete rotation, period T = 1.25 s

Initial moment of inertia $I_i = 2.25$ kgm^2

Final moment of inertia $I_f = 1.8$ kgm^2

(a) The initial angular speed $\omega = \omega = \frac{2\pi}{T} = \frac{2(3.14)}{1.25\ s} = 5\ rad/s$ = 5 rad/s

(b) Applying the conservation of momentum

$I_i \omega_i = I_f \omega_f$

$$f = \frac{I_i \omega_i}{I_f} = \frac{2.25\ kg/m^2}{1.8\ kg/m^2} = (5\ rad/s^2) = 6.25\ rad/s$$

Activity 17

If you stop a spinning raw egg for the shortest possible instant and then release it, the egg will start spinning again. If you do the same to a hard-boiled egg, it will remain stopped. Try it. Explain it.

Examples

A horizontal disk with moment of inertia I1 rotates with angular speed ω1 about a vertical frictionless axle. A second horizontal disk having moment of inertia I2 drops onto the first, initially not rotating but sharing the same axis as the first disk, Figure 33. Because their surfaces are rough, the two disks eventually reach the same angular speed, ω. Find the ratio ω/ω1.

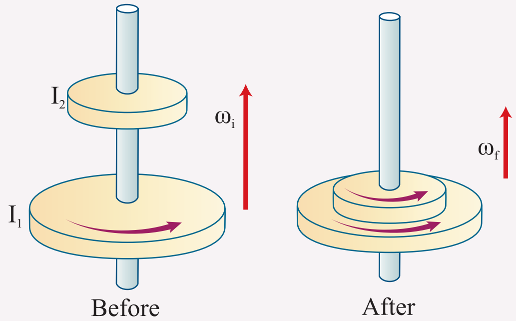


Figure 33.

Solution

For a system of two objects the conservation of momentum can be written as

$$I_1\ \omega_{1i} + I_2\ \omega_{2i} = I_1\ \omega_{1f} + I_2\ \omega_{2f}$$

Since the discs stick together after interaction, we have $\omega_{1f} = \omega_{2f} = \omega$

$$I_1 \omega_{1i} + I_2 \omega_{2i} = (I_1 + I_2)\ \omega$$

$$\omega = \omega = \frac{I_1\omega_{1i} + I_2\omega_{2i}}{I_1 + I_2}$$

With $\omega_{2i} = 0$, we have $\omega = \omega = \left(\frac{I_1}{I_1 + I_2}\right)\omega_{1i} = \left(\frac{I_1}{I_1 + I_2}\right)\omega_1 \Rightarrow \frac{\omega}{\omega_1} = \frac{I_1}{I_1 + I_2}$

Activity 18

Group work

Bicycle Wheel Gyro

Tools and Materials

- Two handles (plastic handles)
- A bicycle wheel
- A low-friction rotating stool or platform (typing or computer chairs often work well)
- A partner

Optional: plastic spoke guards, screw eye (also known as an eye bolt), drill, hook, chain or rope suspended from a large stand or a ceiling.

Set up of materials

1. Screw the handles onto each side of the wheel's axle. You may have to remove the outer nuts to clear enough axle for the handles. You may want to put plastic spoke guards on the hubs first to protect your fingers from the spinning wheel.

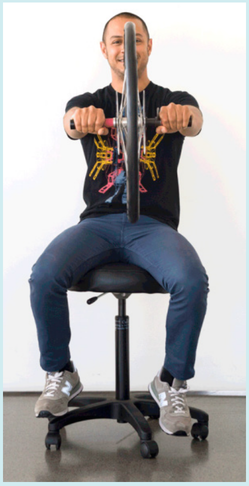


Figure 34.

To Do and Notice

Sitting on the stool or chair, hold the wheel by the handles while another person gets it spinning as fast as possible, Figure 34. Lift your feet off the floor and tilt the wheel. If the stool has sufficiently low friction, it should start to turn. Tilt the wheel in the other direction and see what happens.

2. If you have the screw eye, drill a hole in the end of one handle and mount the screw eye in the hole. (See Figure 35 below for assembly.)

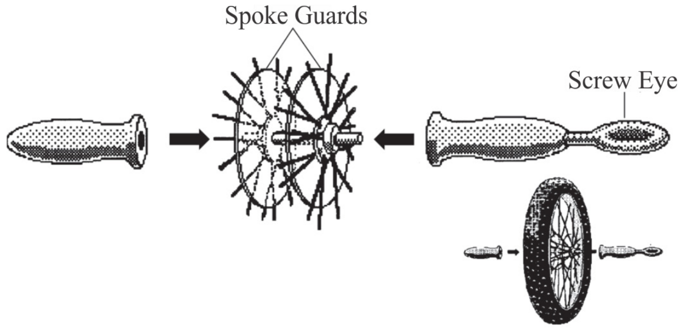


Figure 35. How to assemble parts

Get the wheel spinning, then use the screw eye in the end of the handle to hang the wheel from a hook mounted to the free end of a chain or rope, Figure 36. Hold the wheel so that the axle is horizontal, then release it. The axle will remain more or less horizontal while it moves slowly in a circle. (See diagram; click to enlarge.)

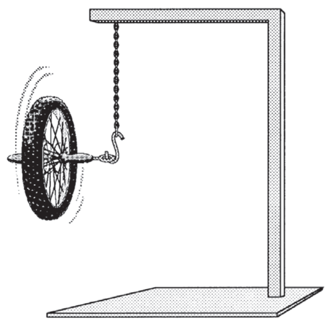


Figure 36. The wheel does not fall while spinning

Exercises

1. State the law of conservation of angular momentum.
2. How can you use the right-hand rule to determine the direction of angular momentum
3. given the position vector **r** and velocity **v** of a particle? (b) given the direction of angular velocity ω?
4. In an isolated system the moment of inertia of a rotating body doubled. What will happen to the angular velocity of the body?

5. A disc is spinning about a vertical axis at 25 rad/s. An identical disc initially non-rotating is placed on the first disc. If the friction force between the discs is high enough that they rotate together, what is the new angular speed of the system?
6. A mouse is initially at rest on a horizontal turntable mounted on a frictionless vertical axle. If the mouse begins to walk clockwise around the perimeter, what happens to the turntable? Explain.
7. Why is it easier to balance on a bicycle when it is moving? Why is it so difficult to balance on a bicycle when it is still?

Experiment on conservation of momentum

Aim

The aim of this experiment is to determine the velocity of a body by means of a ballistic pendulum.

Theory

Conservation Laws and the Ballistic Pendulum

Conservation laws apply here to energy and momentum.

Mechanical energy is the sum of potential energy (PE) and kinetic energy (KE).

Gravitational potential energy if energy due to position.

PE = mgh, where m is mass, g is the acceleration by gravity and h is the change in height measured from the reference point.

Kinetic energy is energy of m5ion. The formula for Kinetic energy is

$$KE = \frac{1}{2} mv^2$$,where m is still mass and v is velocity.

$$ME = PE + KE$$

The law of conservation of mechanical energy states that in the absence of dissipative the total mechanical energy of an object or system of objects remains constant.

Momentum (P) is another quantity that is conserved. Momentum is simply the product of mass and velocity:

$$P = m\mathbf{V}$$

In the absence of unbalanced external force, the amount of momentum before collision equals the amount of momentum after an event.

$$\mathbf{P}_i = \mathbf{P}_f$$

$$M_1\mathbf{U}_1 + M_2\mathbf{U}_2 = M_1\mathbf{V}_1 + M_2\mathbf{V}_2$$

There are Elastic collisions, Inelastic collisions and Perfectly Inelastic collisions.

Perfectly inelastic collision is a collision where one or both objects are permanently deformed and they stick together. Once again, momentum is conserved but KE is not. In the case of a perfectly inelastic collision between two objects the conservation equation is:

$$M_1\mathbf{U}_1 + M_2\mathbf{U}_2 = (M_1 + M_2)\mathbf{V}$$

Where **U** is for velocity before collision and **V** is velocity after collision.

In this experiment you will study an inelastic collision using a Blackwood ballistic pendulum. The colliding bodies are a small metal ball, which is fired from a spring loaded gun, and a metal receptacle, or catcher. The receptacle is also the bob of a simple pendulum. Initially the pendulum is at rest. When the gun fires, the ball collides with the pendulum and is trapped in the catcher which then starts to swing. A ratchet and pawl system catches the pendulum at the height of its swing.

The best way to understand this experiment is to divide it into three separate events. First, the gun fires and the ball of mass m travels horizontally with initial velocity U In the absence of external forces, the horizontal component of its velocity will not change. The horizontal component of the ball's initial linear momentum is:

$$\mathbf{P}_i = m\mathbf{U} \qquad (1)$$

In the second event, the ball collides with the "catcher" of mass M and is trapped by the spring. The system loses kinetic energy in the deformation of the spring and the creation of sound. Linear momentum, however, must be conserved. The pendulum, of mass (M + m), moves with a new horizontal velocity, V. The momentum of the system is now:

$$P_f = (M + m)V \qquad (2)$$

Since the two momenta are equal, we can solve for U:

$$U = \frac{(M+m)}{m}V \qquad (3)$$

Finally, the system acts like a simple pendulum. It moves upward and is caught by the ratchet at the highest part of its swing. The ratchet and pawl are designed to exert negligible force on the pendulum while it is moving upward, so mechanical energy is conserved. This means that the pendulum's kinetic energy at the bottom of its swing must equal its potential energy at the top of its swing. The change in height, h, of the center of mass can be easily measured. We can then solve for V:

$$\frac{(M+m)}{m}V^2 = (M+m)gh \qquad (4)$$

Therefore:

$$V = \sqrt{2gh} \qquad (5)$$

Substitute this value for V into equation 3:

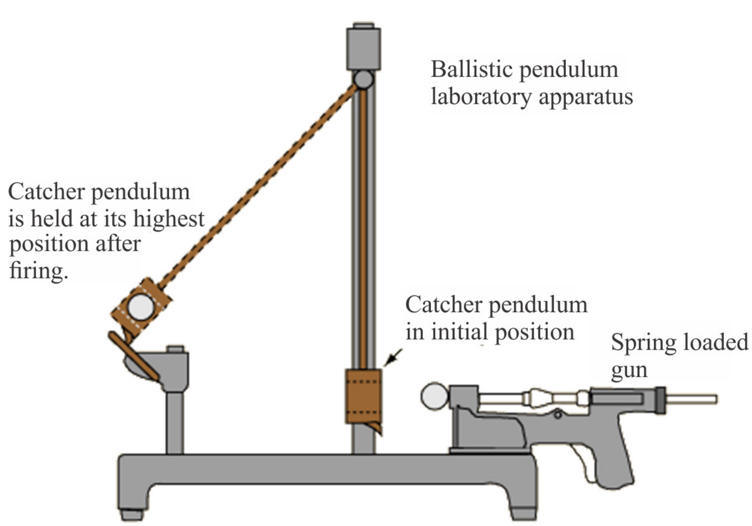
$$U = \frac{(M+m)\sqrt{2gh}}{m} \qquad (6)$$

Apparatus

A spring loaded gun

Catcher pendulum

Set up of apparatus



Procedure

Never point the gun at anyone !!!

Your instructor will show you how to load and fire the gun. Occasionally the ball will miss the catcher; therefore do not fire it if anyone is within range of the ball.

Be careful when you remove the ball from the catcher; push the spring inward to release the ball. The spring will break if you pull it backwards.

Unscrew the set screw that holds the pendulum at the top of the apparatus. Measure the mass of the ball (m) and of the ball and pendulum together (M + m). Reassemble the apparatus. Make sure that the pendulum can swing freely with a minimum of side-to-side motion.

1. Fire the gun and record the number of the notch in which the pendulum comes to rest. Repeat twice. If the notch numbers vary greatly, or if you have other problems, consult your instructor.
2. While the pendulum is hanging freely, measure the height of the center of mass marker from the base of the apparatus.
3. Average the notch numbers and raise the pendulum to that notch. Measure the height of the center of mass marker from the base of the apparatus. The difference between the two heights is h.
4. Use equation 6 to find the initial velocity of the ball.

SUMMARY

- Linear momentum is a vector quantity and it is defined as the product of mass and velocity.
- Newton's second law of motion can be restated as net force is equal to the time rate of change of momentum.
- Impulse is defined as the product of the average force applied and the duration for which the force is acted. It is equal to the change of momentum.
- The area under net force versus time graph provides the impulse or the change of momentum.
- A collision is an interaction between two objects in which a force acts on each object for a period of time.
- Kinetic energy is the energy possessed by a body due to its motion. E_k = $E_k = \frac{1}{2}mv^2$
- In an elastic collision, both momentum and kinetic energy are conserved. In an inelastic collision, momentum is conserved but kinetic energy is not. In a perfectly inelastic collision, momentum is conserved, kinetic energy is not, and the two objects stick together after the collision, so their final velocities are the same.
- For any system, the forces that the particles of the system exert on each other are called internal forces. Forces exerted on any part of the system by some object outside it are called external forces.
- For an isolated system in which there is interaction among particles, the total momentum of the system is constant, even though the individual momenta of the particles that make up the system can change.

- The momentum of an isolated system is conserved, but not necessarily the momentum of one particle within that system.
- The angular momentum of a rotating object is given by **L** = **r** x **P**
- Angular momentum can also be expressed as L = Iω
- Angular momentum is related to torque as $\sum \tau = \frac{\Delta L}{\Delta t}$
- If the net external torque acting on a system is zero, the total angular momentum of the system is constant and is said to be conserved, $L_i = L_f$

Review Exercises

1. During one part of the liftoff of a model rocket, its momentum increases by a factor of four while its mass is halved. The velocity of the rocket is initially 8.5 m/s [up]. What is the final velocity during that time interval?
2. A 3 kg particle has a velocity of $(3\hat{i} - 4\hat{j})$ m/s. (a) Find its x and y components of momentum. (b) Find the magnitude and direction of its momentum.
3. A 0.1kg ball is thrown straight up into the air with an initial speed of 15 m/s. Find the momentum of the ball (a) at its maximum height and (b) halfway up to its maximum height.
4. The blue whale is the largest mammal ever to inhabit on Earth. Calculate the mass of a female blue whale if, when alarmed, it swims at a velocity of 57 km/h, East and has a momentum of 2.15×10^6 kgm/s, East.
5. Two identical carts each of mass 800 g are moving toward each other in opposite directions. One of them is traveling to the right at 3 m/s and the other to the left at 4 m/s. The carts stick together after collision and move together at a common velocity of 0.5 m/s to the left.
   (a) By what factor did the kinetic energy of the system change?
   (b) What fraction of the initial kinetic energy is lost as the result of collision?
6. A 1500-kg car traveling east with a speed of 25 m/s collides at an intersection with a 2500-kg van traveling north at a speed of 20 m/s, as shown in Figure 37. Find the direction and magnitude of

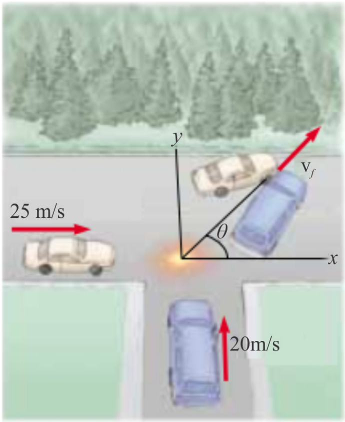


Figure 37. An eastbound car colliding with a northbound van.

the velocity of the wreckage after the collision, assuming that the vehicles undergo a perfectly inelastic collision (that is, they stick together).

7. In a game of billiards, a player wishes to sink a target ball in the corner pocket, as shown in Figure 38. If the angle to the corner pocket is 35°, at what angle θ is the cue ball deflected? Assume that friction and rotational motion are unimportant and that the collision is elastic. Also assume that all billiard balls have the same mass m.

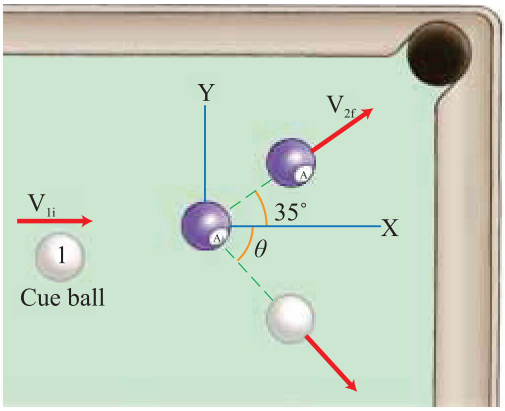


Figure 38. The cue ball (1) strikes the ball (2) and sends it toward the corner pocket

8. A 1.5 kg particle moves in the xy plane with a velocity of $\mathbf{v} = (4.2\hat{i} - 3.6\hat{j})$ m/s. Determine the angular momentum of the particle when its position vector is $\mathbf{r} = (1.5\hat{i} + 2.2\hat{j})$ m.

9. A student stands on a freely rotating platform, as shown in Figure 39. With his arms extended, his rotational angular frequency is 0.25 rev/s. But when he draws his arm in, that angular frequency becomes 0.80 rev/s. Find the ratio of his moment of inertia in the first case to that in the second.

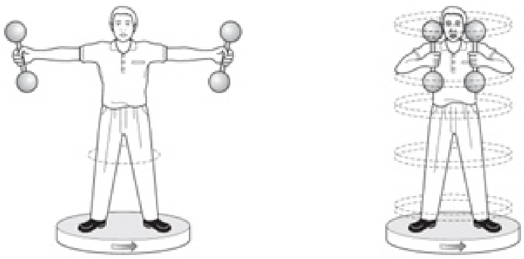


Figure 39.

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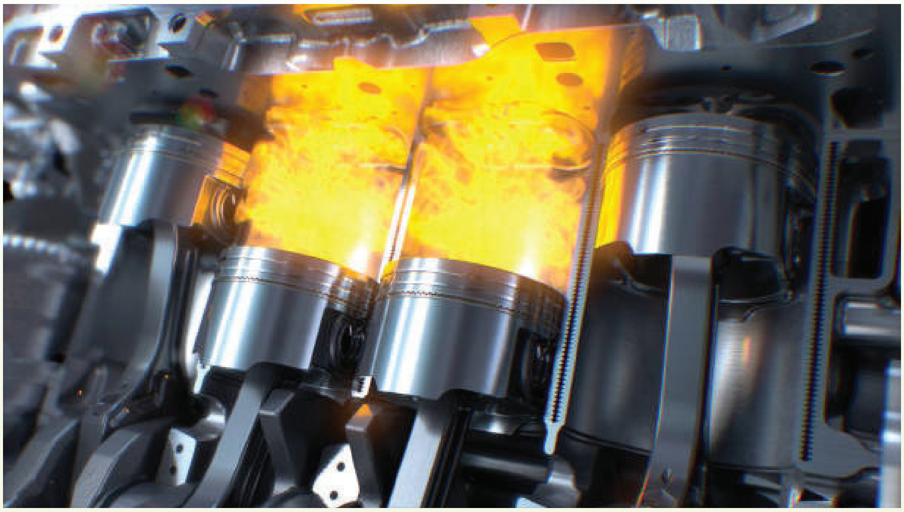
CHAPTER 4

HEAT



Chapter Contents

Chapter Outcome

Learners will able to:

- recognize and appreciate the importance and effect of heat energy in our environment.

Chapter Objectives

Upon completion of this chapter, learners will:

- elaborate the concept and characteristics of heat
- distinguish between specific heat and specific heat capacity.
- distinguish between the methods of heat transfer and the laws of heat exchange.
- discuss heat transfer in a vacuum flask.
- use a simple calorimeter to conduct an experiment to measure heat transfer.
- calculate the specific heat of a solid sample using the calorimeter experiment data.
- analyze possible sources of error in a calorimetry experiment.
- determine the specific heat capacity of a given the solid by method of mixtures.
- distinguish between latent heat of fusion and vaporization.

Introduction

In this unit we will discuss about temperature and heat which are a fundamental aspects of many physical situations. Temperature and heat are very different concepts and in this unit we make a clear distinction between them. We will also learn how to measure heat energy and the factors on which heat energy for a given body depends. We will then learn the difference between specific heat capacity and heat capacity. We will also discuss the mechanisms by which heat can be transferred from one body to another (or from one region to another). Finally we will discuss how heat is used in calorimetery, and how it is involved in changes of states of matter.

4.1 Heat

The terms "temperature" and "heat" are often used interchangeably in everyday language. In physics, however, these two terms have very different meanings. In this section we'll define temperature and see that heat refers to energy transfer caused by temperature differences and learn how to calculate and control such energy transfers.

Key terms

- Heat is a form of energy that flows from a body at a higher to a body at a lower temperature.

Temperature and Heat

In grade 10, under the topic kinetic theory, you have learnt that particles of any substance are in random motion. In solids, particles are vibrating. In liquids and gases, particles are moving freely. The faster the motion of particles of a substance the hotter is the substance. When an object is hot the molecules have a lot of energy and move fast. When an object is cold, the molecules have little energy and move slowly. If a hot object with high-energy atoms comes in thermal contact with a cool object with a relatively low-energy atoms, the atoms of the hot object will give some of their energy to the atoms of the cool object. This can be demonstrated as follows.

Take three small containers. Label them as A, B and C. Put cold water in container A and hot water in container B. Mix some cold and hot water in container C. Now dip your left hand in container A and the right hand in container B. After keeping the hands in the two containers for 2–3 minutes, put both the hands simultaneously in container C. Do both hands get the same feeling?

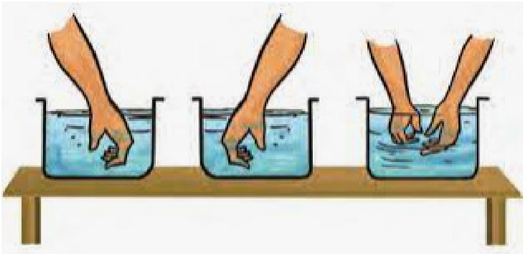


Figure 1. Feeling water in three containers

Warning! Make sure that the hot water is not so hot that it won't burn your hand.

From this demonstration we see that, our left hand which was in the cold water feels hotter, while our right hand which was in the hot water now feels colder. That is, the hotter object cools down and the colder object warm up. This means the colder water gains energy, while the hotter water loses energy.

When we place two objects at different temperatures in thermal contact with each other, the particles at the boundary collide and transfer kinetic energy from the faster to the slower particles. Thus, there is a net transfer of energy from the hotter body to the colder one. This energy that flows from a warm body to a cool body by virtue of temperature differences is called **heat**. Heat is thus a form of energy.

Two objects are said to be in **thermal contact** with each other if energy can be exchanged between them. Note that, the energy transfer will continue until both objects are at the same temperature. When the two objects attain the same temperature, then the flow of heat stops. This level is what we call **thermal equilibrium**.

Everyday examples of sources of heat energy: the sun, a hot stove, automobile fuels, any fire, hot water and geothermal are sources of heat energy.

Heating Safety rules

1. Turn off heat sources when they are not in use.
2. Point test tubes away from yourself and others when heating substances in them.
3. Use the proper procedures when lighting a Bunsen burner.
4. To avoid burns, do not handle heated glassware or materials directly. Use tongs, test-tube holders, or heat-resistant gloves or mitts.

5. Make sure you wear protective gear, as needed. Basics include a lab coat and safety goggles. You may also need gloves, hearing protection, and other items, depending on the nature of the experiment.

Safety Symbols: Hot surfaces have the symbol shown in the figure.

HOT SURFACE

Relationship between heat and temperature

Heat and temperature are two different but closely related concepts. Heat is the cause but. Temperature is the effect. Heat is a form of energy but temperature is the property of a body which indicates the hotness or coldness of a body. Note that they have different units: temperature has units of degrees Celsius (°C) or Kelvin (K), and heat has units of energy, Joules (J).

Heat transfer is a result of temperature difference. Heat transfer occurs only in the direction from a hot object to a cold object. Thus temperature determines the direction of heat flow i.e., it determines whether the body will receive heat from another body or give heat to the body.

Temperature is a measure of the average kinetic energy of the atoms or molecules in the system. It is a measurement of how fast the atoms or molecules are moving within an object. The faster they move, the hotter is the object.

Did you know?

The First Technologies are originated from fire (heat). It was the experiences of people of the Stone Age in tool-making that ultimately led to modern technologies. The control of fire by early humans was a critical technology enabling the evolution of humans. Fire provided a source of warmth and lighting, protection from wild animals (especially at night), a way to create more advanced hunting tools, and a method for cooking food and preserve meat with smoke. Later in the 19^{th} century, the energy of burning fuels led to the invention of Steam Engine which lead the world to the industrial revolution. There's evidence that humans used fire in these ways as far back as 2 million years ago.

What are the importance of Heat Energy in our Environment?

In our daily life heat energy offers many services. It is used

- as a source of power (generate electricity), in geothermal power plants, for warming our houses on cold morning days.
- in the industry for food processing, manufacturing of glass, paper, textile and to melt metals so that it can be shaped to the required design …etc.
- for cooking, for making tea or coffee.
- in temperature maintenance of warm-blooded animals.
- in the form of heating pads (thermotherapy). It is believed that heat increases blood circulation, promotes healing.
- for drying washed clothes.
- for sterilization.
- in incubation, to grow chicken in the egg.
- When the heat from the sun contacts water on the earth, it evaporates by forming water vapor, which causes the water cycle. The water cycle is a phenomenon responsible for the rain and life on earth.
- Heat energy is the base for all automotive vehicles like cars, buses, trucks, etc.

Activity 1

1. List what heat can do in our daily lives.
2. List how heat is used in factories and plants to make our daily lives convenient.
3. Share your ideas with your classmates.

Quantity of Heat

We have seen that heat is a form of energy that is shared between two bodies at different temperature. We use the symbol Q for amount (quantity) of heat. The factors that affect the amount of heat energy gained by a cold body or lost by a hot body can be shown through an experiment.

Experiment: Verifying the relationship between heat and mass

The quantity of heat is related to the mass of the substance being heated. We will now illustrate this concept experimentally.

The materials required: a large bowl containing boiling water, heat source, two thermometers, two identical glass beakers and a timer.

Procedure:

- Pour some boiling water and put the bowl on an electric hot-plate and stir it continuously so that its temperature remains close to 100°C.

- Now mark the two identical laboratory glass beakers A and B. Pour 500 g of water at room temperature into beaker A and 1000 g of water at room temperature into beaker B.
- Support the beakers side by side with their bases just in contact with the hot water as shown in Figure 2 so that they are heated equally.
- Record the reading on each thermometer at each subsequent half-minute. The water in each beaker must be vigorously stirred before the reading is taken.

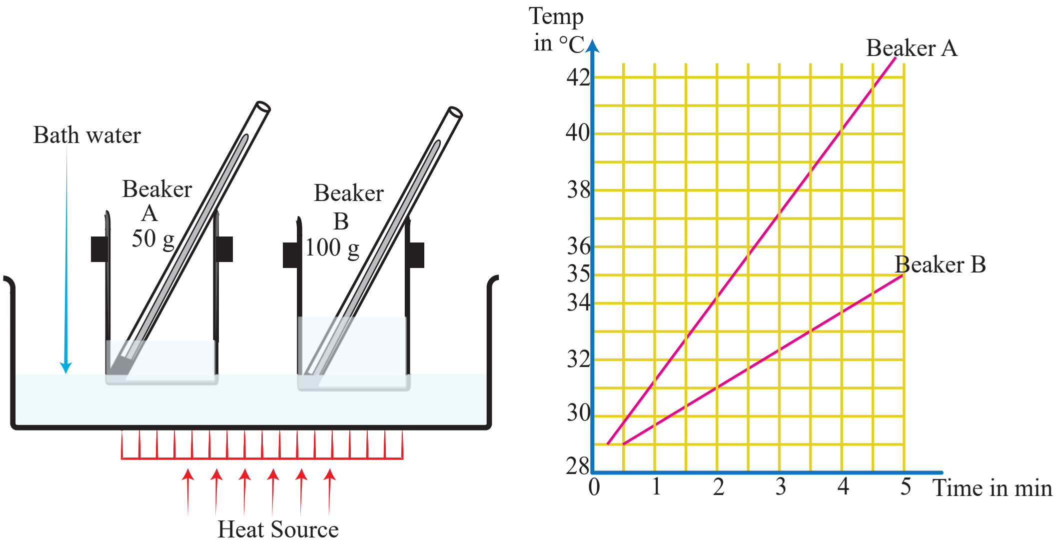


Figure 2. The relationship between heat and mass.

Table 1 Temperature of waters in beakers A (500g) and beaker B (1000g)

Time (min)	Temperature of beaker A	Temperature of beaker B
0.0		
0.5		
1.0		
1.5		
2.0		
2.5		
3.0		
3.5		
4.0		

4.5		
5.0		

Plot the two sets of readings on the same graph as in Figure 2 (b)

Questions

1. Which water becomes hotter quickly? Beaker A or beaker B? Why?
2. If the rate of heat supply is equal for both beakers, on what does the rise in temperature depend?
3. Draw a horizontal line say at 35°C as shown in the Figure 2. What time does beaker A and beaker B take to rise to this temperature?

For the same temperature record of both beakers does it take the same amount of heat? Which beaker takes more time?

As more time is taken more heat is required. Referring to Figure 2, it takes 5 minutes for the 1000g water to reach to a temperature of 35°C, but it takes around 2.5 minutes for the 500g water to rise to the same temperature (35°C). The heat gained or lost is directly proportional to the mass of the substance for a fixed rise in temperature.

$$Q \quad \alpha \quad m$$

For a fixed mass of a given substance heat is directly proportional to a change in temperature. For Example it is found experimentally that 4,200J of heat is required to raise the temperature of 1kg of water by 1°C. Therefore, it requires 42,000J of heat energy to raise the temperature of 1kg of water by 10°C.

$$Q \quad \alpha \quad \Delta T$$

Putting these two observed facts together, we find that for a given substance, the heat (Q) it gains or loses is directly proportional to the product of the mass (m) and the change in temperature (ΔT)

$$Q \quad \alpha \quad m\,\Delta T$$

$$\Rightarrow \quad Q = mc\Delta T$$

Where the proportionality constant c is called the **specific heat capacity**. The SI unit of heat is joule. Calorie is a commonly used non – SI unit of heat. The unit calorie is defined as the amount of heat necessary to raise the temperature of one gram of water by one degree celsius. That amount equals 4.20 J. Thus,

1 Calorie = 4,200 J

Examples

How much energy does it take to raise the temperature of 100 g of copper by 10°C? (Specific heat of copper = 386J/kg°C)

Solution

$c = 0.386$ J/g.°C, $m = 0.1$ kg; $\Delta T = 10$°C

The amount of heat energy, Q, absorbed by the copper is

$Q = cm\Delta T = 0.386\ \text{J/g.°C} \times 100\ \text{g} \times 10\text{°C}$

$= 386\ \text{J}$

Examples

A 50 g piece of aluminum is at 20°C. If 500 calorie of energy is transferred to the aluminum, what is its final temperature? (Specific heat of aluminum = 900J/kg°C)

Solution

$Q = 500$ calorie $= 500$ calorie $\times 4.20$ J/calorie $= 2100$ J, $c = 900$J/kg °C

$m = 50\ \text{g} = 50 \times 10^{-3}\ \text{kg}$

Solving the equation $Q = mc\Delta T$ for ΔT, we find

$$\Delta T = \frac{Q}{cm} = \frac{2100\text{J}}{(900\ \text{J/kg°C})(50\times10^{-3}\text{kg})} = 46.7\text{°C}$$

$T_f = T_i + \Delta T = 20\text{°C} + 46.7\text{°C} = 66.7\text{°C}$

Exercises

1. Discuss the characteristics of heat.
2. 2000J of energy is needed to heat 1kg of paraffin through 1°C. How much heat is needed to heat 2kg of paraffin through 10°C?
3. A 1.5 kg copper block is given an initial speed of 3.0 m/s on a rough horizontal surface. Because of friction, the block finally comes to rest. (a) If the block absorbs 85 % of the initial kinetic energy as heat energy, calculate its increase in temperature. (b) What happens to the remaining energy?
4. An aluminum saucepan of mass 400 g, containing 750 g of water, is heated on a stove. How much energy would be required to bring the water from a room temperature of 20°C to the boil?

4.2 Specific heat and Specific capacity

Key terms

- Specific heat capacity (c) is the heat energy required to raise the temperature of unit mass by one degree (°C or K).
- Heat capacity (C) is the heat energy required to raise the temperature of a body by one degree (°C or K).

In this section you will learn the relationship and difference between specific heat capacity and heat capacity of a body.

Specific Heat Capacity

The amount of heat required to cause the same rise in temperature for a given mass of different substances is different. For Example 1kg of water requires 4,200 J of heat to increase its temperature by 1°C, but 1kg of silver requires only 234J of heat to increase its temperature by the same 1°C. Thus, every substance has a unique value of heat to increase a unit mass of it by 1°C or 1K. This quantity is known as the **specific heat capacity.** The heat needed to change the temperature of a unit mass of a substance by one degree (1°) is called the specific heat capacity (c) of the substance.

$$c = \frac{Q}{m\Delta T}$$

The SI unit of specific heat capacity c, is J/kg.K or J kg.°C

Note that change in temperature is the same in both Kelvin and Celsius scales. For Example ΔT = 10 K= 10°C, whereas 10°C = 283K.

Different substances have different values of specific heat capacity. This means, every substance requires a unique amount of energy per unit mass to change its temperature by 1°C. Hence specific heat capacity is the characteristics of that substance. The greater a material's specific heat, the more energy must be added to a given mass of the material to cause a particular temperature change.

Water has the highest value of specific heat capacity. The specific heat capacity of water is equal to 4,200 J/(kg.K). Because of its high specific heat capacity; water is used as a cooling agent in machines.

The specific heat capacity of some common substances is given below in table 2.

Table 2 The specific heat capacity of different substances.

Substance	C (J/kg.°c)
Aluminum	890
Brass	378
Copper	386
Gold	132
Ice (0°C)	2142
Iron	480
Mercury	140
Water	4200
Lead	128
Silver	235

Heat Capacity (C):- In the case of specific heat capacity we measure the heat required per unit mass of that body. If we want to consider the total mass of the body instead of a unit mass, we use the term heat capacity. The **heat capacity** denoted by capital C of a particular sample of a substance is defined as the amount of heat energy needed to raise the temperature of that sample by 1K (or 1^0C).

$$C = \frac{Q}{\Delta T}$$

The SI unit of C is J/kg or J/^{0}C

Since $\textit{specific heat}\,(c) = \frac{Q}{m\Delta T}$ and $\textit{heat capacity}\,(C) = \frac{Q}{\Delta T}$, thus the heat apacity of a body is the product of the mass of the body with its specific heat capacity.

Heat capacity = mass × specific heat capacity

$C = mc$

Examples

What is the specific heat of a metal if 135 kJ of heat is needed to raise 5.1 kg of the metal from 20°C to 30°C?

Solution

$Q = 135\text{ kJ} = 1.35 \times 10^5\text{ J}$, $\Delta T = 30°C - 20°C = 10°C$, $m = 5.1\text{ kg}$

From the definition of specific heat, we find

$$c = \frac{Q}{m\Delta T} = \frac{1.35\times10^5 J}{(5.1\text{ kg})(10°C)} = 2.65\times10^3 J/kg°C$$

Examples

It requires 200J of heat energy to raise the temperature of 80g of a certain substance from 20°C to 25°C. What is the value of the specific heat capacity and heat capacity of the substance?

Solution

$Q = 200J$; $m = 80g = 8 \times 10^{-2}kg$; $T_i = 20°C$; $T_f = 25°C$

$$c = \frac{Q}{m\Delta T} = \frac{200J}{8\times10^{-2}kg\,(25°C - 20°C)} = 500J/kg.°C$$

$$C = mc = 8\times10^{-2}kg\,(500\frac{J}{kg°C}) = 40J/°C$$

James Prescott Joule (1818 – 1889)

James Prescott Joule (1818 – 1889) was an English physicist, mathematician and brewer. He is best known for establishing the relationship between mechanical work and heat transfer. Joule studied the nature of heat, and established that the various forms of energy are basically the same and can be changed one into another. This led to the law of conservation of energy, which in turn led to the development of the first law of thermodynamics.

His first experiments concerned electric motors with a view to replacing the steam engines in the brewery with electric ones. This led him to his first law: Joule's Law in 1840. According to Joule's law, the heat generated in an electric wire is proportional to the current squared multiplied by the resistance (**$P = I^2R$**). Joule was the first scientist to identify the property of ferromagnetic materials in 1842. In 1843 he published his value for the amount of work required to produce a unit of heat, called the mechanical equivalent of heat. In 1852 Joule and William Thomson (later Lord Kelvin) discovered that when a gas is allowed to expand without performing external work, the temperature of the gas falls. This "Joule-Thomson effect" was used to build a large refrigeration industry in the 19th century. He also worked with Lord Kelvin to create the absolute temperature scale now known as the "Kelvin scale." He is also credited with the first-ever calculation the velocity of a gas molecule.

Exercises

1. Define the terms specific heat capacity and heat capacity.
2. What s the relationship between specific heat capacity and heat capacity?
3. What is the heat capacity of a 15 kg copper?
4. When 860 J of heat energy is supplied to 0.4 kg of a substance, its temperature raises from 25°C to 50°C. Determine the specific heat capacity of the substance.

4.3 Heat Transfer and the Laws of Heat Exchange

In this section you will learn the various processes by which heat flows from one body to another. Along the way, you will apply the law conservation of energy or the law of heat exchange in solving problems. We will then discuss how heat is used in calorimetery, how it is involved in changes of states of matter, and finally we will discuss about the heat energy converters.

Heat Transfer

We have defined heat as energy that flows. As far as two objects or systems are in thermal contact heat will transfer from the hotter to a cooler region. Heat energy can be transferred from one place or body to another in three different ways. These are: **conduction**, **convection**, and **radiation**.

Heat transfer: Conduction

If you hold one end of a copper rod and place the other end in a flame (Figure 3), the end you are holding gets hotter and hotter, even though it is not in direct contact with the flame. When we heat one end of an object (say a metal bar) the molecules there vibrate faster and faster. As a result they collide with their slower moving neighbors. During the collision they transfer some of their kinetic energy to molecules having less kinetic energy. These in turn transfer some of their energy by collision to their neighboring molecules. In this way energy is transferred from a high temperature region to a lower temperature region. A process of heat transfer in which energy is passed Heat transfer by conduction from molecule to molecule through a material by successive molecular collision is called **conduction**.

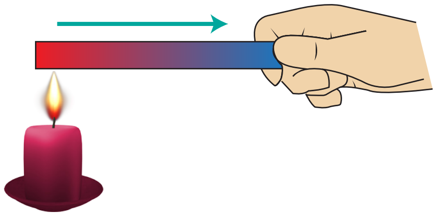


Figure 3. Heat transfer by conduction

Although heat is transmitted through a material medium, there is no transfer of the particles of the medium in the process of conduction.

ACTIVITY 2

1. What is the reason for good conductor of metals?
2. Which one is a better conductor? copper or iron?

Copper is a good conductor of heat than iron. This is demonstrated in Figure 4. Take a long bar made of copper at one end and iron at the other, joined together with a rivet. At regular intervals along it attach ball bearings with candle wax. Heat the bar strongly at its midpoint with a Bunsen burner. Record your observations.

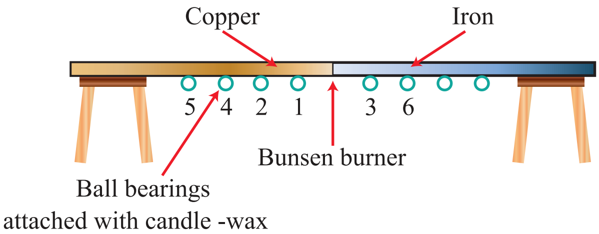


Figure 4. How to show that copper is a better conductor of heat than iron.

As heat is conducted along the bar in both directions, the wax melts and the ball bearings drop off. The iron conducts the heat, but not as well as the copper: the numbers on Figure 4 indicate a likely order in which the balls will fall.

Heat transfer: Convection

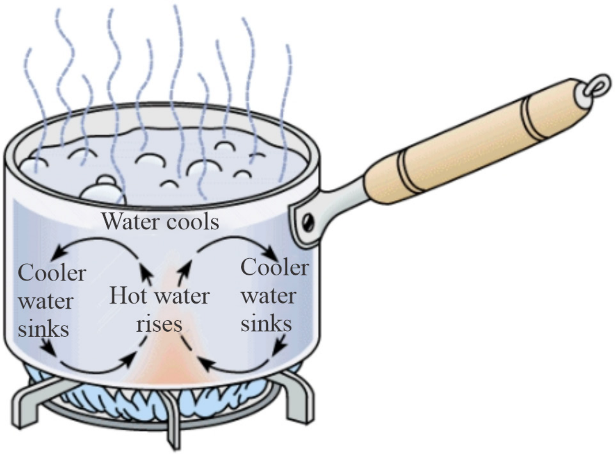


Figure 5. Heat transfer by convection

Suppose you want to boil a tea using a tea kettle, and place the kettle on a store (Figure 5). When the water is heated it expands and hence its density decreases. This causes the hot water to rise up, because it is surrounded by more dense cold water. At the same time the colder water moves down toward the source and takes the place of the displaced hot water. In this way the whole water in the kettle will get heat from the source. This means of heat transfer is called **convection**.

During convection heat is transferred from one place to another by the actual movement of a heated fluid. Only liquids and gases (fluids) can transfer heat by convection, because it is only in these materials that the constituent particles are able to move independent of each other. Familiar Examples of convection include hot-air and hot-water home heating systems, the cooling system of an automobile engine, and the flow of blood in the body.

There are two types of convection. If the carrier flows due to differences in density (caused by thermal expansion), the process is called **natural or free convection**. This type of convection is called free or natural convection, because the fluid is not forced to move by an external agent.

In another type of convection called **forced convection**, the fluid (hot or cold) is forced to move from one place to another by a pump. This method is used in nearly all modern house heating or cooling systems. In a hot water heating system, hot water from a boiler is pumped through pipes. The air in our room will then be heated.

ACTIVITY 3

Observing convection

Convection can be observed by taking a larger beaker of water and dropping in just one small crystal of potassium permanganate (a chemical that dissolves in water to give purple coloring).

Procedure

1. Fill the beaker with water. Put individual crystals of potassium manganite on the bottom of beaker.
2. Heat the beaker gently under the crystal and watch how the colored water rises to the surface.
3. Note the path that the colored water takes from the heater to the top of the water and back down again.

Figure 6. Convection currents in water

Heat transfer: Radiation

Conduction and convection require the presence of matter as a medium to carry the heat from the hotter to the colder region. But a third type of heat transfer occurs without any medium at all. This form of energy transfer through vacuum by use of electromagnetic waves is referred to as **radiation**. This is how energy reaches us from the sun. It is proved experimentally that all bodies at temperature above absolute zero are continuously radiating energy. When the wave strikes an object, it gets hotter.

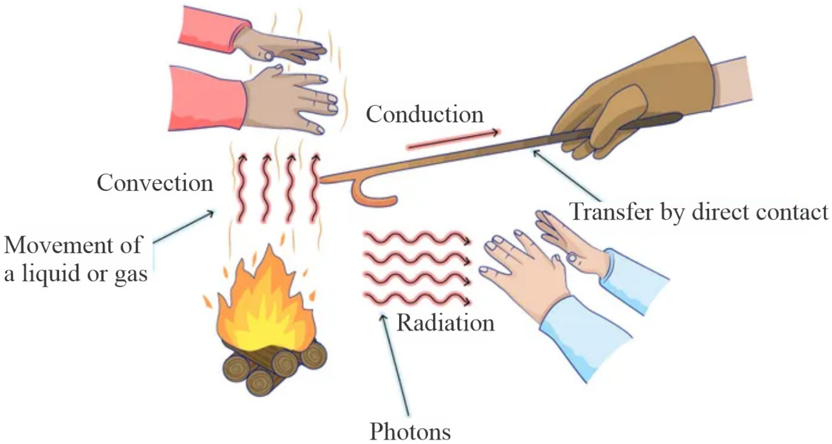


Figure 7. Heating by conduction, convection and radiation.

How Does Hot Objects Emit Radiation?

As we heat an object, we increase the average kinetic energy of the particles in that object. So, the hotter we make the object, the higher the kinetic energy (vibrational, rotational, and translational) of that object gets. These particles are charged particles in some case, and when we accelerate a charged particle in any way it creates a changing electric field which creates an electromagnetic radiation.

Absorption, Emission, and Reflection of Radiation

Energy from a radiation can be either absorbed or reflected by an object. Thus, if the object is a good absorber of radiation then it is a poor reflector and vice versa. That means objects that are good reflectors of radiation are poor absorbers. Note also that when we heat up objects, they emit the energy that they have gained as radiation. This means that a good absorber of radiation is also a good emitter and a poor absorber of radiation is a poor emitter.

Blackbody Radiation

All objects with a temperature above absolute zero emit energy in the form of electromagnetic radiation. Different objects emit different amount of radiation. A blackbody is a body which absorbs all radiation falling on it, It reflects poor but transmits or radiates as much as it absorbs.. It is an idealized object because objects don't actually absorb all the radiation that falls on them. Thus, a blackbody radiation contains all the wavelength (or frequencies) of the electromagnetic spectrum. The radiation that such an idealized object would emit when hot is known as **blackbody radiation**.

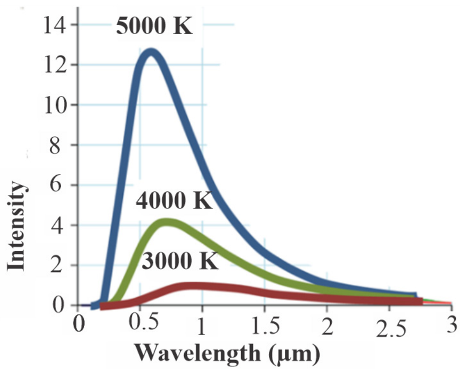


Figure 8. Black body radiation curves.

The graph below denotes the spectrum of radiation a black body would emit at different temperature. At a temperature of 3,000K the object emit visible light with a wavelength corresponding to red color, while at 5,000K, it emit light of green color. This shows that the distribution of the thermal energy radiated by a blackbody depends **only on its temperature**. As temperature increases the maximum intensity is shifting to the left. That is why hot objects appear to glow like a hot oven, light bulb filament, or the Sun and other stars.

As seen in Figure 8, the cooler objects peaks at a higher wavelength than the hotter objects do. This means that as an object gets hotter it not only radiates more intense light but it also starts to radiate lower wavelength light. The Sun is a good Example of a black body because it emits radiation of all wavelengths. In addition to the Sun, Carbon black is also an example of a black body.

Activity 4

Discuss in group

A concrete floor and a carpet have the same temperature. If you stand bare foot on concrete floor and on carpet that are found at the same temperature, you feel as if the floor is colder than the carpet. Explain why.

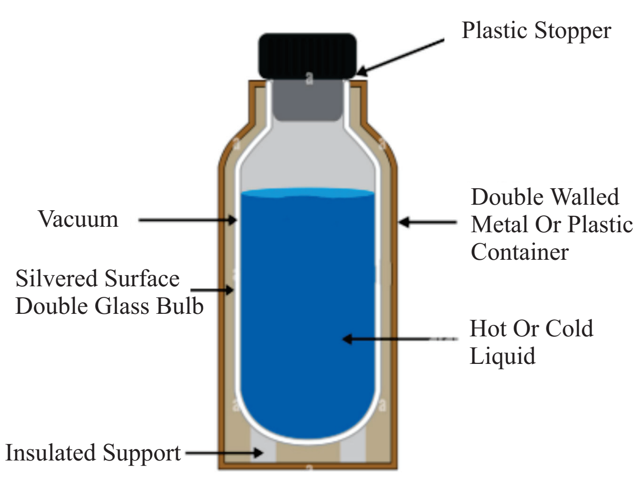
How does a vacuum flask work?

A vacuum flask, also known as thermos, consists of two flasks, placed one with in the other and joined at the neck. The gap between the two flasks is made to be nearly vacuum. The vacuum prevents conduction and convection. The tight stopper prevents air from entering or leaving the flask, so convection isn't possible either. The silver coating on the inner bottle prevents heat transfer by radiation. A hot drink put inside the flask has no chance for the heat to escape from the vacuum flask and it will continue to remain hot even after being kept for long hours. A vacuum flask is invented by Sir James Dewar in 1892. For this reason it is also known as the Dewar's flask.

A vacuum flask have the following components:

1. Screw-on stopper.
2. Outer plastic or stainless steel case.
3. Outer layer of glass coated with reflective material (or stainless steel in unbreakable flasks).
4. Vacuum.
5. Inner layer of glass (or stainless steel in unbreakable flasks).
6. One or more supports keep inner vacuum container in place.

Vacuum Or Dewar Flask



7. Additional insulation reduces heat losses and cushions flask against impacts.

ACTIVITY 5

Discuss the following questions in group.

a. In our home we use a vacuum flask to keep hot drinks like tea to keep hot. Can we use it to keep cold drinks cold?

b. Why your tea goes cold as it stays longer?

The Law of Heat Exchange

When two bodies at different temperature are mixed, the hotter body loses while the colder one gains heat energy. It follows that, from the principle of conservation of energy, provided that no heat is lost to the surrounding, the amount of heat lost by the hotter body is equal to the amount of heat gained by the colder body.

Heat lost by hot body (bodies) = Heat gained by cod body (bodies)

This statement is known as **the law of heat exchange.**

Examples

If 200 cm^3 of water at 100°C is poured into 150 g glass cup initially at 20°C, what will be the final temperature of the mixture when equilibrium is reached? Assuming no heat flows to the surroundings.

Solution

Mass of water is: $m_w = \rho v = (10^3 \text{ kg/m}^3)(200 \times 10^{-6}\text{m}^3) = 0.2$ kg and its specific heat capacity is: $c_w = 4200$ J/kg. K, $m_g = 150$ g = 0.15 kg and $c_g = 840$ J/kg.°C

Applying the law of heat exchange (conservation of energy, we have

Heat lost by water = Heat gained by cup

$$m_w c_w (100 - T_f) = m_g c_g (T_f - 20°C)$$

$$(0.20 \text{ kg})(4200 \text{ J/kg°C})(100 \text{ °C} - T_f) = (0.15 \text{ kg})(840 \text{ J/kg.k})(T_f - 20°C)$$

$$840(100 - T_f) = 126(T_f - 20)$$

$$84000 - 840 T_f = 126 T_f - 2520$$

$$86520 = 966 T_f$$

$$T_f = 89.6 \text{ °C}$$

Notice that ΔT is a positive quantity on both sides of our conservation of energy equation. On the left we have "heat lost", and ΔT is the initial minus the final temperature (100°C − T_f), whereas on the right, the "heat gained", ΔT, is the final minus the initial temperature.

Examples

What mass of water at 25°C must be allowed to come to thermal equilibrium with a 3.00 kg gold bar at 100°C in order to lower the temperature of the bar to 50°C? (c_{gold} = 129 J/kg°C).

Solution

$$\begin{pmatrix}\text{Heat lost}\\ \text{by gold bar}\end{pmatrix} = \begin{pmatrix}\text{Heat gained}\\ \text{by water}\end{pmatrix}$$

$$m_w c_w \Delta T_w = m_g c_g \Delta T_g$$

$$m_w = \frac{m_c\ c_g\ \Delta T_g}{c_w\ \Delta T_w}$$

$$m_w = \frac{(3\text{kg})(129\ \text{J/kg°C})(100°\text{C} - 50°\text{C})}{(4200\ \text{J/kg.°C})(50°\text{C} - 25°\text{C})} = 0.184\ \text{kg}$$

Exercises

1. What mass of cold water at 10°C must be added to 20kg hot water at 80°c by someone who wants to have bath at 50°C?
2. When 0.2kg water at 85°C is mixed with 0.5kg of liquid at 15°C the final temperature is 45°C. What is the specific heat of the liquid?
3. A 200g water at 80°c is mixed with 400g of water at 10°c. Assuming no heat is lost to the surrounding. What is the final temperature of the mixture?
4. What is the difference in temperature between water at the top and bottom of a water fall, if the water fall is 50m high?

Calorimetery

A Calorimeter is a metallic vessel on which two or more substances at different temperature will be mixed. It is an insulated container that allows no heat loss to the surrounding. One widely used type of calorimeter consists of an insulated container of water, a stirring device, and a thermometer (Figure 9).

Figure 9. Calorimeter

After the two objects are mixed in the calorimeter, then by applying the law of heat exchange the specific heat capacity of one of the substances will be determined. This method of determining the specific heat capacity of substances using a calorimeter is known as

calorimetry. For this reason calorimetry is also known as **the method of mixtures**. It is an experimental method of determining the specific heat of a substance.

Experiment: Calorimeter heat experiment

The specific heat capacity of a solid can be determined by an electrical method and a method of mixture.

1. Electrical method

Theory

By definition, the specific heat of a substance is the amount of heat required to raise the temperature of 1 kilogram of the substance by 1°C. Each substance has its own specific heat, and this quantity is therefore an "identifying" property that object. It can be used to help identify a substance, just like the melting point or density.

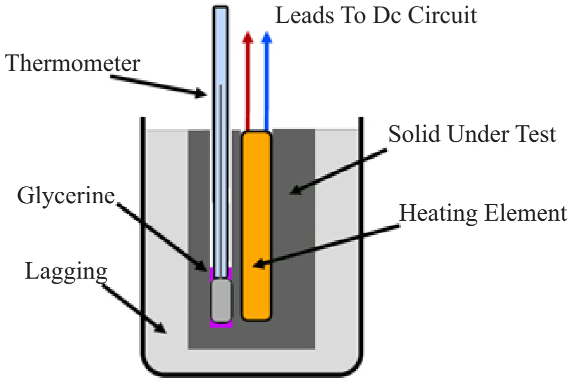


Figure 10. Arrangement for determining latent heat of fusion.

The amount of heat transferred (Q) into or out of a sample of a substance, such as water, can be determined if the mass (m) of the sample and the substance's specific heat (c) are known, and the sample's change in temperature (ΔT) is measured.

That is, $Q = m\,c\Delta T$

The solid under test is a lagged cylinder with holes drilled for the thermometer and the heater element. A little glycerin or oil is added to the thermometer hole to improve thermal contact.

Materials Required: calorimeter, water, thermometer, stirrer and the solid.

- Two holes are drilled into the specimen solid of mass m.
- A thermometer is inserted in one of the holes and an electric heater into the other hole. The holes are then filled with a good conducting fluid, e.g. oil to ensure thermal contact.
- The apparatus is insulated and initial temperature T_i is recorded.
- The heater is switched on at the same time a stop watch is started.
- The steady values of ammeter reading, I and voltmeter reading, V are recorded.

- After considerable temperature rise, the heater is switched off and stop watch stopped.
- The highest temperature T_f recorded and time t taken noted.

Energy supplied by current = VIt

Heat required to raise the temperature of the block by ΔT, Q = mcΔT.

⇒ VIt = mcΔT.

Since the mass m of the solid m is known and V, I, t, ΔT are measured. So the specific heat capacity of the solid (c) can be calculated.

$$c = \frac{VIt}{m\Delta T}$$

Note. The small heat capacities of the heater and thermometer have been neglected.

2. Method of mixture

Theory

The theory behind this specific heat test is based on the conservation of energy. Heat is a form of energy, and in this case, it will be transferred between the sample and water. We will be measuring the change in temperature of the water in the calorimeter, which lets us calculate the change in heat of the water in the calorimeter. From the law of heat exchange this energy should be equal to the change in heat of the sample. The following steps are used in a laboratory experiment in order to determine the specific heat capacity of substances.

1. Using balance obtain the mass of the solid sample, say m_s.
2. Weigh the empty calorimeter (m_c), whose specific heat capacity is known, c_c.
3. Fill half of the calorimeter with cold water and weigh them($m_c + m_w$). The difference between step 3 and step 2 will give the mass of the water, m_w.
4. Using a thermometer, measure the temperature of the water in the calorimeter (T_c). Note that the calorimeter is in direct contact with the water, and hence their temperature is the same.

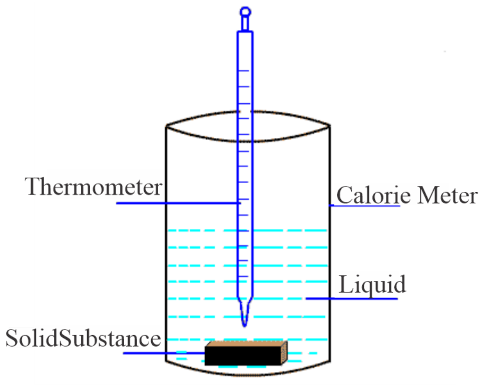


Figure 11. Determining the spaecific heat of a substance using the method of mixture.

5. Put the solid sample in a beaker of boiling water for about 15 minutes so that it will have the temperature of the boiling water. Record the temperature of the solid (T_s).
6. Use the tongs to carefully pick up the metal sample and quickly transferred into the water in the calorimeter. Stir it and measure the temperature of the mixture when it becomes stable (T_f).

Data

Mass of solid (m_s) =

Mass of calorimeter + stirrer (m_c)=

Mass of calorimeter + stirrer + water ($m_s + m_c$) =

Initial temperature of water = calorimeter (T_c) =

Final temperature of mixture (T_f) =

Temperature of boiling water = Temperature of solid = 100^0C

Specific heat capacity of water = (4,200J/Kg.K)

Calculations

Every quantity is recorded except the specific heat capacity of the solid (c_s). Using the law of heat exchange one can evaluate (c_s). In this experiment the solid loses heat and the calorimeter and the water gain the heat. The basic equation will be:

Heat lost by sold = Heat gained by calorimeter and stirrer + Heat gained by water

$$Q_s = Q_c + Q_w$$

$$m_s c_s (\Delta T)_s = m_c c_c (\Delta T)_c + m_w c_w (\Delta T)_w$$

$$c_s = \frac{m_c c_c (\Delta T)_c + m_w c_w (\Delta T)_w}{m_s (\Delta T)_s}$$

$$c_s = \frac{m_c c_c (T_f - T_c)_c + m_w c_w (T_f - T_c)}{m_s (T_s - T_f)}$$

$$c_s = \frac{(m_c c_c + m_w c_w)(T_f - T_c)}{m_s (T_s - T_f)}$$

Questions

1. Write the possible sources of error.
2. If your solid is of known specific heat capacity (iron, copper, lead …), compare what you have got with the theoretical value. Calculate the percentage error.

Safety

This experiment does involve using hot plates that can heat to several hundred degrees Celsius. Handle hot glassware with either crucible tongs or "hot hands," keep long hair tied back and wear safety gloves. Even though no flames are used,

Examples

In an experiment similar to that described above, 120 g of water was used. The water was heated for 5.0 minutes, during which time the average current was 0.77 A and the potential difference was 11.4 V. The temperature of the water rose from 21.2 °C to 26.3 °C. What value do these data give for the specific heat capacity of water?

Solution

The energy supplied by the electrical heater resulted to the increase in the temperature of water. Hence,

$$mc\Delta T = VIt$$

$$\Rightarrow c = \frac{VIt}{m\Delta T} = \frac{11.4\text{ V} \times 0.77\text{A} \times 300\text{s}}{0.12\text{kg} \times 5.1°\text{C}} = 4302.9\text{J/kg.°C}$$

Examples

In order to determine the specific heat of an unknown metal, a 0.150 kg sample of the metal is heated to 530°C. It is then quickly placed in 400g of water at 10°C, which is contained in a 200g aluminum calorimeter cup. The final temperature of the mixture is 30°C. Calculate the specific heat of the metal.

Solution

$m_m = 0.150$ kg, $T_m = 530°C$, $m_w = 400$ g $= 0.4$ kg, $T_w = 10°C$, $c_w = 4200$J/kg.k,

$m_{Al} = 200$ g $= 0.2$ kg, $T_{Al} = 10°C$ (same as the water) and $c_{Al} = 900$ J/kg.k.

Applying the conservation of energy, we find

$$\begin{pmatrix}\text{Heat lost}\\ \text{by metal}\end{pmatrix} = \begin{pmatrix}\text{Heat gained}\\ \text{by water}\end{pmatrix} + \begin{pmatrix}\text{Heat gained}\\ \text{by calorimeter cup}\end{pmatrix}$$

$$m_m c_m \Delta T_m = m_w c_w \Delta T_w + m_{Al} c_{Al} \Delta T_{Al}$$

$$c_m = \frac{m_w c_w \Delta T_w + m_{Al} C_{Al} \Delta T_{Al}}{m_m \Delta T_m}$$

Initially, the water and the cup are at the same temperature. So, the change in temperature for both is the same $\Delta T_m = \Delta T_{Al}$.

$$c_m = \frac{(0.4\ \text{kg})\ (4200\ \text{J/kg.°C})\ (30°\text{C} - 10°\text{C}) + (0.2\ \text{kg})\ (900\ \text{J/kg°C})\ (30°\text{C} - 10°\text{C})}{(0.150\ \text{kg})\ (530°\text{C} - 30°\text{C})}$$

$$c_m = \frac{33600\ \text{J} + 3600\ \text{J}}{75\ \text{kg.°C}}$$

$$c_m = \frac{37200\ \text{J}}{75\ \text{kg.°C}} = 496\ \text{J/kg.°C}$$

Exercises

1. A 100 g aluminum calorimeter contains 250 g of water. The two substances are in thermal equilibrium at 10°C. Two metallic blocks are placed in the water. One is a 50 g piece of copper at 80°C. The other sample has a mass of 70 g and is originally at a temperature of 100°C. The entire system reaches a final temperature of 20°C. Determine the specific heat of the unknown second sample.
2. A calorimeter has a mass of 55g. Cold water is poured into the calorimeter, thus calorimeter and cold water have a mass of 120g at a temperature of 17°c. When hot water at a temperature of 45°c is poured into the calorimeter, the calorimeter and the mixture have a mass of 190g. If the final temperature of the mixture becomes 30°c, what is the specific heat of the calorimeter?
3. A 1.5 kg metal rod at a temperature of Tm = 100 °C, is dropped into 1.2 kg water contained in a 2.5 kg iron calorimeter at a temperature of Tw= 20°C. The system attains thermal equilibrium at a temperature of 24 °C. If no heat enters or leaves the system, what is the specific heat capacity of the metal?

4.4 Latent Heat of Fusion and Vaporization

In this section you will learn what happens when a change of state takes place from ice to water (condensation) and from water to vapor (vaporization). During change of state the temperature of the object does not change even though it either gains or losses heat. The latent heat of fusion and latent heat of vaporization of ice and water will be determined experimentally.

Phase Change

We use the term **phase** to describe a specific state of matter, such as a solid, liquid, or gas. A **phase of matter** is when all the physical properties within a material are uniform. At a given phase, a material will have the same density and refractive index. The transitions from one state to another are accompanied with either absorption or liberation of heat and change of volume. Change of state is not accompanied with change in temperature. Which means during change of state the speed and hence the kinetic energy of the molecules also remains constant. The absorbed or liberated heat energy is used in breaking or forming the bond between the molecules which in turn affects the potential energy of the molecules.

Key term

- The latent heat of fusion of a substance is the energy involved in changing the state of unit mass of the substance at the melting/freezing point. $Q=mL_f$
- The latent heat of vaporization of a substance is the energy involved in changing the state of unit mass of the substance at the boiling point. $Q=mL_V$

Substances may exist in three states: solid, liquid and gas states. For Example, water exists in the solid state as ice, in the liquid state as water, and in the gaseous state as steam or vapor. A transition from one phase to another is called a **phase change.** A change from one phase to another takes place under conditions of phase equilibrium between the two phases, and for a given pressure this occurs at only one specific temperature.

We can represent these conditions on a pressure versus temperature graph is called **phase diagram** (Figure 12). Only a single phase can exist at each point, except for points on the solid lines, where two phases can coexist in phase equilibrium.

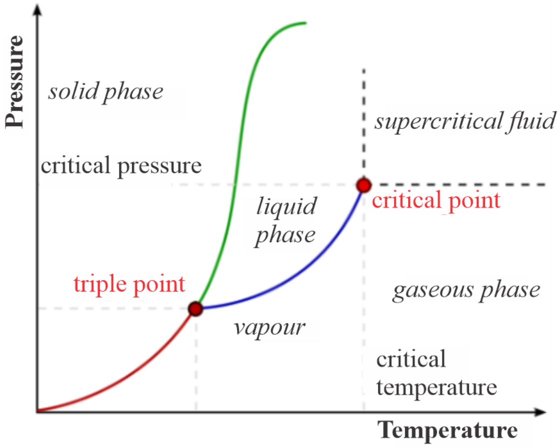


Figure 12. Phase diagram

Triple Point and Critical Point

The phase diagram in Figure 12, shows whether a substance exists as a vapor, liquid, or solid at a given temperature and pressure. In the diagram, we have two specific points: the triple point and the critical point.

Triple Point: The temperature and pressure at which the fusion curve, the vaporization curve and the sublimation curve meet and all the three phases of a substance coexist is called the **triple point** of the substance. For example the triple point of water is represented by the temperature 273.16 K and pressure 6.11×10^{-3} Pa.

KEY TERM

- Triple point is a point on a p - v diagram where the three phases of a substance can co exist.
- Critical point is a a point on a p - v diagram where the critical temperature and pressure of a substance meet.

Critical Point: The critical point, some times called **critical state**, occurs where the critical temperature and critical pressure of a substance meet. Above this point clear phase boundaries cease to exist. The critical point occurs at one specific temperature and pressure. For water the critical point is about 647 k and 22MPa. At the critical point, the properties of the gas and liquid phases merge together giving only one phase.

They are one and the same and are referred as a **supercritical fluid**. Above the critical temperature, it is impossible to form a liquid regardless of any increase in pressure.

Change of State

A substance can exist in any of the three states: Solid, liquid, and gas. In general when heat is added to or removed from a substance its temperature changes – increases or decreases. However, there are situations where the flow of heat does not result to a change in temperature. This occurs whenever a substance changes from one phase to another.

For Example when ice at $0^{o}c$ is heated, the addition of more heat does not increase the temperature but causes the ice to melt. Once the ice is melted, adding more heat will cause the temperature of the resulting water to increase. A similar situation occurs when the temperature reaches to $100^{o}C$ where water changes its phase to vapor. Adding more heat at this time does not increase temperature but causes more vaporization until the whole liquid is changed into a vapor. The quantity of heat associated with a change of state without the corresponding change in temperature is known as **latent heat**. It is called latent or hidden heat because the heat does not show its presence by producing a change in temperature.

Specific latent Heat of Fusion (L_f)

The change of state of a substance from solid to a liquid state is known as **fusion** or **melting**. The reverse process i.e. the change from the liquid to solid state is called **freezing** or **solidification**. This happens at melting point of the substance. The amount of heat needed to change a unit mass of the substance from the solid to the liquid state without change of temperature is known as the **specific latent heat of fusion (L_f)**.

Specific latent heat of fusion = $\frac{\textit{Heat needed to melt the solid}}{\textit{mass}}$

$$L_f = \frac{Q}{m}$$

The unit of specific latent heat of fusion is J/kg.

The same amount of heat is released when unit mass of the liquid solidifies at the same temperature. The specific latent heat of fusion of ice is

L_f of ice = 3.34×10^5 J/kg = 80 Cal/g

This means that 3.34×10^5J of heat is required to melt 1kg of ice at 0^oC to water at 0^oC. The amount of heat needed to convert a solid of mass m into liquid state is, then given by: $Q = mL_V$.

Experiment: Measuring the specific latent heat of fusion of ice

1. Determining the Latent heat of fusion of ice by electrical method

Title: Measuring Specific Latent Heat of Fusion of Ice

Objective: To enable the student to measure the specific latent heat of fusion of ice by an electric method.

Materials required: electric heater, stop watch, balance, ice holding container, liquid collecting container, ice, lagging.

1. Place the electric heater in the melting ice packed in the funnel.
2. Switch on the heater and the stop-watch.
3. Collect the water from melting ice for several minutes.
4. Switch of the heater and the stop watch.
5. Record the time of the watch as "t".
6. Find the mass of the collected water and record it as "m".
7. Evaluate the latent heat of fusion from $L_f = \frac{pt}{m}$.

When an electric heater is immersed in an ice, the electrical energy flowing in heater will be totally converted into heat energy in the ice. From electricity, the electrical energy is given by

Electrical energy = Pt = VIt.

From heat, we know that the heat gained by the ice is given by $Q = mc(\Delta T)$.

Assuming there no heat lost to the surroundings of the ice, we can say the heat energy gained by the ice is equal to the electrical energy.

Heat energy gained by ice = Electrical energy given by heater

That is $VIt = mL_f$.

By solving for "L_f" we obtain $L_f = \dfrac{VIt}{m}$

2. Determining the Latent heat of fusion of ice by the method of mixtures

Title: Experiment to Measure the Heat of Fusion of Ice

Aim: The aim of this experiment is to measure the heat of fusion of ice. This is the amount of heat energy needed to change one gram of ice at its melting point (0°C) into one gram of water at the same temperature.

Apparatus: Calorimeter, balance, set of metric basses, thermometer, warm water, ice, paper towels

Method

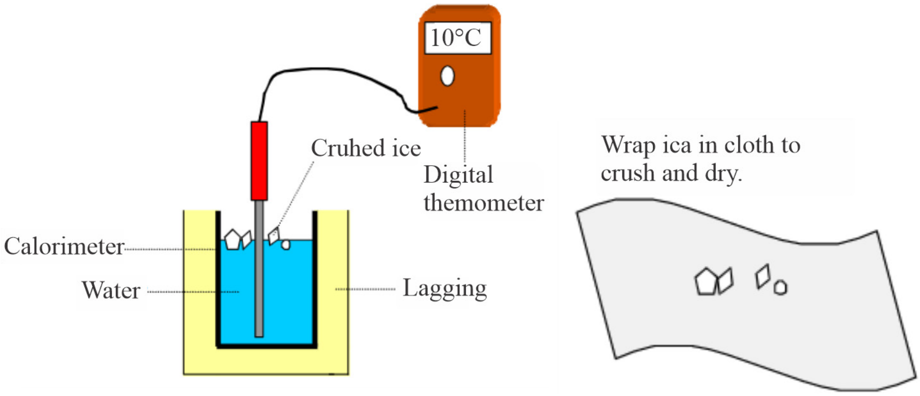


Figure 13. Determining the latent heat of fusion of ice using the method of mixture.

1. Find the mass of the calorimeter, m_c.
2. Half fill the calorimeter with water warmed to about 10^0C above room temperature. Find the combined mass of calorimeter and water (m_2). The mass of water $m_w = m_2 - m_c$.
3. Record the initial temperature of calorimeter plus water (T_1).
4. Surround the ice cube with kitchen paper or a cloth and crush them and dry them with kitchen paper or cloth.
5. Add pieces of dry crushed ice, a little at a time, to the calorimeter. Do this until the temperature of the water has fallen by about 20^0C.

6. Record the lowest temperature T_2 of the calorimeter plus water plus melted ice. The rice in temperature of ice $\Delta T_1 = T_2 - 0^0C$ and fall in temperature of the calorimeter plus water $\Delta T_2 = T_1 - T_2$
7. Find the mass of the calorimeter plus water plus melted ice, m_3. The mass of melted ice $(m_i) = m_3 - m_2$.

Calculations

Assuming no heat loss:

Heat gained by ice + Heat gained by the resulting water = Heat lost by calorimeter + Heat lost by the water

$$Q_i + Q_{rw} = Q_c + Q_w$$

$$m_i L_f + m_{rw} c_{rw} (\Delta T)_1 = m_c c_c (\Delta T)_2 + m_w c_w (\Delta T)_2$$

$$L_f = \frac{m_c c_c (\Delta T)_2 + m_w c_w (\Delta T)_2 - m_{rw} c_{rw} (\Delta T)_1}{m_i}$$

Questions

1. What is the percentage error of your experiment?

$$\%\ \text{error} = \frac{\text{Actual value - Experimental value}}{\text{Actual value}} \times 100\%$$

2. Write the possible sources of error?

Examples

How much heat is required to completely melt a 4.4 kg ice at 0 °C?

Solution

$m_i = 4.4$ kg, $L_f = 334{,}000$ J/Kg

$Q_{gain} = mL_f = 4.4$ kg x 334000 J/kg = 1469600 J = 1.5 MJ

Examples

How much ice (L_f= 80cal/g) at 0°c must be added in order to lower the temperature of 1 liter of water (c = 1cal/g.°c) from 70°c to 20°c? (density of water = 1g/cm³.)

Given: $L_f = 80$ cal/g; $T_{ii} = 0°c$; $V_w = 1$ liter = $1000cm^3$

$m_w = \rho_w V_w = 1gcm^3\ (1000cm^3) = 1000g$

$T_{iw} = 70°c$; $T_f = 20°c$

Solution

Heat gained in melting the ice	+	Heat gained to increase the temperature of the resulting water from 0 to 20°c	=	Heat lost by Water in decreasing its temperature from 70°c to 20°c

$$m_i L_f + m_i C_w \Delta T_{rw} = m_w C_w \Delta Tw$$

$$m_i = \frac{m_w c_w \Delta T_w}{L_f + C_w \Delta T_{rw}} = \frac{1000g\,(1cal/g.°c)(70-20)°c}{80\ cal/g.°c + 1cal/g.°c(20-0)°c} = 500g$$

Specific latent Heat of Vaporization

The change of the state of a substance from a liquid phase to a vapor is called **vaporization**. The reverse process i.e. the change from vapor to liquid phase is called **condensation** or **liquefaction**. The amount of heat energy require to change the phase of a unit mass of a substance from a liquid to a vapor (gas) phase at constant temperature (boiling point) is called its **Specific latent heat of vaporization (Lv)**. It is also the energy given off when the substance changes its state from vapor to liquid phase. Mathematically:

$$L_v = \frac{\textit{Heat needed to vaporize}}{\text{mass}} = \frac{Q}{m}$$

The specific latent heat of vaporization of water = $2.26 \times 10^6 J/kg = 540 Cal/g$.

That is, 2.26×10^6J of heat is needed to vaporize
1kg of water at 100°C to vapor at 100°C. The amount of heat needed to convert a liquid of mass m into gaseous state is, then given by: $Q = mL_v$.

Table 4.3 Shows the specific latent heat of fusion (L_f) and specific latent heat of vaporization (L_v) of some substances.

Table 3 Specific latent heat of fusion and vaporization

Substance	Melting point	L_f		Boiling point	L_v	
		J/kg	Cal/g		J/kg	Cal/g
Nitrogen	– 210°c	0.26×10^5	61	– 196°c	2.01×10^5	48
Oxygen	– 219°c	0.14×10^5	3.3	– 183°c	2.10×10^5	51
Alcohol, ethyl	– 114°c	1.0×10^5	25	78°c	8.5×10^5	204
Water	0°c	3.33×10^5	80	100°c	22.6×10^5	540

Mercury	– 39°c	0.12×10^5	2.8	357°c	2.7×10^5	65
Lead	327°c	0.25×10^5	5.9	1750°c	8.67×10^5	208
Silver	961°c	0.88×10^5	21	2193°c	23.0×10^5	558
Iron	1808°c	2.89×10^5	69.1	3023°c	63.4×10^5	1520
Tungsten	3410°c	1.84×10^5	44	5900°c	48.0×10^5	1150

Experiment: Measuring the Specific Latent heat of Vaporization of Water.

Theory

1. Determining the Latent heat of vaporization of water by electrical method

Objective: To enable students' to measure the specific latent heat of vaporization of water by electrical method.

Materials required: Container to hold water, water, lagging, electric heater, stop-watch, balance.

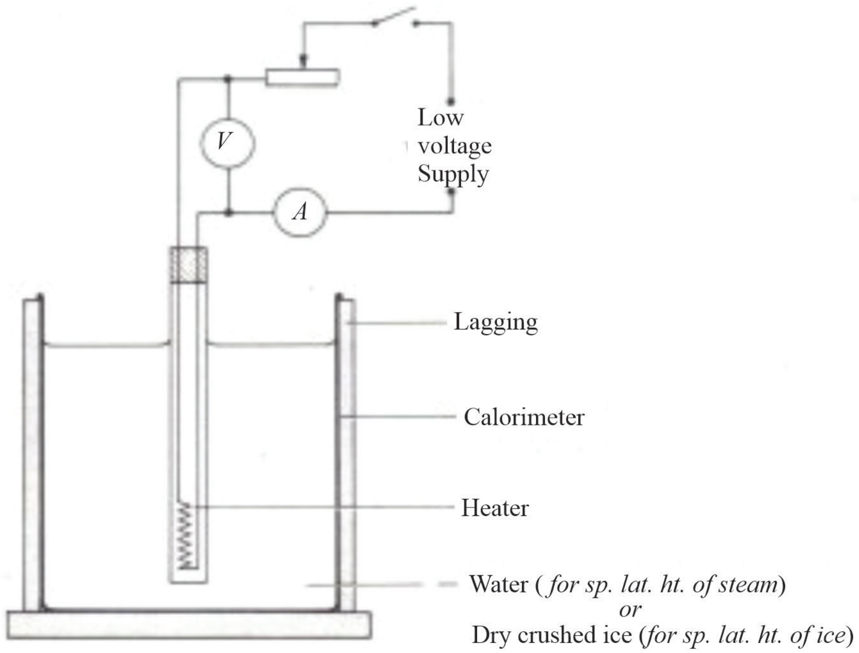


Figure 14. Specific laterat heat of vaporzation

Method I

A calorimeter provided with a lid is weighed, first empty, and then when half full of water. An electric immersion heater is placed in the water and connected in circuit as shown in Figure 14.

Procedure

- The lid is removed, the heater is immersed and the current switched on and adjusted to a suitable value.
- As soon as the water starts to boil, a stop clock is started.
- The current is kept constant by the rheostat, and switched off 15 minutes later.
- The immersion heater is then removed and the calorimeter immediately covered with its lid to prevent further loss of vapor.
- Having removed the polystyrene jacket the calorimeter is cooled by immersion in cold water and then dried with a cloth and reweighed.
- The various readings to be taken are detailed below.

Calculations

- Mass of calorimeter, lid, water = m_1
- Current = I
- P.d across heater coil = V
- Timing of boiling = t
- Mass of calorimeter, lid, water after boiling = m_2
- Mass of vapor (m) = $m_1 - m_2$

Energy gained by water = Electrical energy given by heater. That is

$$VIt = mL_V$$

By solving L_V we obtain

$$L_v = \frac{VIt}{m}$$

2. Determining the Latent heat of vaporization of a liquid by a method of mixture

Objective: To enable students' to measure the specific latent heat of vaporization of water by of mixture.

Apparatus: Calorimeter, thermometer, Bunsen burner, water, lagging (foam), steam trap, rubber tubes, flask, cork stopper.

Procedure:

1. Set up apparatus as shown in figure 15.
2. Put the dry, empty calorimeter on a balance scale to find mass of calorimeter. m_{cal}.
3. Fill calorimeter about half full of cool water, about 10^0C - 15°C below room temperature.

4. Find the mass of the calorimeter and water, m_1
5. The mass of the cool water can now be calculated by $m_w = m_1 = m_{cal}$.
6. Record the temperature of the calorimeter plus water T_1.
7. Boil water in round bottom flask.
8. Place calorimeter in insulation and lid to prevent heat loss and gain to the surrounding.
9. To ensure the steam used is dry, we use a steam trap. (in the absence of a steam trap, we can use a rubber tubing insulated with cotton wool, but the first steam will condense and we must wait until only steam is coming out of the delivery tube before inserting it under the cool water in the calorimeter).
10. Immerse the end of the steam tube into the water.
11. Heat the water until about 10^0C - 15°C above room temperature.
12. Record the final temperature T_2 of the calorimeter plus water plus condensed steam. The fall in temperature of the steam $\Delta T_1 = 100 - T_2$.
13. The rise in the temperature of the calorimeter plus water $\Delta T_2 = T_2 - T_1$.
14. Once experiment is completed immediately find the mass of the calorimeter, water and steam, m_2, so to determine mass of steam. (The mass of the condensed steam $m_s = m_2 - m_1$).

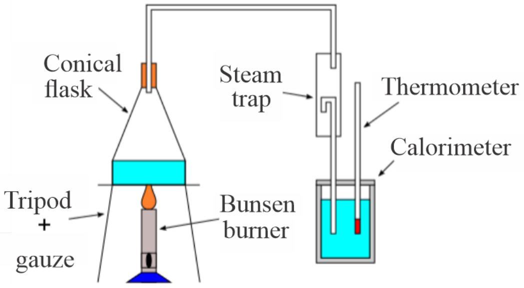


Figure 15. Latent heat of vaporization

Calculations

Heat lost by steam = heat gained by water and calorimeter

$$Q_s + Q_{rw} = Q_c + Q_w$$

$$m_s L_v + m_{rw} c_{rw} (\Delta T)_{rw} = m_c c_c (\Delta T)_c + m_w c_w (\Delta T)_w$$

$$L_v = \frac{m_c c_c (\Delta T)_c + m_w c_w (\Delta T)_w - m_{rw} c_{rw} (\Delta T)_{rw}}{m_s}$$

Questions

1. What is the percentage error of your experiment?

$$\% \text{ error} = \frac{\text{Actual - Experimental value}}{\text{Actual value}} \times 100\%$$

2. Write the possible sources of error?

Examples

Using the heat of vaporization for water is 2260 J/g, calculate the energy needed to boil 50.0 g of water at its boiling point of 100°C.

Solution

L_v = 2260 J/g , m = 50 g

$Q = mL_v$ = 50 g x 2260 J/g = 113000 J = 113 kJ

Activity 6

Discuss in group why evaporation of water has a cooling effect and condensation has a warming effect.

Heating and Cooling Curves

To summarize our discussion assume that ice at a temperature less than 0^0C is being heated. Its temperature continuously increases until it starts to melt at 0^0C (melting point of ice). This temperature remains constant until all of the ice melts to water. Once all of the ice becomes water at 0^0C the temperature again starts to increase until it starts to boil at 100^0C (boiling point of water). This temperature in turn remains constant until all of the water vaporizes to gas (vapor). If the gas continuous to be heated starting above 100^0C its temperature increases as required. The diagram that shows all these processes is known as the **heating curve**, See Figure 16. It is also possible to draw the cooling curve using the same concept.

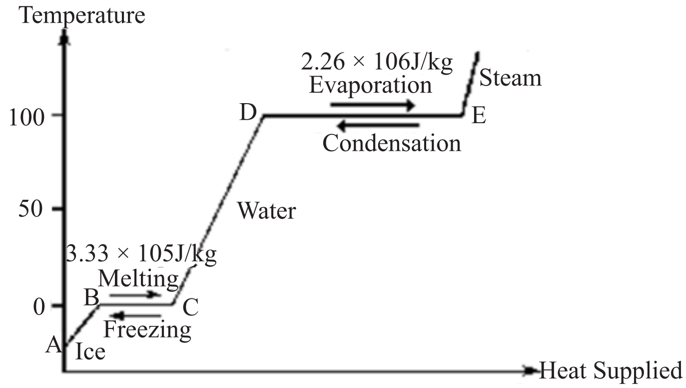


Figure 16. Heating curve of water

Examples

How much heat energy is required by 20g of ice at – 10°c until it becomes steam at 120°c?

Solution

First: Find the heat required to change the temperature of 20g of ice at -10°c to 20g of ice at 0°c

m = 20g ; c = 0.5 cal/g.°c ; T_i = -10°c ; T_f = 0°c

Q_1 = mcΔT = 20g (0.5 cal/g.°c) (0 – (-10°c)) = 100cal

Second: Find the heat required to change 20 g of ice at 0°c to 20g of water at 0°c

m = 20g ; L_f = 80 cal/g

$Q_2 = mL_f$ = 20g (80 cal/g) = 1600 cal

Third: Find the heat required to increase the temperature of 20 g of water from 0°C to 100°C

m = 20g ; C = 1 cal/g.°c ; T_i = 0°c ; T_f = 100°c

$Q_3 = mc\Delta T = mc\ (t_f - t_i)$

Q_3 = 20g (1 cal/g.°c)(100-0)°c = 2000 cal

Fourth: Find the heat required to change 20g of water at 100°c to vapor at 100°c

m = 20g ; L_v = 540 cal/g

$Q_4 = mL_v$ = 20g (540 cal/g) = 10,800cal

Fifth: Find the heat required to increase the temperature of 20g of steam from 100°c to 120°c

m = 20g ; C = 0.5 cal/g°c ; T_i = 100°c ; T_f = 120°c

$Q_5 = mc\Delta T = mc\ (T_f - T_i)$

Q_5 = 20g (0.5 cal/g°c) (120°c – 100°c) = 200cal

Finally: Find the sum to determine the total heat required

$Q = Q_1 + Q_2 + Q_3 + Q_4 + Q_5$

Q = 100cal + 1,600cal + 2,000cal + 10,800cal + 200cal = 14,700cal

Examples

How much steam ($L_v = 2.4 \times 10^6$ J/kg) at 100°c is needed to raise the temperature of 280g of water (C = 4200 J/kg.°c) from 30°c to 80°c?

Given: $L_v = 2.4 \times 10^6$ J/kg ; T_{is} = 100°c; m_w = 280g = 0.28kg

C_w = 4200 J/kg.°c ; T_{irw} = 30°c; T_f = 80°c

Solution

Heat lost by Steam in condensing	+	Heat lost by the resulting water in decreasing its temperature from 100 to 80°c	=	Heat gained by water in increasing its temperature from 30°c to 80°c

$$m_s L_v + m_s C_w \Delta T_{rw} = m_w C_w \Delta T_w$$

$$m_s = \frac{m_w C_w \Delta T_w}{L_v + C_w \Delta T_{rw}}$$

$$m_s = \frac{0.28\text{kg}(4200\text{J/kg.°c}(80-30)\text{°c}}{2.4\times10^6\ \text{J/kg} + 4200\ \text{J/kg.°c}\ (100-80)\text{°c}} = 0.0237\ \text{kg}$$

Heat Engines

A heat engine is a device which transforms the chemical energy of a fuel into thermal energy and uses this energy to produce mechanical work. The majority of world's power is still produced directly or indirectly from heat provided by burning fuel such as coal and oil. The energy conversion follows a pattern outlined below.

Chemical energy in fuel⇒Heat energy⇒Mechanical energy + Wasted heat

Heat engines are classified into two types-

(a) External combustion engine
(b) Internal combustion engine.

External combustion engine: Steam Engine

In external combustion engine burning fuel takes place outside the engine. Examples: in the steam engine or a steam turbine plant, heat from burning wood, coal or oil outside the engine boils water to generate steam into the engine. When water is changed to steam under normal atmospheric pressure it expands to about 1700 times. In a boiler however, the steam is confined and exerts pressure in all directions. Steam under pressure may do work by exerting its force against the blades of a steam turbine. In either case, the heat of burning fuel is transformed to work through the use of steam. Nuclear power plants use steam generated by boiling water using nuclear reactors. The steam for this purpose may be heated to 723 K and have a pressure of the order of 2.5×10^7 Pa.

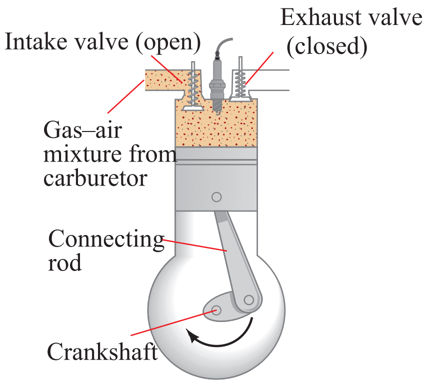
Figure 17. Steam Engine

Internal Combustion Engine

An **internal combustion engine** (**ICE**) is a heat engine in which the combustion (burning) of a fuel occurs within the combustion chamber. ICEs are typically powered by fossil fuels like natural gas or petroleum products such as gasoline, diesel fuel or fuel oil. If burning takes place inside the engine, the pressure built up by burning the fuel is converted to mechanical work by exerting a force which pushes a piston or turns the blades of a turbine.

The fact that fuel can be burn inside the engine, unlike the steam engine, reduces the energy loss outside the engine and increases its efficiency. Gasoline engine, diesel engine and jet propulsion engine are examples of internal combustion engines.

a)



b)

Figure 18. a) Gasoline (petrol) engine b) Diesel engine

Exercises

1. Explain how (a) an internal combustion engine and (b) external combustion engines work.
2. Calculate the amount of heat required to convert 150 g of water at 100 °C completely to steam at 100 °C.
3. A 50 g ice cube at 0°C is heated until 45 g has become water at 100°C and 5.0 g has become steam at 100°C. How much energy was added to accomplish the transformation?
4. Calculate the heat required to change:
   (a) 10 gram of ice at 0^0C to water at 0^0C.
   (b) 10 gram of water at 50^0C to steam at 100^0C.
   (c) 50 gram of ice at 0^0C to stem at 100^0C.
   (d) 40 gram of ice at – 10^0C to steam at 100^0C.

Summary

- Heat is thermal energy transferred from a hotter system to a cooler system that are in contact.
- Temperature is a measure of the average kinetic energy of the atoms or molecules in the system.
- The heat released or absorbed by a body is calculated using the equation: $Q = mc\Delta T$
- Specific heat capacity is the number of joules of heat energy required to raise the temperature of 1 kg of a substance by 1 k ($c = \frac{Q}{m\Delta T}$, measured J/kg/k).
- The three mechanisms of heat transfer are conduction, convection, and radiation.
- *Conduction* occurs within a body or between two bodies in contact.
- *Convection* depends on motion of mass from one region of space to another.
- *Radiation* is heat transfer by electromagnetic radiation, such as sunshine, with no need for matter to be present in the space between bodies.
- Heat capacity is the heat required per unit rise in temperature ($c = \frac{Q}{\Delta T}$, measured in J/k).
- Calorimetry is an experimental method of determining the specific heat capacity of a body by mixing two or more bodies in a calorimeter and apply the law of heat exchange.
- A phase change occurs when a substance changes state.
- The critical point is where the liquid and gas phases become indistinguishable.
- The triple point is where all three states coexist together.
- Latent heat is energy that is supplied to a substance but does not result in a rise in temperature but does result in a change of phase.
- Latent heat of fusion ($L_f = \frac{Q}{m}$) at melting point.
- Latent heat of vaporization ($L_v = \frac{Q}{m}$) at boiling point.
- In an internal combustion engine, a mixture of fuel and air is burned in a closed cylinder, inside the engine, contrary to the external combustion engine in which the burning of fuel takes place outside the engine.

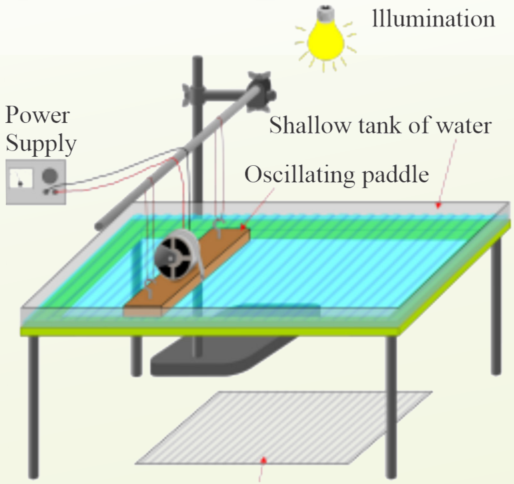
Review Exercises

1. Explain, why on a hot day a dog breath quickly while hanging out its tongue.
2. How much heat is given out when an iron ball mass 2kg and specific heat capacity 440 J/kg.k cools from 300°c to 200°c?
3. The temperature of a metallic bar of mass 500g rises by 10°C when it absorbs 8kJ of heat energy. Determine the specific heat capacity of the bar.
4. A tank holding 60kg of water (C = 4200 J/kg.k) is heated by a 3kW immersion heater. How long will it take to rise the temperature of the water from 10°C to 60°C?
5. A 0.01 kg lead bullet moving at 300 m/s embeds itself in a large block of wood. All of the kinetic energy lost by the lead bullet is transferred into heat which is shared equally by the bullet and the block of wood. Determine the temperature change of the bullet.
6. A 100g ice cube at 0°C is dropped into 1.0 kg of water that was originally at 80°C. What is the final temperature of the water after the ice has melted?
7. How much heat must be added to convert 200 g of ice of water at 0 °C to steam at 100 °C completely?
8. What mass of steam at 140°C is needed to warm 250 g of water in a 100g glass container from 40°C to 50°C?
9. How much heat is required to convert a 1g ice at -30°C to water vapor at 120°C? Plot the temperature versus heat graph.
10. A copper penny at 25°C drops from a height of 40 m to the ground. If 50% of the potential energy goes into increasing the internal energy, what is its final temperature?

P11CH05

CHAPTER 5

WAVES



Chapter Contents

Chapter Outcome

Learners will able to:

- recognize and appreciate the importance of the nature of wave's characteristics and components in daily activities.

Chapter Objectives

Upon completion of this chapter, learners will:

- analyze characteristics, concept and components of waves.
- elaborate on the properties and the categories of waves.
- design methods of production and transmission of sound wave and its application.
- compute the speed of sound relative to its temperature.
- analyze the Doppler Effect.
- distinguish between
  - Loudness and intensity of sound;
  - Intensity and intensity level;
  - Music and noise:
  - Stringed and non-stringed music instruments.
- analyze vibrations in strings and tubes (pipes).

Introduction

The idea of energy has to be explored as much as possible because life on earth much depends on the vital use of energy. It is the transfer of energy that gets things done. In this unit we will investigate an important way of transferring energy. One way of transferring energy is by wave motion. All waves transfer energy. The idea of waves is important in understanding sound. Sound plays so important role in our lives that we communicate each other using it.

Periodic vibrations can cause disturbances that move through a medium in the form of waves. Many kinds of waves occur in nature, such as sound waves, water waves and electromagnetic waves. As a wave propagates, it carries energy. This chapter deals about the types, characteristics and properties of waves.

5.1 Nature Characteristics and Properties of Waves Types of Waves

The characteristics of waves (wave speed, amplitude, period, frequency, and wavelength) and the properties of waves (reflection, refraction, diffraction, interference and polarization) will be discussed in this section.

Key terms

- Wave: A disturbance in a medium without carrying the particles of the medium.
- Medium: A material through which a wave travels.

Definition of waves

Waves are disturbances which originate from some vibrating source and propagate through a medium and vacuum. A **medium** is the substance through which a wave can propagate. Water is the medium of water waves. Air is the medium through which we hear sound waves. The electric and magnetic fields are the medium of light. The most important property of wave is it transfer energy from one point to another. Waves transfer energy without transferring the particles of medium.

When a finger is dipped, or a stone is thrown into the water, the water particles are pushed away (disturbed) from their rest position. The surface is now disturbed. It is higher than normal in some places and lower than normal in others. The disturbed water at the point of impact disturbs the water next to it, which in turn disturbs the water next to it, and so on. If you place a cork or anything that floats on water at the middle of the disturbed water it bobs up and down and stays in its position as the ripples pass beneath it. It doesn't move along with the disturbance. Thus when the disturbance moves forward the particles of the medium vibrate up and down about their mean position of rest but they do not move forward along with the wave.

KEY TERMS

- Pulse: A single disturbance
- Crest: The highest point of a transverse wave.
- Trough: The lowest point of a transverse wave.

When the water surface is disturbed only once, we create a pulse through the water. A single non-repeating disturbance traveling in a medium is called a **pulse**. But when the water is disturbed repeatedly at regular intervals, we create a wave in the water. Thus a pulse is a single disturbance while a wave is a disturbance that is repeated at regular intervals. Examples of common waves that we encounter in our daily life are sound and light. All waves can be characterized by the following characteristics: amplitude, wavelength, period, frequency, and speed.

Did you know?

Ocean waves are caused by wind. When the wind blows over the surface of the ocean, it creates friction, transferring energy from the wind and into the water. The actual wave height is dictated by the wind speed and the size of the area hit by the wind.

Characteristics of Waves

The most basic wave characteristics that are used to describe a wave are amplitude, wavelength, frequency, period and speed.

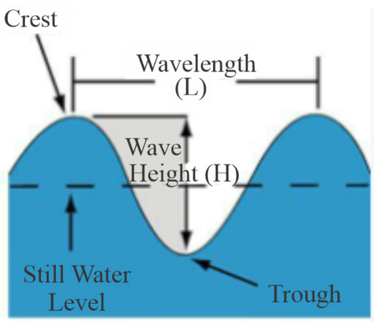


Figure 1. Crest and trough in a water wave.

1. Crest and Trough

Waves in water consist of moving crests and troughs. A crest is a place where the water rises higher than when the water is still. A trough is a place where the water sinks lower than when the water is still. The crest is the highest point of the wave and the trough is the lowest point of the wave. Figure 1 shows the crests and troughs on a wave.

Amplitude

The maximum displacement of the wave from the equilibrium (or rest position) is called the amplitude of the wave. It is the maximum height from the center line to the crest or the trough. Amplitude is directly related to the amount of energy carried by a wave. The greater the force that produces a wave, the greater the amplitude of the wave and the greater the energy carried by the wave. Sounds with greater amplitude will be louder; light with greater amplitude will be brighter. The symbol A is used for the amplitude of a wave. The unit of amplitude is meter (m).

Did you know?

The tallest wave ever measured is at Lituya Bay, just off the coast of Alaska.

The breaking waves from this tsunami created a mountain of water that stretched to a massive 1,720 feet or 524 meters and devastated everything in its path.

Wavelength

The wavelength (λ) is the distance between any two adjacent points which are in phase. It could be the distance between two adjacent crests or troughs. It is the distance the wave travels in one complete cycle. The wavelength is usually represented by the Greek letter lambda (λ).

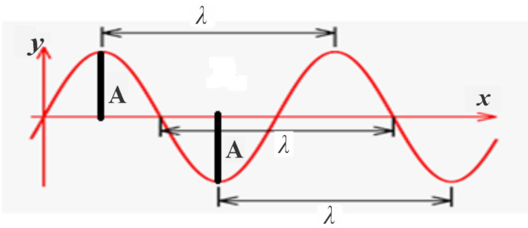


Figure 2. Wavelength of a wave

Period

The period (T) is the time taken for a wave to make one complete vibration or it is the time taken for a wave to move a distance of one wavelength. The period is usually represented by the upper case "T." As the period is time, it is measured in units of time such as seconds or minutes.

Frequency

The *frequency* of a wave, *f*, is a measure of how many waves pass by, or how many complete oscillations occur, in one second. The frequency of a wave is the same as the frequency of the vibrating body which produces the wave. The higher the frequency, the shorter wavelength and greater energy. Frequency is often represented by the lower case "f." The unit of frequency is hertz (Hz) which is equal to one wave per second.

We can easily see that period and frequency are inversely related. The period is the reciprocal of the frequency and vice versa.

$$T = \frac{1}{f} \quad or \quad f = \frac{1}{T}$$

KEY TERMS

- Amplitude: The maximum displacement from the equilibrium position.
- Wavelength: The distance a wave travels after one full cycle.
- Period: The time taken for one full cycle.
- Frequency: The number of complete oscillations made per unit time.
- Speed: The distance the wave travels per unit time.

Speed

The speed of a wave is how fast the disturbance of the wave is moving. The speed of a wave refers to the distance traveled by a given point on the wave in a given interval of time. Knowing wavelength and wave period, the **wave speed** (*v*) can be calculated by dividing wavelength (λ) by period (T).

$$v = \frac{\lambda}{T}$$

Since, $\frac{1}{T} = f$, it implies that the speed (*v*) of a wave is equal to the product of frequency and wavelength.

$$v = \frac{\lambda}{T} = f\lambda$$

The above equation is known as the wave equation.

Note that the speed of a wave depends on the properties of the medium through which the wave propagates and not on the mechanism that is generating the wave.

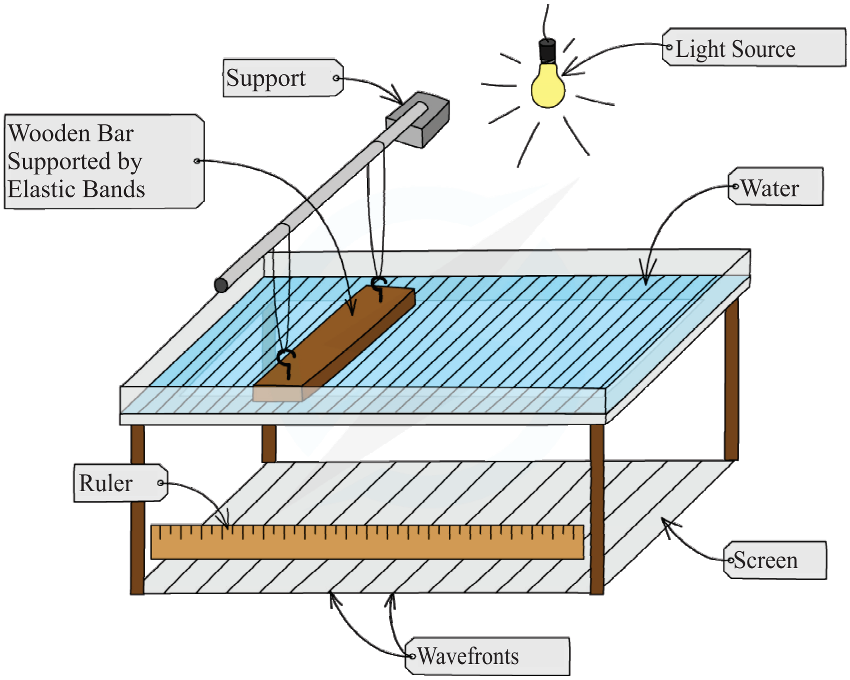
For example, the speed of sound waves depends on the pressure, density, and temperature of the air through which they propagate, and not on what is making the sound.

Experiment: Calculating the speed of a water wave using a ripple tank.

Aim of the experiment: To measure the frequency, wavelength and speed of waves in a ripple tank.

Materials required:

1. Ripple tank plus accessories
2. Suitable low voltage power supply
3. Meter ruler



Read these instructions carefully before you start work.

Set up the ripple tank. A large sheet of white card or paper needs to be on the floor under the tank.

Method:

1. Set up the ripple tank as shown in the diagram with about 5 cm depth of water.
2. Adjust the height of the wooden rod so that it just touches the surface of the water.

3. Switch on the lamp and motor and adjust the speed of the motor until low frequency waves can be clearly observed.
4. Place a meter ruler at right angles to the waves shown in the pattern on the card.
5. Measure across as many waves as possible. Then divide that length by the number of waves. This gives the wavelength of the waves.
6. Count the number of waves passing a point in ten seconds then divide by ten to record frequency (the frequency of waves is also determined by frequency of vibration of the dipper).
7. Calculate the speed of the waves using the equation:

   Wave speed = frequency × wavelength

   $v = f\lambda$

Examples

If a wave has a wavelength of 20 meters and a period of 4 seconds, what is its speed?

Solution

$$v = \frac{\lambda}{T} = \frac{20\ m}{4s} = 5.0 m/s$$

Examples

A wave has a distance of 1.6m between consecutive crests. If it takes 4 sec for one complete vibration, what are (a) the frequency and (b) the speed of the wave?

Solution

$\lambda = 1.6m;\ T = 4s$

(a) $f = \frac{1}{T} = \frac{1}{4s} = 0.25\ Hz$

(b) $v = \lambda f = 1.6m \times 0.25\ Hz = 0.4\ m/s$

Examples

The FM radio station broadcasting for Addis Ababa and the surrounding has a frequency of 97.1MHz. What is the wavelength of this radio wave?

Solution

$f = 97.1MHZ = 97.1 \times 10^6 HZ,\ C = 3 \times 10^8 m/s$

$$\lambda = \frac{v}{f} = \frac{3\times10^8 m/s}{97.1\times10^6 Hz} = 3.1m$$

Exercises

1. Define a wave motion.
2. Does a wave motion involve the motion of the medium as a whole, as the wave progresses in the medium? Explain.
3. Define (a) pulse, (b) wave.
4. Define the following terms: (a) crest, (b) trough, (c) amplitude, (d) frequency, (e) period, and (f) wavelength
5. The distance between a crests and adjacent trough of a wave motion is 1m. A person counted 11 crests passing a given point in the medium in one second. What is the speed of the wave?

Wave fronts and Rays

Wave front is defined as the imaginary surface constructed by the locus of all points of a wave that have the same phase. This could be where all the crests are, where all the troughs are or any phase in between. Wave fronts are useful for showing how waves move in two dimensions. The length between two lines on a wave front is exactly one wavelength.

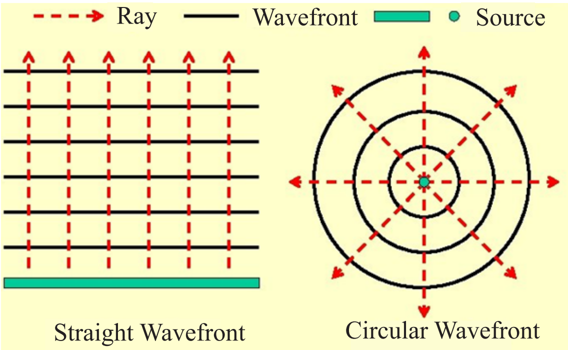


Figure 3. (a) Plane wave front and (b) spherical wave front

Key terms

- Wave front: A wave front is the set of all locations in a medium where the wave is at the same phase.
- Ray: Rays are lines that show the direction of propagation of a wave.

The direction of travel of the wave fronts is shown by a straight line with an arrow. It is called a ray. A ray is a line extending outward from the source and representing the direction of propagation of the wave at any point along it. Wave fronts and rays are always perpendicular to each other.

Properties of Waves

The properties of a wave include: reflection, refraction, diffraction, interference and polarization.

Key terms

- Reflection of waves: The turning back of a wave in reaching to a boundary through which they cannot pass.

Reflection of waves

When a wave is incident on the boundary separating the two media it is reflected (turned) back into the first medium without any change in the speed or wavelength of the wave. This phenomenon is known as the **reflection of waves**. Sound waves bounce off walls, light waves bounce off mirrors, and radar waves bounce off planes. Bats use reflection while they fly at night and avoid things as small as telephone wires. Figure 4 Shows the incident and reflected wave fronts represented by their crests.

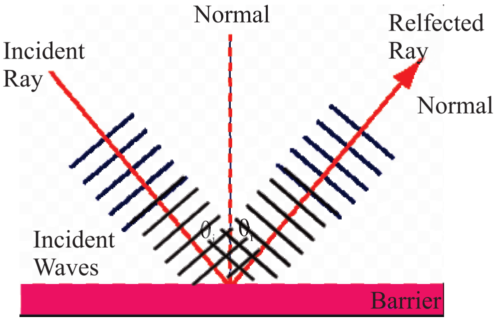


Figure 4. Incident and Reflected Ray.

Activity 1

Compare the wavelength, frequency and velocity of the incident and reflected wave.

Refraction of Waves

Key terms

- Refraction: The bending of a wave in moving from one medium to another of different density.

The speed and wavelength of a wave depends on the medium through which the wave travels. Therefore, when a wave approaches a boundary obliquely (at an angle), their direction is changed, because of the change in the speed of the wave. This change in the direction of waves at the boundary between two different media is known as **refraction**. During refraction, the speed and wavelength of the wave changes, but the period and frequency of the wave remain the same.

When you look at a pencil that emerges from water it looks like it is bent. This is because the light from below the surface of the water bends when it leaves the water. Your eyes project the light back in a straight line and so the object looks like it is a different place.

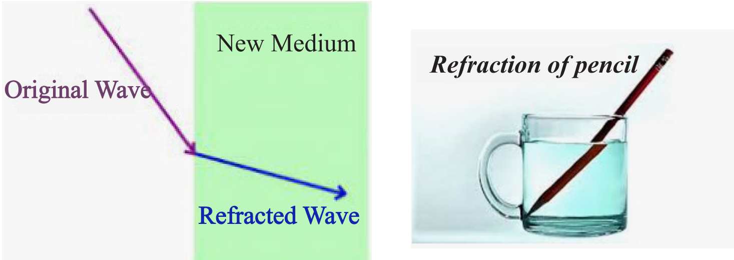


Figure 5. Refraction of waves

Refraction of Water Waves

A deep water and a shallow water acts like a different media. It is observed that as the waves pass into the shallow water from the deep water the wavelength of the wave decreases (see Figure 6).

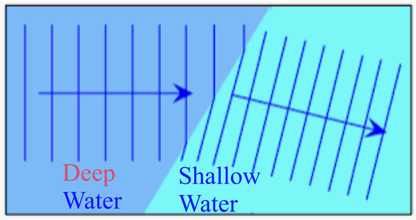


Figure 6. Refraction of water Wave.

ACTIVITY 2

From Figure 6 we see that the wavelength of a water wave shortens when it enters the shallow water. What does this tells you about the speed of water wave in deep and shallow water?

KEY TERMS

- When a wave slows down in the new medium it bends towards the normal.
- When a wave speeds up in the new medium it bends away from the normal.

There are two types of refraction: **Refraction towards the normal** and **refraction away from the normal.** When the speed of a wave decreases on entering the second medium, its wavelength also decreases. Therefore, the angle of refraction will be less than the angle of incidence ($r < i$). As a result the wave refracts towards the normal, see Figure 7 (a). But if the speed of a wave increases on entering the second medium, its speed and hence it wavelength increases. Thus, the angle of refraction will be greater than the angle of incidence ($r > i$). As a result it refracts away from the normal, see Figure 7 (b).

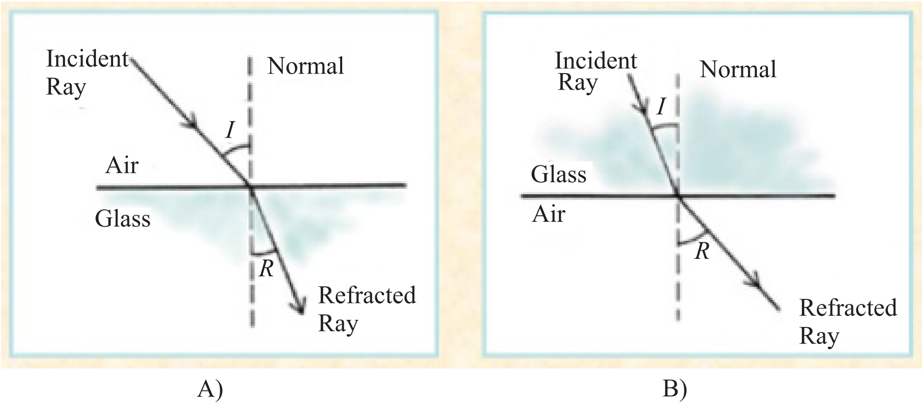
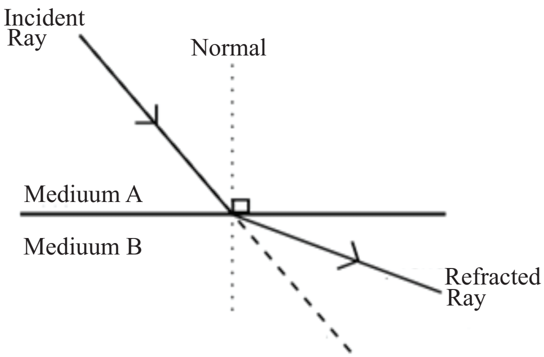


Figure 7. a) Refraction towards the normal b) Refraction away from the normal

Activity 3

1. How will (a) mechanical waves, and (b) electromagnetic waves refract towards the normal and away from the normal?
2. A wave that moves from medium A to B is refracted as shown in the figure below. In which medium is the speed of the wave greater and in which medium is smaller?



Examples

Water wave propagates with a speed of 0.8m/s and wavelength 4cm in the shallow section of the ripple tank. When the wave enters into the deep section its wavelength becomes 6cm. What is the speed of the wave in the deep section?

Given: $V_1 = 0.8$ m/s, $\lambda_1 = 4\text{cm} = 0.04\text{m}$, $\lambda_2 = 6\text{cm} = 0.06\text{m}$.

Solution

Since the frequency of the wave is the same in both media.

$$f_1 = f_2$$

$$\frac{V_1}{\lambda_1} = \frac{V_2}{\lambda_2}$$

$$\Rightarrow V_2 = \frac{V_1\lambda_2}{\lambda_1} = \frac{0.8\text{m/s} \times 0.06\text{m}}{0.04\text{m}} = 1.2\text{m/s}.$$

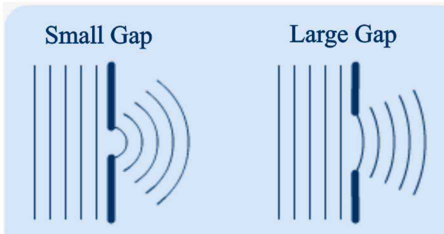
Diffraction of Waves

Diffraction happens when waves move through a gap or around an obstacle. The wave fronts change their shape when they pass through the gap. They spread out behind the gap, and change their direction. When the gap is very narrow as in Figure 8 (a), the gap seems to act as a source of circular waves. When the gap is wider as in Figure 8 (b) the effect are not very noticeable. This change of direction of the wave fronts when they pass through a gap is called **diffraction**. Diffraction effects are greater when the opening between the objects is about the same size as or smaller than the wavelength of the waves.

If you place a solid obstacle in the straight waves, you will see that the wave bend behind the obstacle as in Figure 8 (c). They are able to pass into the region behind the obstacle, even though they are not passing through it. Thus we can also define diffraction as the bending of a wave front into the region behind the obstacle. It is because of diffraction that sounds can sometimes be heard around corners and in the shadow of buildings. It is also why radio signals, particularly those with a long wavelength, can be received in the shadow of hills.

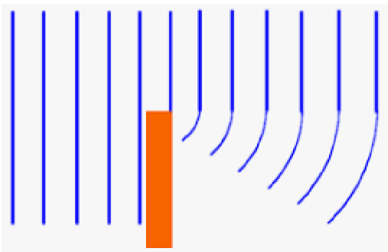
KEY TERMS

- Diffraction of waves: is the spreading out of a wave after passing through a small opening or bouncing of a wave around an obstacle.



a) Diffraction of waves through small opening

b) Diffraction of waves through large opening



c) Diffraction around an obstacle

Figure 8.

The frequency of the wave depends on the frequency of the source generating the wave, which is constant in this case. Therefore, during diffraction the speed and wavelength do not change, but the direction of the wave changes as they pass the edge of the obstacle.

Note that waves that have a longer wavelength are diffracted more strongly than waves of smaller wavelength.

ACTIVITY 4

1. Suppose your friend is calling you being behind a house through which there is no gap for sound to pass through. Then how do you hear his call?
2. Why is the diffraction of sound more easily observable than the diffraction of light?

Interference of Waves

When two or more identical waves (same wavelength, amplitude and frequency) travel in the same medium at the same time, they interfere each other. Such a mixing up of waves is called **interference**.

When the two waves are in phase (the crest of one meets the crest of the other or the trough of one meets the trough of the other) they reinforce each other. In such cases the interference is said to be **constructive.** When the interfering waves are out of phase (the crest of one coincides with the trough of the other) they tend to cancel each other and the interference is said to be **destructive**.

The amplitude of the resulting wave will depend on the amplitudes of the two waves that are interfering. During constructive interference the amplitude of the resulting wave is larger than the amplitude of either of the interfering waves, see Figure. 9 (a). During destructive interference the amplitude of the resultant wave is less than the amplitude of the interfering waves, see Figure. 9 (b). If they have the same amplitude, the resultant wave will have zero amplitude while they cross over each other. Such kind of interference is known as **complete destructive interference**.

The Superposition Principle

The amplitude of the resultant wave is determined by use of the **superposition principle** stated below.

"When two or more waves travelling in the same medium at the same time interfere, the resulting displacement of the new wave is the algebraic sum of the displacement caused by the individual waves."

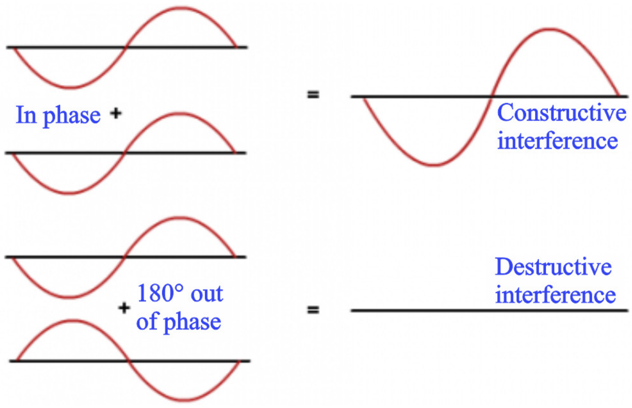


Figure 9. Constructive and destructive interference.

Interference of Water Waves

A vibrating dipper on a water surface sends out circular waves. Two sets of circular waves are produced in this way in a ripple tank. These two sets of waves pass through each other continuously and produce an interference pattern (see Figure 10).

Key terms

- Interference of waves: when two or more identical waves travel in the same medium at the same time they mix up together

Demonstration:

To study the properties of water waves with a ripple tank:

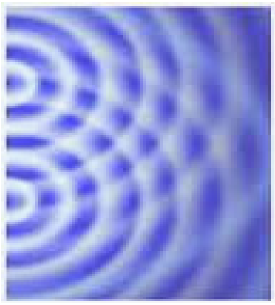


Figure 10. Interference of water waves

A ripple tank is a device used in demonstrating wave properties such as reflection and refraction. It consists of a shallow transparent tray of water with a point light source above it and a white screen on the floor below (Figure 11). Before adding water, the tray is leveled with a spirit-level to ensure a uniform water depth of rather less than 1 cm.

Straight parallel waves may be produced by a horizontal metal strip, or circular waves by a vertical ball-ended rod. When either of these is dipped into the water, a pulse of ripples is sent across the surface. The bar is moved up and down by the vibrations of a small electric motor having an eccentric metal disc on its rotating spindle. A rheostat in the motor circuit controls the speed and hence the frequency of the waves sent out. Owing to the lens effect of the wave crests and troughs, the light source produces a bright and dark wave pattern on the white screen.

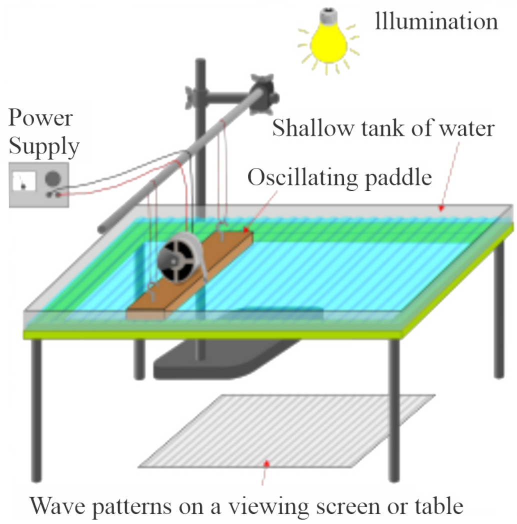


Figure 11. The ripple tank

Reflection

Place in a barrier (metal bar) somewhere in the tank. Then send the ripples towards the barrier. The ripples reflect from the bar. Reflection of straight wave front by a straight barrier is shown in the figure Figure 12.

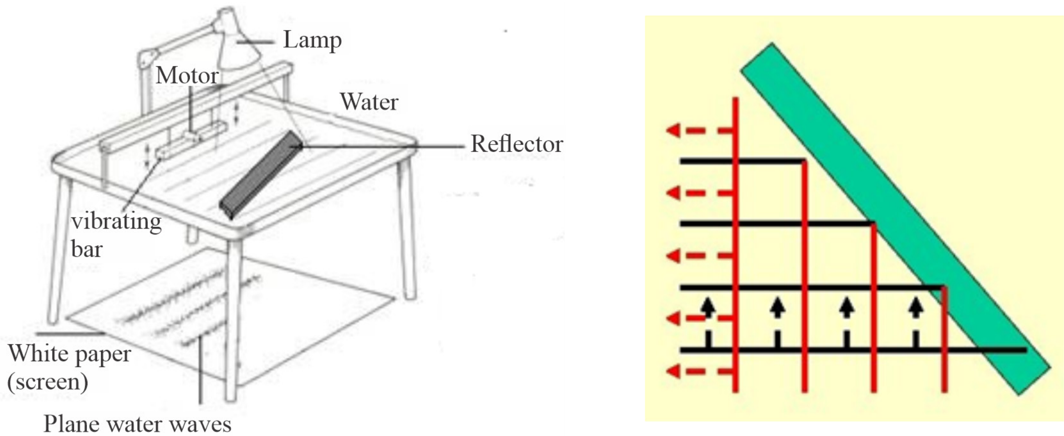


Figure 12. Reflection in a ripple tank

Refraction

Refraction of water waves can be observed in a ripple tank if the tank is partitioned into a deep and a shallow section. A piece of glass slab is placed in the ripple tank so that two regions with different depth are created. Waves traveling from the deep end to the shallow end can be seen to refract.

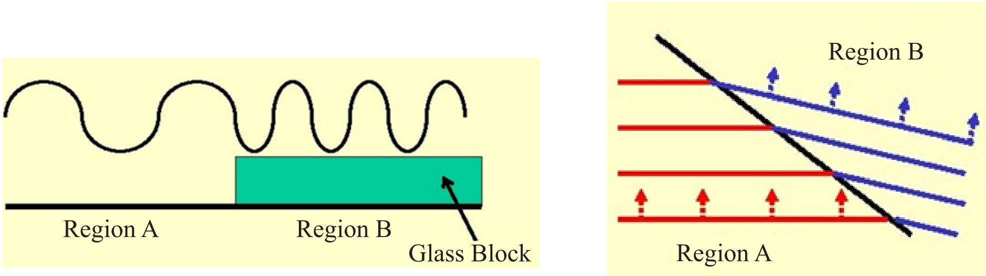


Figure 13. Refraction in a ripple tank

Diffraction

Put two solid barriers in a ripple tank one pair with a narrow gap and the other with a wider gap between them. Then let a straight wave to pass through the gaps. The waves that comes through the hole no longer looks like a straight wave front. It bends around the edges of the hole. If the hole is small enough it acts like a point source of circular waves.

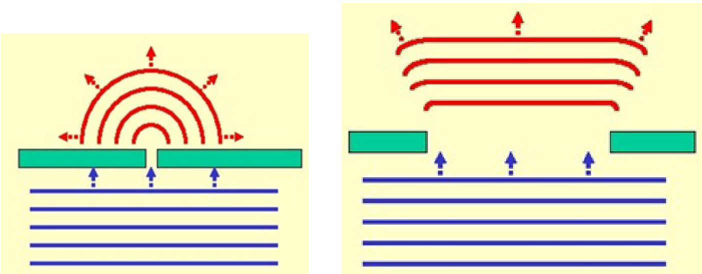


Figure 14. (a) Barrier with a small gap (b) Barrier with a wider

Interference

Plane waves in a ripple tank strike two narrow gaps. Each gap produces circular waves beyond the barriers, and the result is an interference pattern. The pattern is the same as would be produced by two dippers vibrating in phase at the gaps, but fainter.

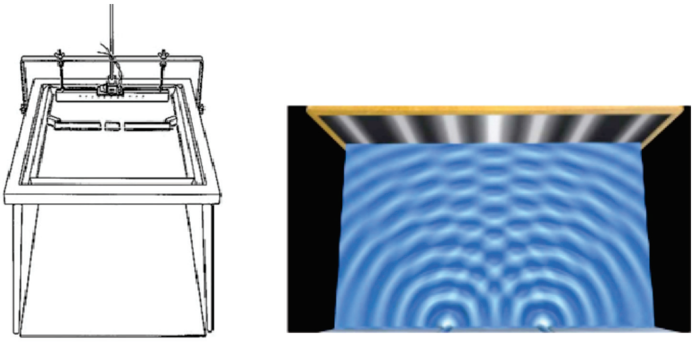


Figure 15. (a) Interference of water waves with (b) two dippers

Polarization

A wave that is vibrating in more than one plane is referred to as unpolarized. Polarized waves are waves in which the vibrations occur in a single plane. The process of transforming unpolarized wave into polarized wave is known as **polarization**.

Key terms

- Polarization is the process of transforming unpolarized wave into polarized wave

Light waves are often polarized using a polarizing filter. Only transverse waves can be polarized. Longitudinal waves, such as sound, cannot be polarized because they always travel in the same direction of the wave.

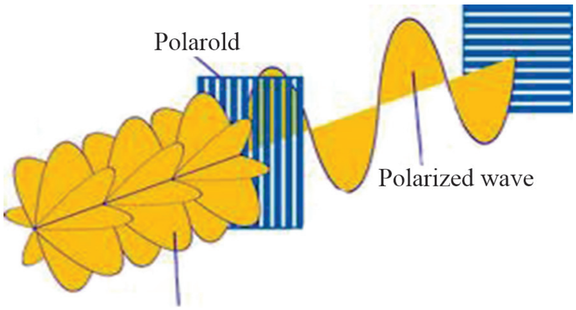


Figure 16. Polarization of wave

Exercises

1. Define the terms: (a) reflection; (b) refraction; (c) diffraction; (d) interference; and (e) polarization.
2. Explain how refraction towards the normal and refraction away from the normal takes place.

3. What is the difference between diffraction through a small opening and a large opening?
4. When are the waves interfering (a) constructively? (b) destructively?

Types of Waves

Key terms

- Mechanical waves: Waves that require a material medium to transmit the disturbance.
- Electromagnetic waves: Waves that do not require material medium (that can propagate through vacuum)

Not all waves are the same type. Some can travel through vacuum but others cannot. Some waves travel parallel to the direction of propagation of the particles of the medium while others move perpendicular to the direction of propagation of the medium. These and other types of waves will be discussed in this section.

1. Based on their ability or inability to travel through a vacuum, waves are classified into two: mechanical and electromagnetic.

A **mechanical wave** is a wave that is not capable of transmitting its energy through a vacuum. Mechanical waves require a medium in order to transport their energy from one location to another. Sound waves, water waves and waves that travel along a spring (slinky) or a string are all examples of mechanical waves. Mechanical waves cause oscillations of particles in a solid, liquid or gas and must have a medium to travel through. All mechanical moves can not travel through vacuum.

Electromagnetic waves are waves that do not require a material medium to transmit the disturbance. They can propagate through transparent materials and can also propagate easily through vacuum. Light wave, radio and TV waves, microwaves, infrared, UV – rays, x – rays and gamma rays are all electromagnetic waves.

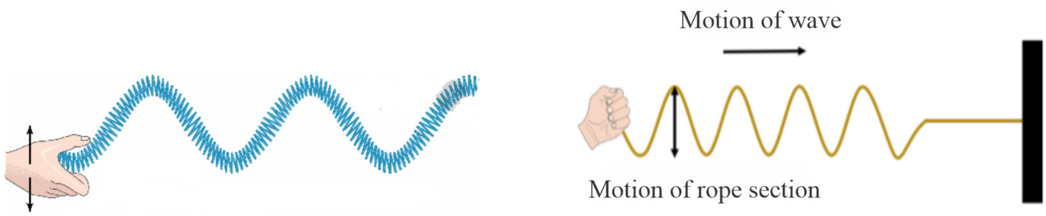
All electromagnetic waves have the same velocity in moving through vacuum (or air). This speed is what we commonly refer as speed of light $= 3.0 \times 10^8\ m/s$. Electromagnetic waves are produced by the periodic changes that take place in magnetic and electric fields and therefore known as Electromagnetic Wave.

2. Based on the basis of the direction of movement of the individual particles of the medium relative to the direction that the waves travel waves can be classified as transversal or longitudinal.

Key terms

- Transverse waves are waves in which the particle of the medium oscillate perpendicular to the direction of propagation of the wave.

A **transverse wave** is a wave in which particles of the medium move in a direction **perpendicular** to the direction that the wave moves. For example, if you attach a horizontal spring to a wall and move the other end up and down vertically while the wave travels horizontally along the spring (Figure 17). In this case, the particles of the medium move perpendicular to the direction that the pulse moves. This type of wave is a transverse wave.



a) b)

Figure 17. Transverse wave in a) spring b) string

You will also get the same kind of wave if you disturb one end of a string up and down, the other end fixed to something. Light waves, waves that travel along ropes and waves across the surface of water, and all electromagnetic waves are transverse.

Parts of transversal waves

For transversal waves the wavelength is the distance between two consecutive crests or troughs or the distance between any two points which are in the same phase (having the same position and direction of propagation, see Figure 18.

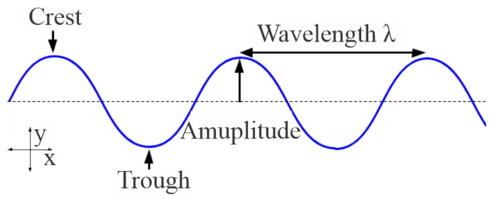
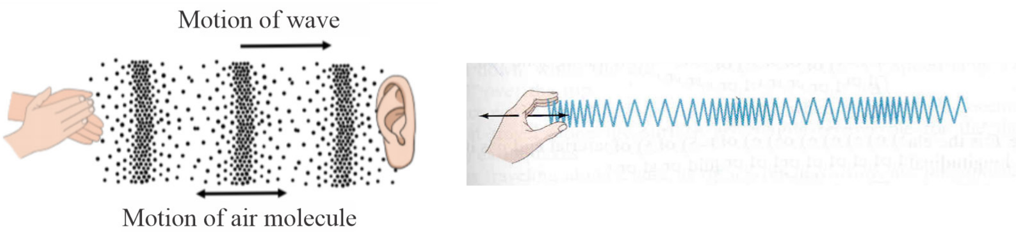


Figure 18. Characteristics of a transverse wave.

Longitudinal waves

A **longitudinal wave** is a wave in which particles of the medium move in a direction **parallel** to the direction that the wave moves. In the production of sound waves by clapping your hands, the air molecules oscillate about an equilibrium position in the same direction as the wave propagates, Figure 19(a).



a) b)

Figure 19. A longitudinal wave in a) air b) spring

Longitudinal waves are also produced by a spring. Fix one end of a long spring (slinky) to a rigid body. Then move the free end back and forth along its length in a regular pattern. The wave will travel along the length of the spring parallel to the disturbance see Figure. 5.19 (b).

Key terms

- Longitudinal waves: Waves in which the particle of the medium oscillate parallel to the direction of propagation of the wave.

In longitudinal waves no crests and troughs are produced. But in course of their movements, the particles come closer together in some places than their normal separation in the medium. In this region a **compression** is formed. In other places the particles move farther apart than their normal separation and a **rarefaction** is formed. The distance from a compression to a compression, or from rarefaction to rarefaction gives the wavelength, λ, of the wave, see Figure 20.

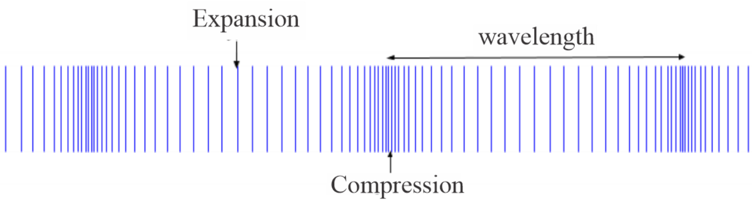


Figure 20. Compression and Rarefaction in a longitudinal wave.

1. **Based on their appearance waves are classified as traveling wave and standing wave.**

A traveling wave is a disturbance that travels through a medium. Consider the ripples (waves) made by a rock dropped in a pond (Figure 5.21). The ripples travel outwards from where the rock was dropped. The individual water molecules will move in small circles about an equilibrium position, but they do not move along with the waves.

Figure 21. A water wave

Standing waves

Standing waves are waves that do not appear to be propagating. They are also called *stationary waves*. These waves arises when a wave meets its own reflection under the right circumstances. Standing waves do not transport energy through the medium. For example, a vibrating string on a violin is a standing wave.

Figure 22. A standing wave by a string

Examples

The speed of sound in water is 1500 m/s. If a tuning fork vibrating at 600Hz is immersed in water, what is the wavelength of the resulting wave?

V = 1500m s, f = 600Hz

$$\lambda = \frac{V}{f} = \frac{1500\text{m/s}}{600\text{Hg}} = 2.5\text{m}$$

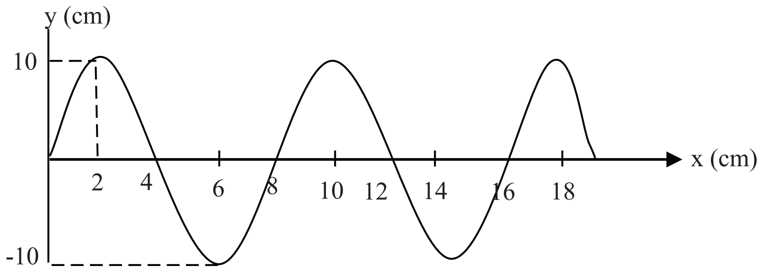
Activity 5

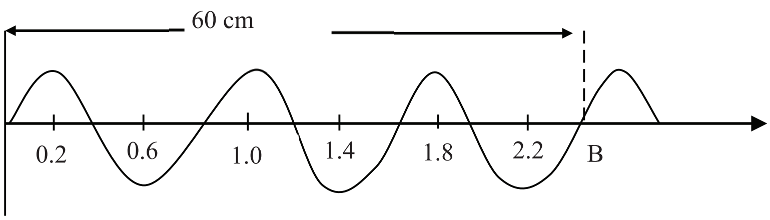
Longitudinal waves can be set up through solid, liquid or gases, but transverse waves are produced only in a solids and liquids but not in gases. Explain why.

Exercises

1. What are the requirements for the production of mechanical waves?
2. Distinguish between:
   (a) mechanical and electromagnetic waves
   (b) Longitudinal and transversal waves
   (c) Travelling and standing waves.
3. Give at least three examples of:
   (a) mechanical waves.
   (b) Electromagnetic waves.
   (c) Transversal waves.
   (d) Longitudinal waves.
4. How is energy transferred by mechanical waves?
5. Are water waves longitudinal waves or transverse waves? Explain.

6. A wave traveling in the + x direction is shown in the figure. What is (a) the amplitude (b) the wavelength of the wave?
7. If the distance between the points A and B in the figure is 60cm, what is (a) the period? (b) the frequency? (c) the wave length? (d) the speed of the wave?





Sound Waves

KEY TERMS

- Source of sound: A vibrating body that could produce sound.
- Compressions: the particles are close together (high pressure region).
- Rarefactions: the particles are spread apart (low pressure region).

Sound waves are the most important example of longitudinal waves. In this section we discuss the characteristics of sound waves, how they are produced, what they are, and how they travel through matter. We then investigate what happens when sound waves interfere with each other.

Sound

Sound is a form of energy. From the source of sound this energy is propagated through the surrounding medium in the form of waves. When the vibration of the source has a frequency between 20Hz and 20,000Hz these compressional waves are able to cause a sensation of hearing and are referred to as **sound waves**.

In the previous section, we have seen that sound is a mechanical longitudinal wave. Thus when the source of sound vibrates, it makes compressions and rarefactions which spread out through medium, which may be a solid, liquid or gas. Sound cannot travel through a vacuum because it needs particles to vibrate.

Did you know?

With the help of sounds, most animals can detect dangers and hazards before they affect them. Dogs and cats can hear much higher pitched sounds than we can. Dolphins, for example, can't hear sounds as low as we can, but can hear high sounds of over 100,000 Hz. Animals that have large ears can hear better as compared to animals with small ears. Flies cannot hear any kind of sound. Not even their own buzzing.

The production and Transmission of Sound

Any source of sound (like a tuning fork, a drum, a guitar,. . .) is always in a state of vibration. When the source of sound vibrates it causes successive compressions and rarefactions. When the molecules are pushed together by a vibrating source a **compression** area (a region of higher pressure) will be formed. These compressed molecules of the air will pass the compression to the adjacent molecules of the air. These molecules in turn compress the next adjacent molecules and so on. In this way this compressed wave travels away from the source into the surrounding air. Next to the compression we have **rarefactions** where the air molecules are far apart. This expansion of few air molecules into a free space causes **rarefaction** (a region of low pressure), see Figure 23.

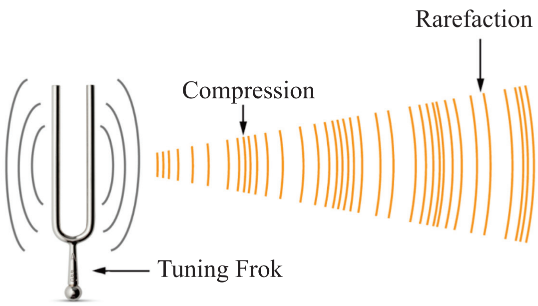


Figure 23. When a source of sound vibrates it forms successive compressions and rarefactions.

Sound is produced by vibrating bodies. The objects that produce sound are called **sources of sound.** Vibrating strings such as violin and human vocal cords; and vibrating plates and membrane such as drum and loudspeakers are some of the sources of sound.

Did you know?

Sound energy can be used in medicine as well as for therapeutic purposes. For example, sound vibrations are utilized in ultrasounds.

Activity 6

Take a drum. Touch it when not in use. Again touch it when producing sound. What do your hands feel when drum is beaten and produce sound? Can you feel the skin of the drum vibrating? Place small pieces of papers on the drum and play it. Observe what happens to the pieces of paper when the drum is beaten. What did you conclude from your observation

Activity 7

Observing sound propagation in solids

(a) Hold one end of a meter stick against your ear. Let your friend slightly scratch at the other end. You may hear the scratching quite clearly although it is hardly audible through the air medium.

(b) Your may have played with a tin-can-telephone during your child hood. It could be a surprising experience to those students who use the telephone for the first time.

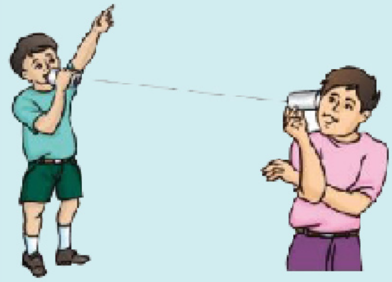


Figure 24. Tin-can-telephone

Activity 8

Propagation of sound through liquids

Figure 25. Sound throught liquid

Take a glass and fill it with water. Take a bell and ring it inside the water. Ask your friend to listen to the sound by keeping his / her ears touching walls of the glass. The conclusion is that sound propagates through matter in all the three states – solid, liquid and gas.

Activity 9

Sound des not travel through vacuum
To show Sound needs a medium to travel.

Take an electric bell and airtight jar. The electric bell is suspended inside the airtight bell jar. With air still in jar ring the bell. Now take out air slowly by using vacuum pump. Ring the bell again. What difference did you observe?

Observation: Sound of bell can be heard when air is inside the jar. As more and more air is removed from the glass jar, the sound of ringing bell becomes fainter and fainter. And when all the air is removed from the glass jar, no sound can be heard at all. This shows that sound can't travel through vacuum.

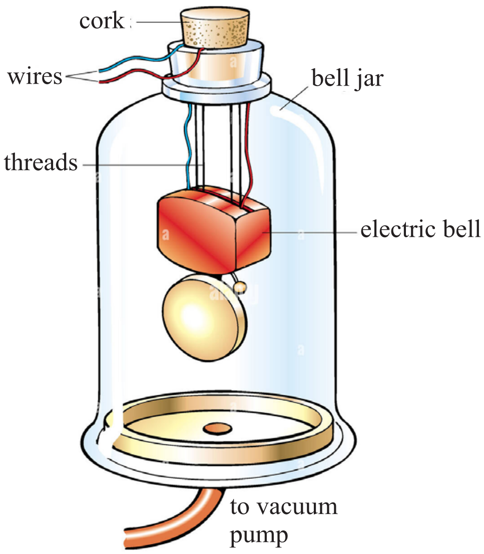


Figure 26. Sound can't travel through vacuum

Characteristics of Sound

Like any kind of waves, sound also have five main characteristics: wavelength, amplitude, frequency, time period and velocity.

Wavelength: In transverse waves we have measured the wavelength of the wave from crest to crest or trough to trough. But in longitudinal waves like sound, we measure the wavelength as the distance between two consecutive compressions or rarefaction which are in same state of vibration, center to center or end to end, see Figure 27.

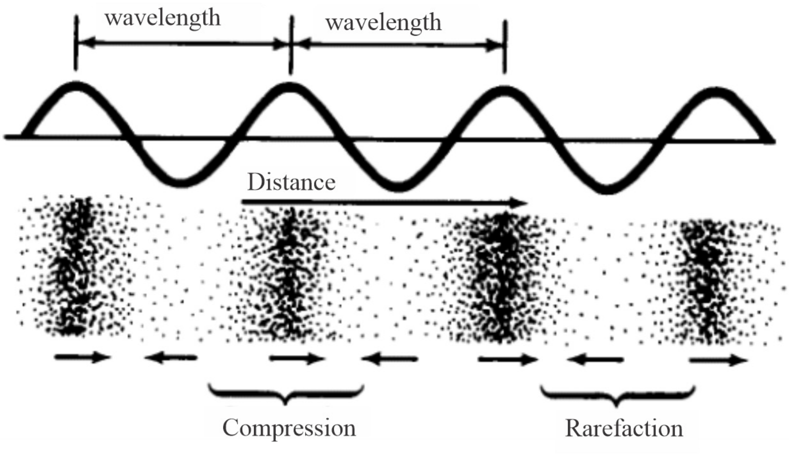


Figure 27. A traveling wave traveling through a spring

Amplitude: The amplitude is the maximum displacement of the particles the sound wave disturbed.

Frequency: The frequency refers to the number of sound waves a source produces per second. Sound frequency is not dependent upon the medium the sound is passing through. The S.I unit of frequency is hertz or Hz. A vibrating body emitting 1 wave per second is said to have a frequency of 1 hertz. Sometimes a bigger unit of frequency is known as kilohertz (kHz) that is 1 kHz = 1000 Hz.

Time Period – The time period is the time required to produce a single complete cycle. Each vibration of the source producing the sound is equal to a cycle. The time period is the reciprocal of the frequency.

Speed of Sound in Different Media

The speed of sound depends on the medium through which it is propagating. We have already seen that sound travels best through media that are dense. Table 1 shows this fact.

Table 1 Speed of sound in different media.

S.No.	Nature of the medium	Name of the medium	Speed of sound (in m s^{-1})
1	Solid	Copper	5010
2		Iron	5950
3		Aluminium	6420

4	Liquid	Kerosene	1324
5		Water	1493
6		Sea water	1533
7	Gas	Air (at 0^oC)	331
8		Air (at 20^oC)	343

As you see from the table the speed of sound tends to increase in a denser medium. Solids are denser than liquids and liquids are denser than gases. Therefore, the speed of sound is greater in solids than in liquids and greater in liquids than in gases. Solids, liquids and gases transmit sound, but sound can't travel through vacuum.

ACTIVITY 10

Discuss in group why Sound waves travel faster in liquids than gases, and fastest of all through solids.

Speed of Sound in Air

The speed of sound in air depends on the temperature of the air. When the temperature of a medium increases, the kinetic energy of all the molecules in the medium increases and hence the speed of sound in moving through this medium also increases. For sound traveling through air, the relationship between wave speed and medium temperature is:

$$v = 331.5\sqrt{1 + \frac{T_c}{273.15^\circ C}}\ m/s$$

where T_c is the temperature in °C and 331.5m/s is the speed of sound at 0°C. This equation is approximated as:

$$v = (331.5 + 0.6T_c)\ m/s$$

From this equation we see that, at a temperature of 0^0C, sound waves travel with a speed of 331.5m/s. The speed of sound increases by 0.6 meters per second (m/s) for every degree-Celsius increase in temperature.

Examples

What is the speed of sound in air at room temperature (20^0C)?

Solution

Using the above equation

v(331.50.620)m/ s343.5m/ s

Speed of Sound in Solids and Fluids

Sound is a mechanical wave. Thus its speed depends on how close the particles of the medium are through which it is moving. The speed, υ,of a sound wave in various materials is related to the elasticity and density, ρ, of the material by the equation

For solids: $\upsilon = \sqrt{\frac{Y}{\rho}}$

Where Y= young's modulus of the solid, and ρ is the density of the solid.

For fluids: $\upsilon = \sqrt{\frac{B}{\rho}}$

Where B = the bulk modulus of the fluid (liquid or gas), and ρ is the density of the fluid.

Examples

Determine the speed of sound in water.

Solution

Water is a fluid, and its bulk modulus is $B_w = 2.1 \times 10^9$ N/m², and its density is $\rho_w = 10^3$ kg.m³. Thus, the speed of sound in water is

$$v_w \sqrt{\frac{B_w}{\rho_w}} = \sqrt{\frac{2.1\times10^9 \text{N/m}^2}{10^3 \text{kg/m}^3}} = 1449.14 \text{ m/s}$$

Examples

Find the speed of a sound pulse in an aluminum bar struck at one end with a hammer.

Solution

Aluminum is a solid and its Young modulus is $Y_{Al} = 7 \times 10^{10}$ N/m², and its density is $\rho_{Al} = 2.7 \times 10^3$ kg.m³. Thus, the speed of sound in Aluminum is

$$v_{Al} \sqrt{\frac{Y_{Al}}{\rho_{Al}}} = \sqrt{\frac{7\times10^{10} \text{N/m}^2}{2.7\times10^3 \text{kg/m}^3}} = 5091.75 \text{ m/s}$$

Reflection of Sound – Echo

Because sound is a wave it will be reflected back from a tall building, a mountain even from the wall of an auditorium. If sound is reflected back from an obstacle to the observer in such a way that he/she hears it distinctly as a repetition of the original sound an **echo** is said to be produced.

Note that sound waves are reflected off hard surfaces like glass and wood. Sound does not reflect off from soft surfaces like cloth and rubber. These surfaces absorb sound. That is why the walls of auditoriums and cinema halls are covered by a soft material like curtain, to decrease the reverberation time (the time taken for sound to die away in a building).

If the echo time (t), that is the time taken for the sound to move from the source to the obstacle, then back to the source is measured, the speed of sound at that temperature is determined by the equation

$$v = \frac{\text{total distance travelled}}{\text{time taken}}$$

The distance the sound travels will be 2s, because the sound wave will travel to the obstacle and reflected back to the source., where is the distance between the soruce of sound and the obstacle. Thus,

$$v = \frac{2s}{t}$$

Applications of Reflection of sound – Echo

Echoes have many uses some of them are mentioned below:

1. **Determination of the velocity of sound in air.** As discussed earlier if the distance between the source and obstacle, and echo time is measured, the velocity of sound at that temperature can be determined by using the equation.

$$v = \frac{2s}{t}$$

2. **Determination of the depth of an ocean (sea)**. The depth of oceans or seas are determined using a device called **sonar**. A sonar makes underwater sound waves, which travels to the bottom of the sea. This wave is then reflected off the seabed and returns to the ship. The time for the sound wave to return back to the ship (echo time) is measured. By use of the speed v of sound in sea water, we can then determine the depth of the sea or ocean using the equation.

$$s = \frac{vt}{2}$$

3. The working of a stethoscope is also based on the reflection of sound. In a stethoscope, the sound of the patient's heartbeat reaches the doctor's ear by multiple reflections of sound.
4. The soundboard is based on the reflection of sound. Sound board is a big concave board and is set in such a fashion behind the stage that speaker is at the

focus. Sound coming from speaker falls over sound board and gets reflected towards the audience. As a result, the audience sitting in the hall even at far distance from the speaker can clearly hear what the speaker is saying.

Examples

A depth measuring device produces sound signals at a frequency of 3,000Hz and having a wavelength of 0.5m in ocean water. These waves are reflected from the bottom of an ocean and received by the device at the top after 2sec. What is the depth of the ocean?

Solution

$f = 3000\text{Hz}; \quad \lambda = 0.5\text{ m}; \quad t = 2\text{ sec}$

$$s = \frac{vt}{2} = \frac{f\lambda t}{2}$$

$$s = \frac{3000\text{Hz} \times 0.5\text{m} \times 2\text{s}}{2} = 1500\text{m}$$

Noise and music

Key terms

- Noise: Unpleasant sound
- Musical note: sounds which are pleasant to hear

We hear different types of sounds around us. Some sounds are pleasant to the ear, whereas some are not. Such unpleasant sounds are called ***noise***. The sounds which are pleasant to hear are called ***music***. Some sources of sound produce pleasant sounds or musical notes while others produce unpleasant sound or noises. Musical notes are sounds that are easy to listen to because they are rhythmic. The waves that carry these notes change smoothly and their wave patterns occur at regular interval of time.

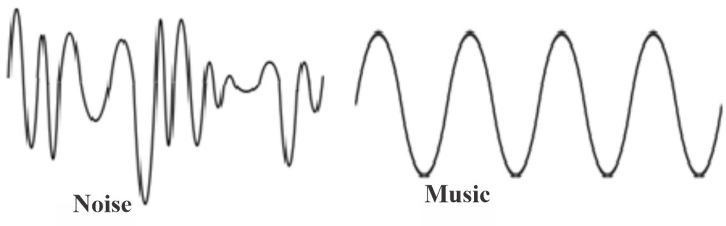


Figure 28. Noise and music

Did you know?

The majority of cows that listen to music end up producing more milk than those who do not.

ACTIVITY 11

Discuss in group why two astronauts talking on the surface of the moon cannot be heard each other.

Loudness and pitch

The **Loudness** of a sound is the magnitude of the auditory sensation produced by the sound. It depends on the amplitude of the sound wave. The greater the amplitude of the sound the louder the sound will be. Actually, loudness is a difficult quantity to measure because it depends on the Judgment of the listener rather than any physical measurement.

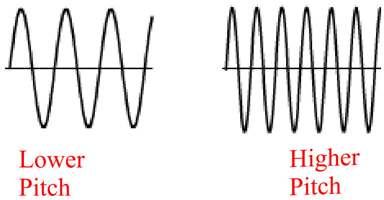


Figure 29. Pitch and frequency

The **pitch** of a sound is the subjective feeling experienced by a listener due to the frequency of the sound. It is associated with the physical characteristics of frequency of vibration. The higher the frequency of the sound waves the higher their pitch.

Audible and Inaudible Sounds

All sound waves cannot produce sensation of hearing. Sound waves that are able to produce sensation of hearing are called **Audible**, and those waves that cannot produce sensation are **Inaudible.**

Audibility depends on frequency. The frequency of audible sound lies between 20Hz to 20,000Hz. The lower limit of audibility is 20Hz and the upper limit of audibility is 20,000Hz. A sound wave whose frequency is less than the lower limit of audibility is called **Infrasonic Wave**. A sound wave whose frequency is greater than the upper limit of audibility is called **Ultrasonic.**

Did you know?

Bats produce ultrasound wave signals. When these signals bounce off objects, they return echoes, which helps them to know whether or not an obstacle is in the way.

Intensity and Intensity Level

Any vibrating source can set up sound waves in air. The greater the energy of vibration the greater the

KEY TERMS

- Intensity is a measure of the rate of sound energy (power) per unit area.

amount of energy carried by the sound wave. So, we can see that the amount of energy carried by a sound wave depends on the amount of energy with which the source vibrates.

The amount of energy (E) transferred per unit time (t) per unit Area (A) perpendicular to the direction of motion is called the **intensity (I) of the wave.**

Since energy per unit time is power, intensity can also be defined as power per unit area.

$$I = \frac{Energy\,/\,time}{Area} = \frac{Power}{Area}$$

$$I = \frac{P}{A}$$

The SI unit of intensity is watt per meter square (W/m^2).

Because sound waves spread out at the same speed in all directions from a vibrating source, they spread out as a spherical wave front. The spherical volume around a vibrating source in a medium will be filled with the energy from the vibrating source. The area of a sphere is given by the equation, $A = 4\pi r^2$. Thus, for a point source that sends out spherical sound waves, the intensity (I) of the sound at a distance, r, from the source is

$$I = \frac{P}{A} = \frac{P}{4\pi r^2}$$

The above equation shows that the intensity of a sound wave is inversely proportional to the square of the distance r between the source and listener, see Figure 30.

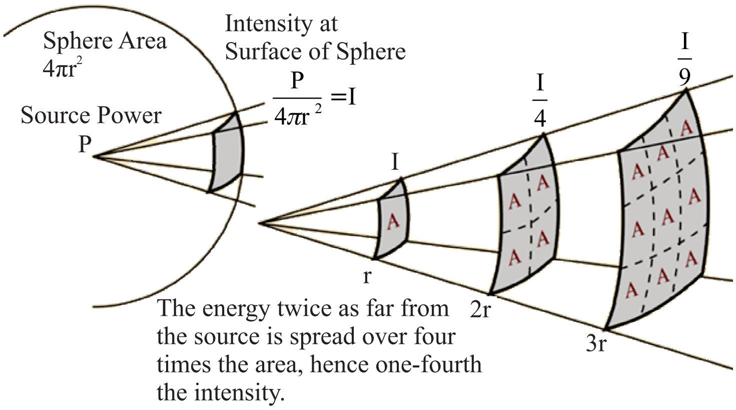


Figure 30. The intensity of a sound is inversely proportional to the square of the distance

Inverse square law

From the above equation of intensity $\left(I = \frac{P}{4\pi r^2}\right)$, you can understand that the intensity of a sound wave is inversely proportional to the square of the distance from the sound source. This principle is known as the inverse square law.

$$I \,\alpha\, \frac{1}{r^2}$$

$$\Rightarrow Ir^2 = \text{constant}$$

The intensity of a sound wave can also be determined using the equation

$$I = \frac{1}{2}\rho\, \upsilon\, \omega^2 A^2$$

Where ρ is the density, υ the speed, ω the angular velocity, and A the amplitude of vibration of the wave. From this equation we see that the intensity of a sound wave is directly proportional to the square of the amplitude of vibration of the source, $I \alpha A^2$.

Examples

The total effective area of the auricle of a listener ear is about $40cm^2$. (a) How much power enters the ear of a person from a sound of intensity $10^{-6} w/m^2$. (b) How long will it take for one micro joule of energy to enter the listener's ear?

Solution

$A = 40cm^2 = 40 \times 10^{-4} m^2 = 4 \times 10^{-3} m^2$; $I = 10^{-6}\ w/m^2$; $E = 1\mu J = 10^{-6} J$

(a) $P = I \times A = 10^{-6}\ w/m^2 \times 4 \times 10^{-3}\ m^2 = 4 \times 10^{-9}\ W$

(b) $P = \frac{W}{t} = \frac{E}{t}$

$$t = \frac{E}{P} = \frac{10^{-6} J}{4 \times 10^{-9} W} = 0.25 \times 10^3 s = 250s$$

Examples

A sound wave with an intensity of $10^{-8}\ w/cm^2$ is incident on an eardrum of area $4 \times 10^{-4}\ m^2$. How much energy is absorbed by the ear drum in 5 min?

Solution

$I = 10^{-8} w/cm^2 = 10^{-8}\ W/(10^{-2}\ m)^2 = 10^{-4} W/m^2$, $A = 4 \times 10^{-4} m^2$, $t = 5min = 300s$

$$I = \frac{E/t}{A}$$

$E = IAt = 10^{-4}\ w/m^2 \times 4 \times 10^{-4} m^2 \times 300sec = 1.2 \times 10^{-5} J$

Examples

The sound from an organ pipe has an intensity of 100 W/m² at a point 1m from the open end of the pipe. What is the intensity of the sound at a distance of 20m?

Solution

$$I_1 r_1^2 = I_2 r_2^2$$

$$I_2 = \frac{I_1 r_1^2}{r_2^2} = \frac{100 w/m^2 \times (1m)^2}{(20m)^2} = 0.25\ w/m^2$$

Intensity Level

The loudness of a sound depends upon the listener. It is quite subjective. So we can't measure it. But it is easy to accept that as the intensity of a sound increases the sound becomes more louder. The relationship between intensity and loudness is logarithmic. If we take the intensity of the least audible sound as a reference level, (the threshold of hearing (I_0), the intensity level of a sound, having intensity I, is given by

$$\text{Intensity level}(\beta) = 10 \log\left(\frac{I}{I_o}\right)$$

Where the reference intensity ($I_o = 10^{-12}\ W/m^2$) is the intensity of the threshold of hearing: $I_o = 10^{-12}\ W/m^2 = 10^{-16}\ W/cm^2$.

We express the loudness of the sound coming from various sources in decibels (dB). If a person is being exposed to the sound of 80dB continuously it may lead to hearing problems. A whisper is about 30 dB, normal conversation is about 60 dB, and a motorcycle engine running is about 95 dB. Loud noise above 120 dB can cause immediate harm to your ears.

Did you know?

The cry of a human baby, is about 115 decibels, it is louder than a car horn.

Examples

What is the intensity level of a sound of intensity 10^{-10} W/m^2?

Solution

$I = 10^{-10}$ w/m^2, $I_o = 10^{-12}$ w/m^2

$$\beta = 10\log\left(\frac{I}{I_o}\right) = 10\log_{10}\left(\frac{10^{-10}\,w/m^2}{10^{-12}\,w/m^2}\right) = 10\log10^{10^2}$$

$$= 10 \times 2 \log 10^{10} = 20dB$$

Examples

What is the intensity of a sound wave whose intensity level is 70 dB?

Solution

$\beta = 70dB$, $I_o = 10^{-12}$w/m^2

$$70 = 10 \log_{10}\left(\frac{I}{I_o}\right)$$

$$7 = \log_{10}\left(\frac{I}{I_o}\right)$$

$$10^7 = \frac{I}{I_o}$$

$$I = 10^7 \times I_o = 10^7 \times 10^{-12} W/m^2 = 10^{-5} W/m^2$$

The ear and hearing

The loudness of a sound depends on intensity and frequency. For a given frequency an increase in intensity produces an increase in loudness, but the sensitivity of the ear is so different in the various frequency ranges. Figure 5. shows a diagram giving the relation between frequency, intensity and hearing. The range of frequencies and intensities to which the ear is sensitive is conveniently represented by a hearing curve shown in Figure 31.

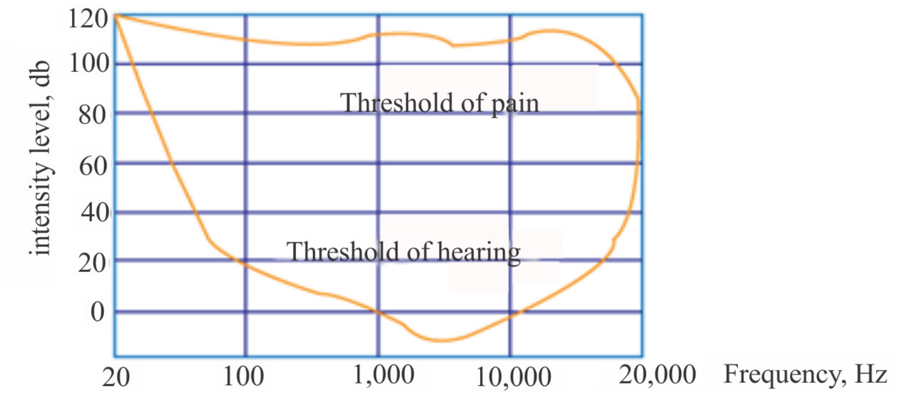


Figure 31. The Hearing curve

From Figure 31 we see that the ear is most sensitive to frequencies between $2000H_z$ and $5000H_z$. Intensities below the line indicating the threshold of hearing are insufficient to produce any sensation of hearing. The curve indicates that the normal ear is sensitive in the frequency range between 20Hz and 20,000Hz. The lower curve called the **threshold of hearing** indicates the minimum intensity level at which sound waves of different frequencies can be heard. The upper limit called the **threshold of pain indicates** the upper intensity level for audible sounds.

The upper curve represents the intensity level of the loudest sound that can be tolerated. This upper limit of audibility is called the threshold of pain. It is about 130dB. The lower limit of audibility corresponds to 5dB. This valve varies from person to person. It depends on age, the shape and health of the ear.

Exercises

1. Define sound.
2. How are sound waves propagated?
3. Why is sound not transmitted through vacuum?
4. Distinguish between
   (a) Intensity and loudness.
   (b) Frequency and pitch.
   (c) A noise and a musical note.
   (d) Threshold of pain and threshold of hearing.
5. What is the range of audio frequencies?
6. What name is given to sound vibrations

(a) Below the audio range?
(b) Above the audio range?

7. What is the speed of sound at 25°C?
8. How long does it take sound to travel 3.5 km in air at a temperature of 30°C?
9. What is the wavelength of sound in air at 30°C if its period is 0.4s?
10. Suppose a man stands at a distance from a cliff and claps his hands. He receives an echo from the cliff after 4 second. Calculate the distance between the man and the cliff. Assume the speed of sound to be 343 m/s.
11. The human ear can just detect sound waves of intensity 10^{-2}W/m^2. Calculate the sound energy incident each second on eardrum of area 20mm^2 at this intensity.
12. What is the wavelength of a 10KH_z sound wave in a bar of iron? (Y= 1.9 $\times 10^{11}$Pa, ρ =7.8 g/cm^3).
13. What is the difference in their intensity level of two sound waves with intensities: 10^{12}W/ cm^2 and 10^8W / cm^2?
14. By how much will the intensity of sound decreases if the distance from a point source is tripled?
15. A loudspeaker radiates sound uniformly in all directions. If the intensity of the sound at a distance of 40m from the source is 4×10^{-6} w/m^2, what will be the intensity at a distance of 80m?
16. A beetle vibrates its wings to give out a sound wave of intensity 4×10^{-3} w/m^2, when you are 5m away from it. (a) How much power is transmitted by this sound wave? (b) What is its intensity at 2m?

The Doppler Effect

Key terms

- Doppler's Effect: The variation in the pitch of a sound due to the relative motion between a source of sound and a listener.

In 1842 Austrian scientist Christian **Doppler,** discovered that when a source of sound and a listener are in motion relative to each other, the frequency of the sound heard by the listener is not the same as the source frequency. This effect works not only for sound but also for light and radio waves. This is the Doppler Effect, which has important applications in medicine and technology.

So far, we have only considered stationary sources of sound and stationary listeners (or observers). In this case, the frequency of sound reaching a listener's ear will be the same as the frequency of the source when the source of sound, the medium and the listener (observer) are all at rest. If however, the source and the observer are in relative motion, the frequency of the wave noted by the observer appears to be different from the true frequency.

Suppose you are standing on a sidewalk and an ambulance is approaching you. As the sound waves move towards you, they compress which increases the frequency resulting in a higher pitch. But, as the ambulance is moving away from you, the sound waves spread further apart so the frequency lowers resulting in a lower pitch (Figure 32). The sound the ambulance is producing is not changing, but the frequency of the sound perceived by our ear changes. This phenomena of variation of the frequency of the sound heard by the relative motion of the source or the listener is known as **Doppler effect**. It is illustrated in the Figure 32.

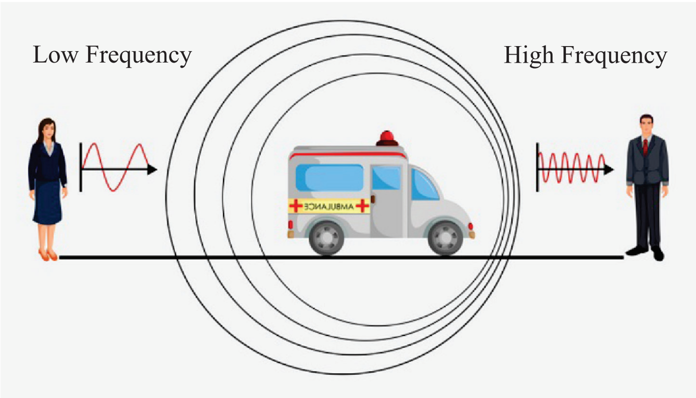


Figure 32. Doppler Effect

Different Cases of Doppler Effect

The apparent frequency due to Doppler Effect for different cases can be deduced as follows.

Case I: When the source and observer are relatively at rest with respect to each other, then the frequency heard by the observer is equal to the actual frequency produced by the source.

$f_L = f_S$

Case II: When a listener moving towards the stationary source.

$$f_L = f_s\left(1 + \frac{V_L}{V}\right)$$

Where V_L is the speed of the listener, and V is the speed of sound.

Case III: A listener moving away from the stationary source.

$$f_L = f_s\left(1 - \frac{V_L}{V}\right)$$

Case IV: When the source of sound is moving toward a stationary listener, the frequency of the sound produced by the source (f_s) and the frequency of the sound received by the listener (f_L) are related by:

$$f_L = f_s\left(\frac{V}{V - V_s}\right) = f_s\left(\frac{1}{1 - \frac{V_s}{V}}\right)$$

Where V is the speed of sound, and V_s is the speed of the source.

Case V: When a source of sound is moving away from the stationary listener, they are related by

$$f_L = f_s\left(\frac{V}{V + V_s}\right) = f_s\left(\frac{1}{1 + \frac{V_s}{V}}\right)$$

Case VI: When a source and a listener moving towards each other.

$$f_L = f_s\left(\frac{V + V_o}{V - V_s}\right)$$

Case VII: A source and a listener moving away from each other.

$$f_L = f_s\left(\frac{V - V_o}{V + V_s}\right)$$

Applications of Doppler Effect

Here we will take a look at some common applications of the Doppler Effect in real life.

1. **To measure the speed of an automobile**

Here is how a police officer uses a radar to know the speed of a vehicle.

- A police officer takes position on the side of the road.
- The officer aims his radar gun at a approaching vehicle.
- The gun sends out a burst of radio waves at a particular frequency.
- The radio waves strike the vehicle and bounces back towards the radar gun.
- The radar gun measures the frequency of the returning waves. Because the car is moving towards the gun, the frequency of the returned waves will be higher than the frequency of the waves initially transmitted by the gun. The faster the car's speed, the higher the frequency of the returning wave.
- The difference between the emitted frequency and the reflected frequency is used to determine the speed of the vehicle. The computer inside the gun performs the calculation instantly and display a speed to the officer.

2. **Radar**

Radar is a device, which transmits and receives radio waves. Radar sends high frequency radio waves towards an airplane. The reflected waves are detected by the receiver of the radar station. The difference in frequency is used to determine the elevation and speed of an airplane.

3. **Sonar (sound navigation and ranging)**

Sound waves generated from a ship fitted with SONAR are transmitted in water towards an approaching submarine. The frequency of the reflected waves is measured and hence the speed of the submarine is calculated.

4. **Doppler echocardiogram**

Doppler echocardiography is a procedure that uses Doppler ultrasonography to examine the heart. An echocardiogram uses high frequency sound waves to create an image of the heart while the use of Doppler technology allows determination of the speed and direction of blood flow by utilizing the Doppler effect.

Examples

(a) A train moving toward a detector at 30m/s blows a 305Hz horn. What frequency is detected by the detector? (b) repeat question (a) if the train were moving away from the detector? Take velocity of sound as 340 m/s.

Solution

V_s= 30m/s, f_s =305H_z V=340m/s

$$f_L = f_S\left(\frac{1}{1-Vs/v}\right) = 350H_z\left(\frac{1}{1-30/340}\right) = 334.5H_z$$

$$f_L = f_S\left(\frac{1}{1+Vs/V}\right) = 350H_z\left(\frac{1}{1+30/340}\right) = 280.3H_z$$

Demonstration: Demonstrate Doppler Effect Using a tuning fork

In this demonstration, Doppler effect is demonstrated in the classroom using a tuning fork.

Method I

- Attach a string to the stem of a tuning fork.
- Hit the tuning fork with the rubber hammer and rotate it on a horizontal plane above your head.
- The class should be able to hear the changes in pitch.

Method II

- Let one student hold a tuning fork and hit it with the rubber hammer.
- At the same time another student approach and then recede from the fork.
- Observe the difference in the pitch your ears will intercept as you approach and recede from the fork.
- What do you observe, if you move rapidly towards and away from the tuning fork?

Exercises

1. State Doppler's Effect.
2. Write at least two applications of Doppler Effect in real life.
3. A boy in a train moving at 25m/s hears a sound wave of frequency 300Hz from a stationary source. Find the frequency of sound heard by the listener if he is moving (a) towards the source, and (b) away from the source. (Take speed of sound at that temperature to be 334m/s).
4. A car moving at a speed of 10m/s produces a sound wave of frequency 400Hz. Find the observed frequency of a listener standing at the side of the road, if: (a) the car is moving towards the listener. (b) the car is moving away from the listener. (Speed of sound in air at that temperature is 330m/s.)

Vibrations in strings and tubes (pipes)

Key terms

- The property of light travelling in a straight line is called as the rectilinear propagation of light.

Waves on a string and pipes play an important role in music. String instruments produce sound by vibration of the string and wind instruments produce sound by vibration of sound waves in the pipe. In this section we will discuss, how the different modes of vibrations are produced in these instruments. Almost all the musical instruments that produce music can be grouped in three major classes.

1. **String instruments: -** produce musical sounds by vibrating strings. Examples: piano, guitar, violin, flute, cello.

Figure 33. String musical instruments

2. **Wind instruments: -** Produce musical sounds when wind is blown into or through a tube.

Examples: clarinet, flute, trumpets and saxophones.

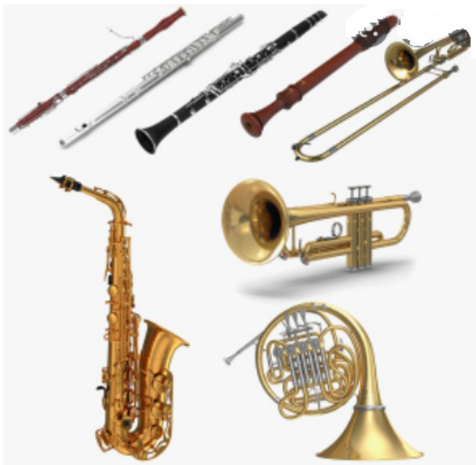


Figure 34. Wind musical instruments

3. **Percussion instruments:-** Produce musical sounds when struck by something.

Figure 35. Percussion musical instruments

Standing Wave on Strings

Let's now consider a string of a definite length L, clamped at *both* ends. Such strings are found in many musical instruments, including pianos, violins, and guitars. When a guitar string is plucked, a wave will be sent in both sides, to the left and to the right. These waves are reflected and re-reflected from the ends of the string, making a standing wave. This standing wave on the string in turn produces a sound wave in the air, with a frequency determined by the properties of the string.

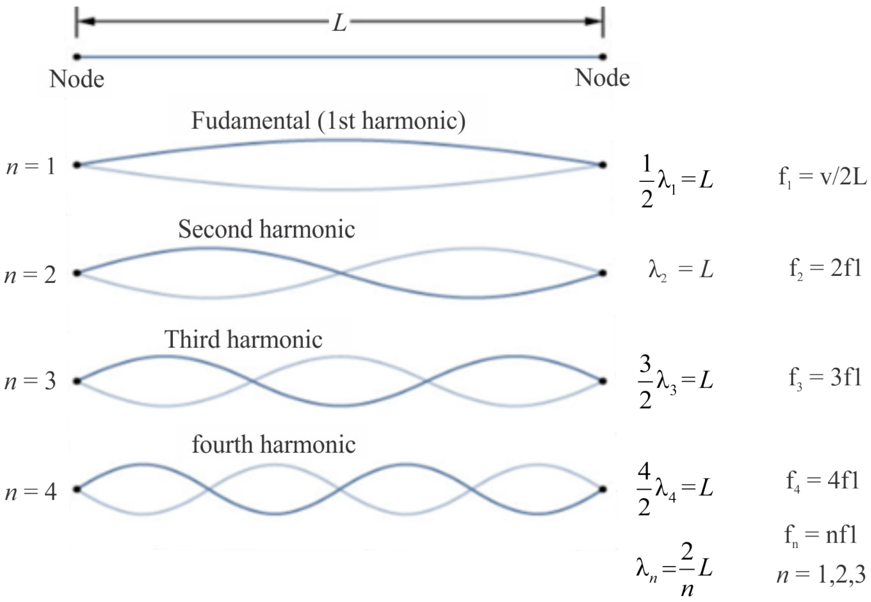


Figure 36. The standing wave pattern of a string

The standing waves on strings cannot be made at any frequency. The special frequencies at which the standing waves are made are called the **resonance**

frequencies. At these frequencies nodes and antinodes are formed in the string, and the wave seems to be standing still.

As the frequency increases different pattern of mode of vibration where the standing waves are formed will appear. These resonance frequencies are named as **the fundamental frequency** of the string or **overtones** of the fundamental frequency. The fundamental frequency is the lowest frequency which have two nodes and an antinodes between them, see Figure 36 (a). All other standing wave patterns that are set up are multiples of this fundamental frequency. These frequencies are known as **harmonics** of the fundamental. The fundamental is the first harmonic. Each of higher harmonics are named by the integer by which the frequency of the fundamental must be multiplied to give its frequency.

Thus the modes of vibration with the frequency 2f, is the 2nd harmonic or first overtone, that with a frequency 3f, is the third harmonic or second overtone, and so on. In all modes of vibration there must be a node at each end of the string.

In general

1. $L = n\,\dfrac{\lambda_n}{2}$

2. $\lambda_n = \dfrac{2L}{n}$

3. $f_n = n\left(\dfrac{v}{2L}\right) = nf_1$

Where n is the number of harmonic or number of segments.

Note that:

- The distance between adjacent antinodes is equal to ½λ.
- The distance between adjacent nodes is equal to ½λ.
- The distance between a node and an adjacent antinode is λ/4.

Examples

A standing wave is formed in a stretched string that is 3m long. What is the wavelength of the first three harmonics?

Solution

$L = 3m, \lambda_n = \dfrac{2L}{n}$

$$\lambda_1 = \frac{2L}{1} = 2\,(3m) = 6m$$

$$\lambda_2 = \frac{2L}{2} = \frac{2\,(3m)}{2} = 3m$$

$$\lambda_3 = \frac{2L}{3} = \frac{2\,(3m)}{3} = 2m$$

Examples

A standing wave is formed in a 4m long string that transmits waves with a speed of 12m/s. What are the frequencies of the first three harmonics?

Solution

$L = 4m; \quad \upsilon = 12m/s$

$$f_1 = \frac{\upsilon}{2L} = \frac{12m/s}{2\,(4m)} = 1.5Hz$$

$f_2 = 2f_1 = 2\ (1.5Hz) = 3Hz$

$f_3 = 3f_1 = 3\ (1.5Hz) = 4.5Hz$

Examples

A string 100cm long is adjusted so that it vibrates in 8 segments when subjected to a frequency of 120Hz. What is the speed of the wave in the string?

Solution

$L = 100cm = 1m\ ,\ n = 8\ ;\ \ f = 120Hz$

$$f = n\left(\frac{v}{2L}\right)$$

$$v = \frac{2Lf}{n} = \frac{2\,(1m)\,(120Hz)}{8} = 30m/s$$

The speed of a wave in a Vibrating string

The wave speed is the speed at which the *disturbance* propagates through the medium. It is not the speed of the individual particles making up the medium. The speed, of transverse waves on a stretched string depends on the properties of the string that affect its elasticity and its inertial properties. For a mechanical wave travelling along a string the speed depends on the tension in the string and the mass per unit length. (μ=m/l) known as the linear mass density (μ) cof the string. Mathematically,

$$\upsilon=\sqrt{\frac{T}{\mu}}$$

Where μ = mass per unit length, and T is the tension in the sting. It is crucial to control the speed of sound in stringed instruments like the guitar, which is why they have tuning knobs at one end (to control T) and the strings are of different mass (to control μ).

The fundamental frequency of a vibrating string can then be determined by the equation

$$f_1=\frac{V}{2L}=\frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

In general for the n^{th} harmonic

$$f_n=nf_1=\frac{n}{2L}\sqrt{\frac{I}{\mu}}$$

Examples

If a string 1.25m long and a mass of 20g vibrates under a tension of 64ON, what is (a) the liner mass density of the string? (b) the speed of the wave and (c) the fundamental frequency produced by this string?

Solution

$$L = 1.25\text{m},\ m = 20\text{g} = 0.02\text{kg},\ T = 640\text{N},\ \mu = \frac{m}{\ell} = \frac{0.02\text{kg}}{1.25\text{m}} = 1.6\times10^{-2}\,\text{kg/m}$$

$$\upsilon = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{640\text{N}}{1.6\times10^{-2}\,\text{kg/m}}} = 200\text{m/s}$$

$$f_1 = \frac{\upsilon}{2L} = \frac{200\text{m/s}}{2(1.25\text{m})} = 80\text{Hz}$$

Examples

A Stretched string vibrates with a frequency of 50Hz. The string is 1.0m long, has of 10g and is stretched with a tension of 400N. What is the wavelength of the wave produced?

Solution

f = 50Hz, l=1.0m, m= 10g = 0.01kg,T = 400N

$$V = \sqrt{\frac{T}{m/\ell}} = \sqrt{\frac{T/\ell}{m}} = \sqrt{\frac{400N \times 1.0m}{0.01kg}} = 200m/s$$

$$\lambda = \frac{\upsilon}{f} = \frac{200m/s}{500Hz} = 4m$$

Standing Wave in Pipes

A longitudinal standing wave is set up in wind instruments which produce sound by a vibrating air column in a pipe. The pipe is called **closed** when it is open at one end and closed at the other, or **open** when it is open at both ends.

Standing Wave in Closed Pipe

If any source of sound is held over an open end of a closed pipe a longitudinal wave is generated which travel through the air column to the closed end of the pipe. On reaching there the wave is reflected back towards the open end. These two identical waves traveling in opposite directions give rise to a longitudinal standing (stationary) wave in the pipe. Since the closed end acts like a rigid barrier a node (N) is formed there, but at the open end an antinode (A) is formed. Note that because it is difficult to represent longitudinal stationary waves in diagram form, they are usually drawn as transverse waves instead.

The different modes of vibration of air in a closed pipe

The standing wave pattern (or mode of vibration) with the lowest frequency has a node at the closed end and an antinode at the open end, see Figure. 37. We can easily find that the length L of the pipe (the distance between a node and the next antinode) is one – fourth of the wavelength of the wave produced, $L = \frac{1}{4}$.

We then increase the length of the pipe to form the next mode of vibration. This is usually made when the player closes the holes on the pipes of some of the instruments, like the flute or Clarinet. When the length of the pipe is increased by one segment we will have two nodes and two antinodes. Thus length of the pipe will be, $L = 3\,(1/4)$. The next mode of vibration will have 3 – nodes and 3 – antinodes.

Thus, $L = 5\,(1/4)$ etc.

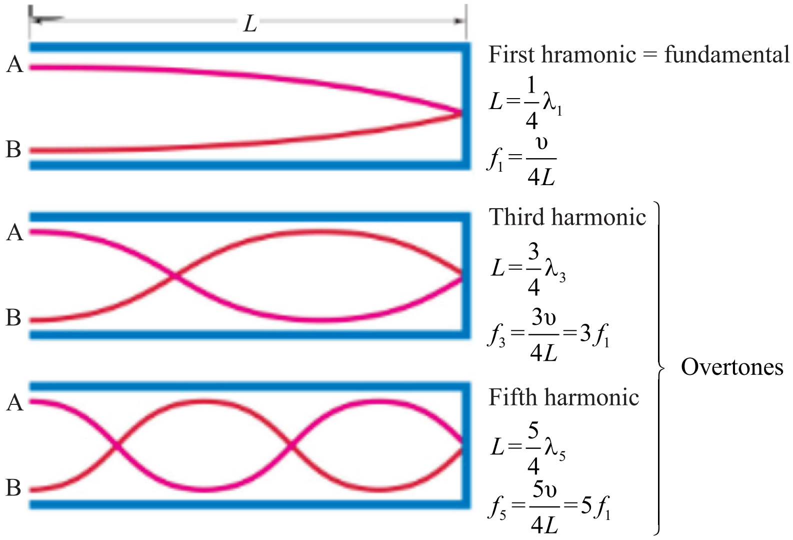


Figure 37. Standing in closed pipe

Now we come up with a general formula for the length of the pipe and the frequency of that mode of vibration.

1. $L = n\left(\frac{\lambda_n}{4}\right)$

2. $f_n = n\left(\frac{v}{4L}\right)$

Where n is the number of harmonic and n = 1, 3, 5, 7 ...

The different standing waves made in a closed pipe have specific names. The standing wave with only one node is called the **first harmonic** or **fundamental note**. All other standing waves made in a closed pipe are called **overtones**. We can also call the standing wave with two nodes having a frequency 3 – times the fundamental as the third harmonic, and the standing wave with three nodes having a frequency 5–times the fundamental the fifth harmonic etc.

$f_1 = \frac{v}{4L}$ – fundamental or first harmonic

$f_3 = 3f_1 = 3\left(\frac{v}{4L}\right)$ – third harmonic or first overtone

$f_5 = 5f_1 = 5\left(\frac{v}{4L}\right)$ – fifth harmonic or second overtone.

This leads to the fact that there are only **odd harmonics** in a closed pipe, no even harmonic.

Examples

A closed organ pipe has a length of 0.8m at room temperature (20°c). What are the (a) frequencies and (b) wavelengths of the first three harmonics?

Solution

L = 0.8m; T = 20°c

$$v = 331.5 + \frac{0.6\,(20°c)}{1°c} = 343.5m/s$$

$$f_1 = \frac{v}{4L} = \frac{343.5m/s}{4(0.8m)} = 107.3Hz,\ f_3 = 3f_1 = 322Hz,\ f_5 = 5f_1 = 536.7Hz$$

$$\lambda_n = \frac{4L}{n}$$

$$\lambda_1 = \frac{4\,(0.8m)}{1} = 3.2m,\ \lambda_3 = \frac{4\,(0.8m)}{3} = 1.07m,\ \lambda_5 = \frac{4\,(0.8m)}{5} = 0.64m$$

Examples

What is the length of a pipe, closed at one end, which gives a fundamental note of 250Hz? (The speed of sound is 340m/s.)

Solution

f = 250Hz; v = 340 m/s

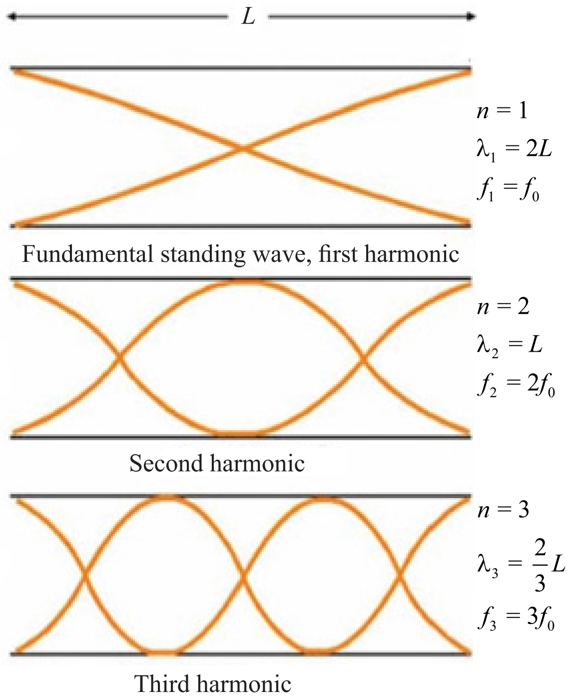
$$\lambda = \frac{v}{f}, \text{ since the pipe is closed } \left(L = \frac{\lambda}{4}\right)$$

$$4L = \frac{v}{f}$$

$$L = \frac{v}{4f} = \frac{340m/s}{4\,(250Hz)} = \frac{340}{1000} = m = 0.34m = 34cm$$

Standing Waves in Open Pipes

Standing waves can also occur in open pipes. In open pipes antinodes will always occur at the two open ends. Therefore, the fundamental note will have one node in the center of the pipe and two antinodes at the two ends, see Figure 38. Thus



n^{th} harmonic: $\lambda_n = \frac{2}{n} L \quad fn = nf_0$

Figure 38. standing wave in open pipes

Examples

What is the frequency of (a) the fundamental (b) the first overtone of an organ pipe open at both ends and of length 125cm? Assume the velocity of sound in air to be 350m/s.

Solution

L = 125cm = 1.25m, V = 350 m/s

(a) $f_1 = \frac{v}{2L} = \frac{350\text{m/s}}{2\,(1.25\text{m})} = 140\text{Hz}$

(b) The first overfore is the second harmonic (n = 2). Thus $f_2 = 2f_1$

$f_2 = 2\,(140\text{Hz}) = 280\text{Hz}$

Examples

What is the shortest length of a tube open at both ends that will resound to a fork of frequency 480Hz, if the velocity of sound in air is 340m/s?

Solution

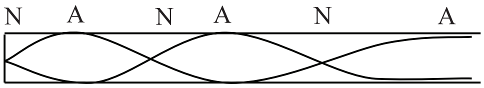
$f_1 = 480\text{Hz},\ v = 340\ \text{m/s}$

$$f_1 = \frac{v}{2L}$$

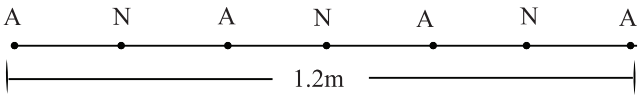
$$\Rightarrow L = \frac{v}{2f_1} = \frac{340\text{m/s}}{2\ (480\text{Hz})} = 0.354\text{m}$$

Exercises

1. Explain why it is not possible to produce a standing wave pattern for all frequencies in every stretched string.
2. Define the terms (a) fundamental frequency, and (b) harmonics
3. An organ pipe closed at one end has a length of 125cm. If the speed of sound is 350m/s, calculate the frequency of (a) the fundamental (b) the first overtone (c) the fifth harmonic.
4. The mode of vibration for a certain organ pipe is shown in the figure. If the length of the pipe is 6m, what is (a) the number of harmonic? (b) The wavelength of the wave?



5. A string fixed at both ends resonates in 3 segments at a frequency of 165Hz. At what frequency would the string resonate in 2 segments?
6. For the modes of vibration shown in the figure determine (a) the wavelength of the wave (b) the fundamental frequency if the speed of sound is 330 m/s.



Beats

When two sound waves having slightly different frequencies are sounded together, the combined sound grows and falls in loudness. The principle of superposition tells us that the waves reinforce one another when they arrive in phase and cancel out when they arrive out of phase. When reinforcement occurs a louder sound is heard and when cancellation occurs a soft sound or no sound at all is heard. This variation in loudness of a sound heard is called **beat**.

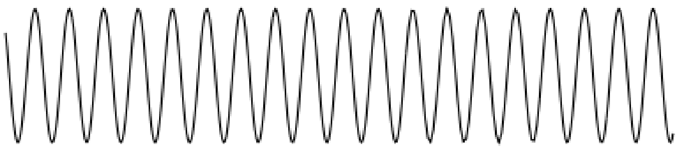
KEY TERMS

- Beats are the periodic and repeating fluctuations heard in the intensity of a sound when two sound waves of very similar frequencies interfere with one another.

The simplest illustration of beat is to draw two different waves and then add them together. You can do this by drawing them yourself to see the pattern that occurs.

Here is wave 1:

Now we add this to another wave, wave 2:



When the two waves are added the resulting wave is drawn below. Notice that the peaks are the same distance apart but the amplitude changes. If you look at the peaks, the peak amplitudes seem to oscillate with another wave pattern.

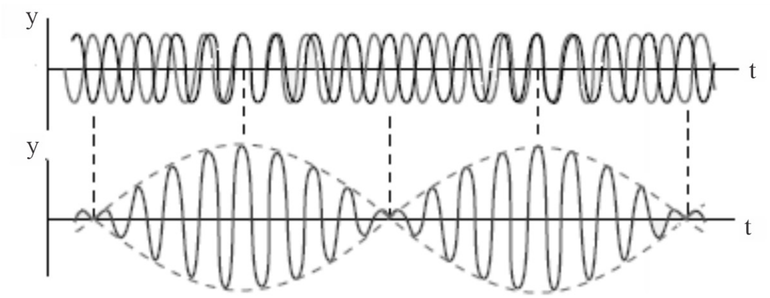


Figure 39. Beat

The beats have a frequency which is the difference between the frequency of the two waves that were added. If two sound waves have frequencies f_1 and f_2, then the number of beats per second, called the beat frequency (f_b) is given by:

$$f_b = |f_1 - f_2|$$

Examples

One tuning fork has as frequency of 445Hz. When a second fork is struck, beat notes occur with a frequency of 3Hz. What are the two possible frequencies of the second fork?

Solution

$f_1 = 445H_z$, $f_b = 3H_z$

First possibility

$$f_b = f_1 - f_2$$

$$\Rightarrow f_2 = f_1 - f_b = 445H_z - 3H_z = 442H_z$$

Second possibility

$$f_b = f_2 - f_1$$

$$\Rightarrow f_2 = f_1 + f_b = 445H_z + 3H_z = 448H_z$$

Exercises

1. If one tuning fork vibrates at 440 Hz and a second one vibrates at 445 Hz, find the beat frequency.
2. A tuning fork produces 8 beats per second with another fork of frequency 340 Hz. What are the two possible frequencies of the former fork?

Experiment Standing Waves in a Closed Tube

Theory:

A tuning fork is held by hand just above the open end of the tube. When the tuning fork is struck by a rubber hammer, it vibrates and longitudinal waves are sent down the air column. These waves are reflected at the water surface and thus produce standing waves. Here, the surface of water will act as the closed end. The sound waves reflected from the water surface change their phase by 180° and therefore are completely out of phase with the incident sound waves. In other words, the amplitude of the standing waves must be zero at the water's surface. Nodes are produced at the water surface and antinodes are produced at the open end.

The length of the water column may be changed by raising or lowering the water level while the tuning fork is held over the open end of the tube. When the frequency of waves in the air column becomes equal to the natural frequency of tuning fork, a loud sound is produced in the air column, indicating the formation of resonance.

Since the distance from a node to antinode is one fourth of a wavelength, the resonance conditions can only be satisfied when the column length (L) is:

$$L = n\frac{\lambda}{4},\ n = 1, 2, 3, 4, 5\ldots$$

where L is the length of the tube and λ is the wavelength.

Objectives:

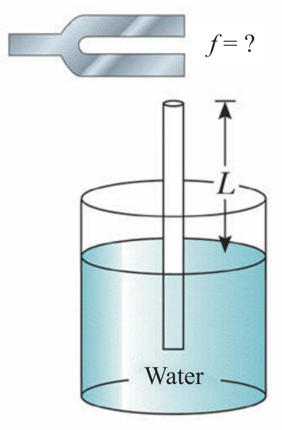
- To observe the resonance phenomenon in a closed cylindrical tube.
- Using tuning forks of known frequency, the velocity of sound in air at room temperature is determined.

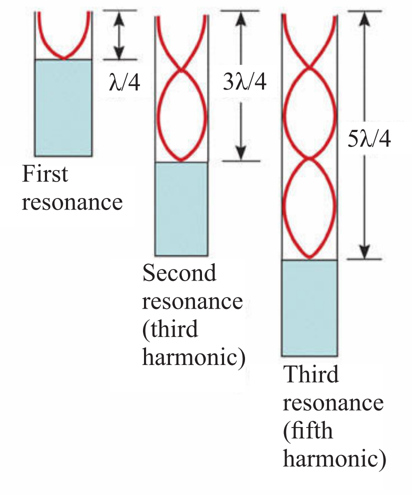
Materials:

- Tuning forks of different frequencies.
- A tube open at both ends (here a graduated cylinder with one end sawed off was used).
- A large jar partially filled with water.

Procedure:

- Fill the tube 2/3 full of water.
- Submerge the tube in the jar of water.
- Strike a tuning fork and hold it over the open end of the tube.
- Adjust the height of the tube until you hear the resonance frequency. When a resonance is found, a pronounced reinforcement of the sound will be heard. Move the water surface up and down several times to locate the point of maximum sound intensity and mark that point with a rubber band on the outside of the tube.
- Lower the water further to find the next resonant length. Continue in this manner as far as the length of the tube will permit. Obtain the lengths $\lambda/4$, $3\lambda/4$, etc. in meters from your measurements. You will need to check to see if your column lengths follow the progression 1, 3, 5, 7, -- since you may have missed a resonance or counted one of the fainter spurious resonances which sometimes occur. Calculate the wavelength and velocity of sound.
- Repeat the procedure for the other tuning forks supplied. Please record the room temperature for reference since the velocity of sound increases with increasing air temperature.





Data:

The first resonance occurs only when the length of air column is proportional to one-fourth of the wavelength of sound waves having frequency equal to frequency of tuning fork. i.e.; For first resonance,

$$\lambda = 4L$$

Thus, using the above equation we can find the wavelength of the sound from the length of the tube that forms resonance. Finally using the frequency of the tuning fork we can easily determine the speed of sound at that temperature.

$$v = f\lambda$$

Conclusion:

Compare your result with the theoretical value of the speed of sound at that temperature.

SUMMARY

- Waves are disturbances which originate from some vibrating source and propagate through a medium and vacuum.
- A medium is the substance through which a wave can propagate.
- A single non-repeating disturbance traveling in a medium is called a pulse.
- The maximum displacement of the wave from the equilibrium position is called the amplitude of the wave.
- The wavelength (λ) is the distance between any two adjacent points which are in phase.

- The period (T) is the time taken for a wave to make one complete vibration.
- The frequency of a wave, *f*, is a measure of how many complete oscillations occur, in one second.
- The speed of a wave refers to the distance travelled by a given point on the wave in a given interval of time ($v = \frac{\lambda}{T} = f\lambda$)
- Wave front is defined as the imaginary surface constructed by the locus of all points of a wave that have the same phase.
- The direction of travel of the wave fronts is shown by a straight line with an arrow called a ray.
- Reflection is turning back of a wave into the first medium when reaching to a boundary through which it can not pass
- The change in the direction of waves at the boundary between two different media is known as refraction.
- The change of direction of the wave fronts when they pass through a gap is called diffraction.
- The mixing up of two or more identical waves that travel in the same medium is called interference.
- The process of transforming unpolarized wave into polarized wave is known as polarization.
- Mechanical waves are waves that require a medium in order to transport their energy from one location to another.
- Electromagnetic waves are waves that do not require a material medium to transmit the disturbance.
- A transverse wave is a wave in which particles of the medium move in a direction perpendicular to the direction that the wave moves.
- A longitudinal wave is a wave in which particles of the medium move in a direction parallel to the direction that the wave moves.
- Standing waves are waves that do not appear to be propagating. They are also called *stationary waves*.
- The speed of sound in air depends on the temperature of the air, and is given by the equation:

$$\upsilon = 331.5\sqrt{1+\frac{T_c}{273.15°C}} \text{ or } \upsilon = (331.5 + 0.6T_c)\text{m/s}$$

- For solids: $\upsilon = \sqrt{\frac{Y}{\rho}}$ For fluids: $\upsilon = \sqrt{\frac{B}{\rho}}$
- The reflection of sound from an obstacle is called echo.
- Unpleasant sounds are called *noise* and sounds which are pleasant to hear are called *music*.
- The Loudness of a sound is the magnitude of the auditory sensation produced by the sound.
- The pitch of a sound is the subjective feeling experienced by a listener due to the frequency of the sound.
- Sound waves that are able to produce sensation of hearing are called audible, and those waves that cannot produce sensation are Inaudible.
- The amount of energy (E) transferred per unit time (t) per unit Area (A) perpendicular to the direction of motion is called the intensity (I) of the wave.

$$I = \frac{Energy\ /\ time}{Area} = \frac{Power}{Area}$$

- The intensity of a sound wave is inversely proportional to the square of the distance of the point of measurement from the sound source. $Ir^2 = constant$
- The intensity of a sound wave is directly proportional to the square of the amplitude of vibration of the source, $I \alpha A^2$.
- The intensity level of a sound, having intensity I, is given by:

$$Intensity\ level\ (\beta) = 10\log\left(\frac{I}{I_o}\right)$$

- The change in the frequency of the sound heard by the listener due to the relative motion between the source and the listener is known as Doppler Effect
- The interference of two identical waves having the same frequency, wavelength and amplitude, which are traveling in opposite directions in the same medium gives rise to standing wave pattern.
- The special frequencies at which the standing waves are made are called the resonance frequencies.
- For a standing wave produced by a vibrating string: $L = n\frac{\lambda_n}{2}$ and $f_n = n\left(\frac{v}{2L}\right) = nf_1, n = 1, 2, 3, \ldots$

- The speed of a wave in a Vibrating string: $\upsilon = \sqrt{\frac{T}{\mu}}$
- For a closed pipe: $L = n\left(\frac{\lambda_n}{4}\right)$ and $f_n = n\left(\frac{v}{4L}\right)$ $n = 1, 3, 5, \ldots$
- Beats are the periodic and repeating fluctuations heard in the intensity of a sound when two sound waves of very similar frequencies interfere with one another.
- The number of beats per second, called the beat frequency (f_b) is given by: $f_b = |f_1 - f_2|$

Review Exercises

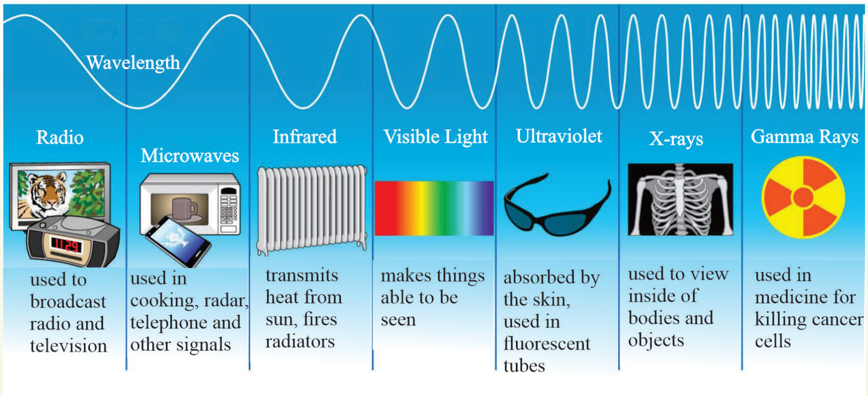
1. What is the difference between a pulse and a wave?
2. What is the product of the period and the frequency of a wave?
3. What type of waves will propagate in (a) solids? (b) Liquids? (c) Gases?
4. What is destroyed when destructive interference occurs?
5. Explain why the speed of sound is generally greater in solids than in liquids and gases.
6. What are the main properties of waves?
7. The distance between a compression and an adjacent rarefaction of a sound wave, traveling at 340 m/s, is 2.5 m. Find the frequency.
8. What is the wavelength of an electromagnetic wave in vacuum if its frequency is 100 M Hz?
9. The velocity of sound in air at 0^0C is 331.5 m/s. At what temperature will the velocity become 360m/s?
10. A person is in between two vertical mountains. The person fires a gun then hears the first echo from the nearby mountain in a time 3 s and hears second echo from far mountain in a time 4 s. What is the distance between the two mountains? Take the speed of sound as 340m/s..
11. A ship using an echo sounding device receives an echo from the bed 0.8 sec after the sound is transmitted. If the velocity of sound in sea water is 1500 m/s, what is the depth of the sea?
12. A point source radiates energy uniformly in a spherical pattern at a rate of 12.56W. What is the intensity of sound energy at a distance of 100m from the source?

13. The intensity of sound from a point source at a distance of 100m is $10^{-6}W/m^2$. What is the intensity from the same source at a distance of 50m?
14. What is the intensity level of the least audible and pain producing sound?
15. The frequency of the fundamental emitted by a closed pipe and an open pipe differ by 20. If the length of the open pipe is 170cm, what is the length of the closed pipe? (The speed of sound = 340m/s).
16. As a train passes you, you hear the frequency of its whistle drop from 1000Hz to 800Hz. What is the speed of the train? Take the speed of sound as 340m/s.

P11CH06

CHAPTER 6

LIGHT



Chapter Contents

Chapter Outcome

Learners will able to:

- recognize the source of light and its effects on mirrors in producing images.

Chapter Objectives

Upon completion of this chapter, learners will:

- determine the sources and importance of radiant energy;
- construct the linear propagation of a light indicating shadows and eclipses;
- analyze the major regions of the electromagnetic spectrum, the photoelectric effect and a principle of a laser;
- examine the effects of burning on the environment;
- analyze and demonstrate the image formation using a mirror.

Introduction

There would be no world if lights were not there, as light provides us with the ability to see things. Plants also need light which is provided by the Sun to prepare their food and to grow. We can say that The Sun is the main source of light for Earth. Light comes from different sources, which are known as light sources. We can say that light is a form of electromagnetic radiation whose wavelength ranges from 380 – 700 nm, which is visible to the human eye. In the modern world, humans are also creating artificial light, which is the most suitable form of electrical energy.

6.1 Nature and Sources of Light

What Is Light?

Light is a form of energy that enables us to see objects surrounding us, it causes the sensation of vision. We see object when light from a source or from a reflecting body enters our eyes. Light can be seen as a form of energy in the electromagnetic spectrum with a range in wavelength from 380 nm on the violet end to 700 nm on the red end of the visible spectrum. There is also light that is not visible to humans. The range just below 380 nm is described as ultraviolet [UV] while the range just above 700 nm is described as infrared [IR]. Even though these wavelength ranges are invisible to the human eye, they are also considered as light. Radio waves and x-rays are also considered as invisible light.

Speed of light

Key terms

- Light is a form of energy that enables us to see objects surrounding us, it causes the sensation of vision.

Light moves at the fastest known speed in the universe. The currently accepted value of the speed of light in empty space is c = 300,000,000m/s. The speed of light is different in different media. The speed of light in a medium is always slower than the speed of light in a vacuum.

The Nature of Light

At the end of the 17th century there were two competing theories concerning the nature of light. In 1690, Christian Huygens (1629–1695) proposed a theory that explained light as a wave phenomenon. Huygens's wave theory would eventually be accepted but not until the late 19th century.

Isaac Newton proposed an alternative theory on the nature of light. He suggested that light was made up of a stream of tiny particles that he called corpuscles (meaning small particles).

Newton's corpuscular theory remained the accepted theory for more than 100 years. In 1801, important experiments were done on the diffraction and interference of light by Thomas Young (1773–1829) and Augustin-Jean Fresnel (1788–1827) that could only be interpreted in terms of the wave theory. The polarization of light was still another phenomenon that could only be explained by the wave theory. Thus, in the 19th century, the wave theory became the dominant theory of the nature of light.

The present stand point of physicists is to accept that light is dualistic in nature, it behaves like both as a particle and as a wave. Today scientists say light is a form of energy made of photons.

Sources of Light

Object which are capable of emitting their own light are called a **source of light**. They are also known as **luminous objects**. Every source of light is a luminous object. Other objects which do not emit light but reflect the light from luminous objects are called **non-luminous objects**. One good example is the moon. The moon just reflects light from the sun.

There are two types of sources of light: Natural sources of light and artificial. Natural light sources produce light naturally without any human involvement, but artificial sources of light are constructed by humans.

Natural Light Sources

Natural sources of light include the sun, stars, fire, and electricity in storms. The Sun is the major source of light for the Earth. There are even some animals and plants that can create their own light, such as fireflies, jellyfish, and mushrooms. Living things that emit light without getting hot are referred to as **bioluminescent**.

Artificial Light Sources

Artificial sources are man – made light sources. Most of the lights that are man-made need an energy source, such as electricity or batteries, to produce light.

(a) Natural (b) Artificial

Figure 1. Sources of light

ACTIVITY 1

1. Name some objects that create light. These are called light sources.
2. Classify them as artificial light sources and natural light sources.

The effects of burning on the environment

KEY TERMS

- Objects which are capable of emitting their own light are called a source of light.
- Objects which do not emit light but reflect the light from luminous objects are called non-luminous objects.
- Natural light sources produce light naturally without any human involvement, but artificial sources of light are constructed by humans.

Burning is one of the means of getting light and heat. We burn wood to get fire, we burn fuel to get energy, we burn grasses for agricultural purposes and we burn wastes to clean our surrounding. These burning have a number of effects on our environment.

Burning is the source of air pollutants. Air pollutants are various chemicals diffused into the atmosphere from both natural and man made burning that change the composition of atmospheres and affect the biotic and abiotic environment.

Burning produces various compounds toxic to the environment including nitrogen oxides, volatile organic compounds (VOCs), carbon monoxide, carbon dioxide and particle pollution.

When fossil fuels are burned, they release nitrogen oxides into the atmosphere, which contribute to the formation of smog and acid rain. Smoke from burning vegetation and organic materials contains toxic gases such as carbon monoxide, carbon dioxide, other greenhouse gases, nitrogen oxides, hydrocarbons, and particulate matter that is small enough to enter the lungs and affect the respiratory system.

Burning of trash, is especially toxic due to synthetic chemicals in coated papers, plastics, and other materials that people commonly throw away. Air pollution has been a major concern throughout the world. Air pollution has always been an issue when it comes to the health of the world. It is a key global risk with the World Health Organization (WHO) estimating 8 million people a year dying prematurely from breathing polluted air.

Exercises

1. Discuss the sources of light and their importance.
2. Explain the nature of light.
3. What is the main source of light for Earth?
4. Is the moon a source of light? Explain.
5. List natural sources of light.
6. List artificial sources of light.
7. What type of energy can you sense with your eyes?
8. Light flash from lightning reaches you before its sound. What does this tells you?

6.2 Properties of Light

Light is all around us. It not only lets us see in the dark, but the properties of light are important to many aspects of our lives. Reflections in rear-view mirrors of cars help to keep us safe. Refraction through lenses of eyeglasses or contact lens helps some people see better. Dispersion of light explains the formation of rainbow and interference of light explains the formation of different colors in a soap bubble.

Key terms

- Reflection is the phenomenon in which light traveling in one medium, incident on the surface of another returns to the first medium.

Reflection of light

Reflection is the phenomenon in which light traveling in one medium, incident on the surface of another returns to the first medium. It is governed by the laws of reflection which states that the angle of incidence is equal to the angle of reflection. Reflection of light from a plane and curved mirrors forms images of different type.

Refraction of light

Refraction is a phenomenon in which there is a change in the speed of light as it travels from one medium to another and there is a bending of the ray of light. This change of direction is caused by a change in speed. For example, when light travels from air into water, it slows down, causing it to continue to travel at a different angle or direction. Mirage, bent pencil in glass of water, rainbow, sunset are some examples of refraction of light.

Figure 2. Refraction of Light

If we dip one end of a pencil or some other object into water at an angle to the surface, the submerged part looks bent. Its image is displaced because the light coming from the underwater portion of the object changes direction at it leaves the water.

Refraction of light can be explained with the help of Figure 3. A ray of light traveling from air falls on the surface of a glass block.

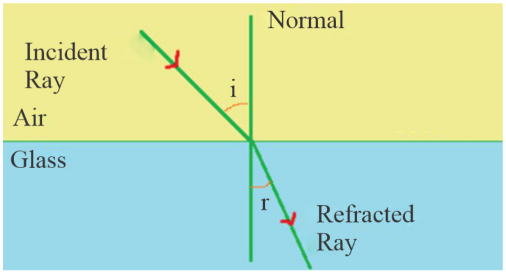


Figure 3. Refraction of Light

At the air glass interface, the ray of light changes direction and bends towards the normal and travels along a different path inside the glass block. The angle 'i' made by the incident ray with the normal is called **angle of incidence**. The angle 'r' made by the refractive ray with the normal is called **angle of refraction**.

Key terms

- Refraction is a phenomenon in which there is a change in the speed of light as it travels from one medium to another and there is a bending of the ray of light.

Types of refraction

Refraction towards and away from the normal

Refraction occurs at the boundary and is caused by a change in the speed of the light wave upon crossing the boundary. The tendency of a ray of light to bend one direction or another is dependent upon whether the light wave speeds up or slows down upon crossing the boundary.

The speed of a light wave is dependent upon the optical density of the material through which it moves. For this reason, the direction that the path of a light wave bends depends on whether the light wave is traveling from a more dense (slow) medium to a less dense (fast) medium or from a less dense medium to a more dense medium.

When light travels from a denser medium to a lighter medium, (for example, from glass to water), it speeds up. In this case, the light bends *away* from the normal line, Figure 4 (a).

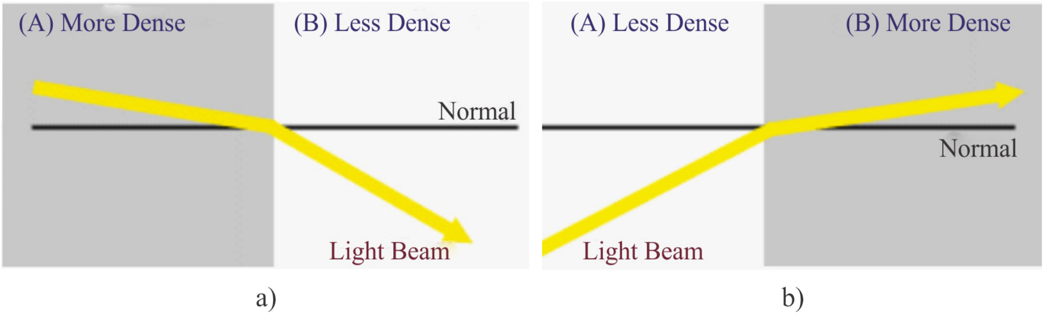


Figure 4. Refraction a) away from the normal b)towards the normal

When light travels from a lighter medium to a denser medium (for example, from air to glass), it slows down. The angle of incidence will be greater than the angle of refraction. Thus, the refracted ray bends towards the normal, see Figure 4 (b).

Note that, during refraction, the frequency of light remain constant, but the speed and wavelength of light change.

Applications of refraction of Light

Refraction makes it possible for us to have optical instruments such as magnifying glasses, lenses, prisms and rainbows. It is also because of the refraction of light that we are able to focus light on our retina. Another important application of refraction is optical fibers which are used for communication with minimum energy dissipation.

Index of refraction is used as a means to measure how much light refracts.

$$Index\ of\ refraction = \frac{Speed\ of\ light\ in\ vacuum}{Speed\ of\ light\ in\ Medium}$$

Dispersion of light

When white light is passed through a glass prism it will be refracted through the prism by different angles which depend on the wavelength of the light. When white light is passed through a glass prism it splits into its spectrum of colors (in order of increasing wavelength violet, indigo, blue, green, yellow, orange and red) and this process of white light splitting into its constituent colors is termed as **dispersion**.

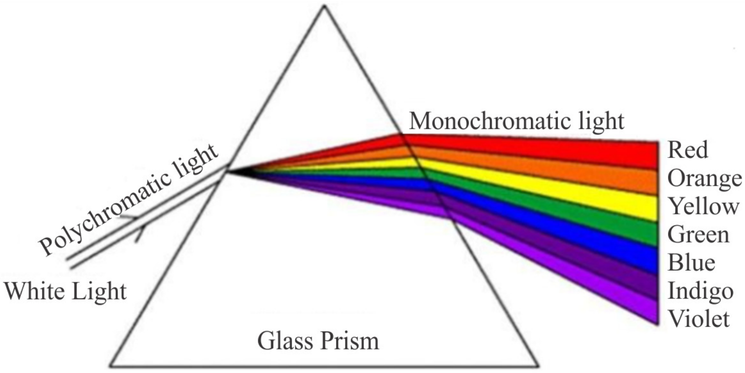


Figure 5. Dispersion of light by a glass prism

Examples of dispersion in daily life: Rainbow formation. Petrol poured on water will show different colors.

What is a White Light?

White light in the sky it is not really a white light it is a mixture of several colors. We can say that white light is the mixture of several colors having different wavelengths and frequency points on the same spot.

KEY TERMS

- The process of splitting a white light into its constituent colors is termed as dispersion.
- The bending of light when it passes around an edge or through a small opening is called diffraction.

Diffraction of light

Light bends when it passes around an edge or through a small opening. This bending is called **diffraction**. The amount of bending depends on the relative size of the wavelength of light to the size of the opening. If the opening is much larger than the light's wavelength, the bending will be almost unnoticeable. If the size of opening or obstacle is near to the wavelength of light, only then we can observe the phenomenon of diffraction.

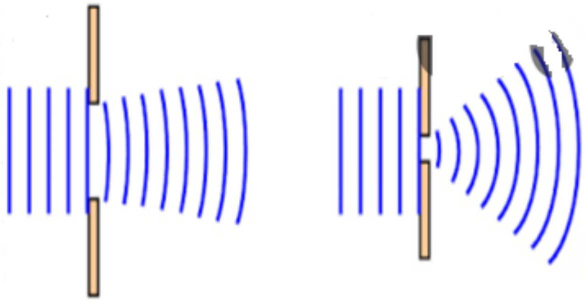


Figure 6. a) Diffraction through narrow gap b) Diffraction through wide gap

ACTIVITY 2

Demonstration of diffraction

- Hold your hand in front of a light source.
- Slowly close your two fingers while observing the light transmitted between them.
- As the fingers approach each other and come very close together, you begin to see a series of dark lines parallel to the fingers.
- The parallel lines are actually diffraction patterns.

Figure 7. Difraction by finger

Polarization of light

Light is the interaction of electric and magnetic fields traveling through space. The electric and magnetic vibrations of a light wave occur perpendicularly to each other. The electric field moves in one direction and magnetic field in another though always perpendicularly. So, we have one plane occupied by an electric field, and another plane perpendicular to it by a magnetic field, and the direction of travel which is perpendicular to both.

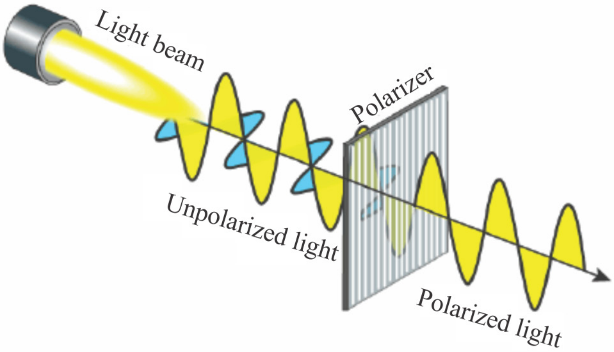


Figure 8. Polarization of light

A light wave that is vibrating in more than one plane is known as **unpolarized light**. Many common light sources such as sunlight, Light Emitting Diode (LED) spotlights, and incandescent bulbs produce unpolarized light.

Because light is a transversal wave it can be polarized. The process of transforming unpolarized light into the polarized light is known as **polarization**. When the electric field vectors are restricted to a single plane by filtration, then the light is said to be polarized with respect to the direction of propagation and all waves vibrate in the same plane. The most common source of polarized light is a laser. In a transverse wave, the direction of the oscillation is perpendicular to the direction of motion of the wave. Hence, polarization is a property applying to transverse waves.

Interference of light

Key terms

- The process of transforming unpolarized light into the polarized light is known as polarization.

When two or more light waves having the same frequency and wavelength and traveling in the same medium meet together at a point, they cancel or enhance the effect of each other at that point. This phenomenon is called interference of light waves. The new wave produced as the result of interference may have the lower, higher or same amplitude.

Types of Interference

Interference of light waves can be either constructive interference or destructive interference.

Constructive interference: When two light waves superpose with each other in such a way that the crest of one wave falls on the crest of the second wave, and the trough of one wave falls on the trough of the second wave, such type of interference is called constructive interference. These waves are in the same phase. Constructive interference results in an increase in amplitude and intensity, or brighter regions called **maxima**.

Destructive interference: When two light waves superpose with each other in such a way that the crest of one wave coincides with the trough of the second wave , destructive interference is said to take place. The phase of these waves are not the same. Destructive interference gives rise to a drop in amplitude and intensity, or dark patches called **minima**.

KEY TERMS

- When two in phase light waves superpose with each other constructive interference take place.
- When two out of phase light waves superpose with each other destructive interference is said to take place.

Conditions for interference

In order to observe sustained interference of light waves, the following conditions must be met.

- The two light sources must be coherent, that is , they must maintain a constant phase relationship and be the same type.
- The two light sources should be monochromatic (single wavelength).

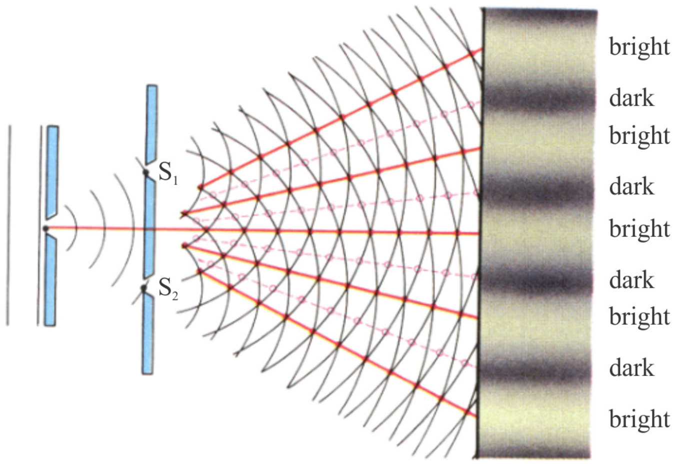


Figure 9. Interference of Light

Examples of Coherent Sources

- Laser light is an example of coherent source of light. The light emitted by the laser light has the same frequency and phase.
- Sound waves are another example of coherent sources. The electrical signals from the sound waves travel with the same frequency and phase.

Exercises

1. Define the 6 basic properties of light (reflection, refraction, diffraction, dispersion, polarization and interference)
2. What happens when light waves bounce off the surface of an object?
3. What happens when light energy bends as it passes from one type of substance to another type?
4. How does light travel?
5. Which objects refract light energy?
6. Which objects reflect light energy?
7. When you look in a mirror, which property of light enables you to see your image?
8. What causes a straw in a glass to appear broken and bent?
9. A light ray is traveling through air. When it enters water, it slows and bends. What kind of refraction is caused?
10. Light traveling from a lighter medium to a denser medium will bend ______ the normal.
11. Which is the property of light that does not change when it travels from one medium to another?
12. What do you call the property of light waves bending as it passes around the edge of the object?

6.3 Propagation of Light

Key terms

- The property of light traveling in a straight line is known as the rectilinear propagation of light.

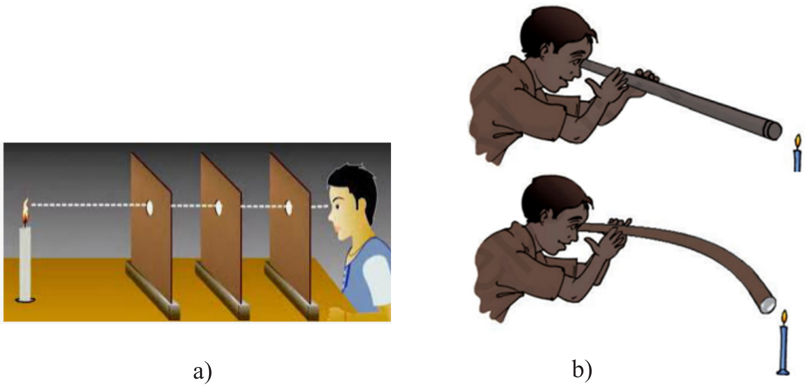
Propagation of light refers to the manner in which light energy moves from one point to another. When an electric lamp is switched on, the lamp is seen from all sides. Light is observed to be sent out from the lamp in all directions.

Rectilinear propagation of light

The property of light travelling in a straight line is known as the rectilinear propagation of light. A light source can be seen if there is a straight line path between the source and our eyes. This can be demonstrated by following experiment.

Experiment

1. Take three pieces of cardboard. Place them one next to the other and make a hole in the middle of each cardboard by using a thick nail. Erect these cards up on the table at a short distance away from each other. Take a candle which is of the same height as the holes in the cards. Light the candle and place it in front of the cards. We see that the light of the candle is visible only when the holes on cards lie in a straight line. If we disturb them the light of the candle disappears. This experiment shows that light propagates in a straight line.



a) b)

Figure 10. Light travels in a straight line

2. Bring a long rubber tube. Light a candle and fix it on a table at one end of the room. Now standing at the other end of the room look at the candle through the pipe. Is the candle visible? Bend the pipe a little while you are looking at the candle. Is the candle visible now? Turn the pipe a little to your right or left. Can you see the candle now? What do you conclude from this? This suggests that light travels along a straight line.

Rays and beams

A ray can be defined as an imaginary straight line drawn in the direction in which light travels. It is represented by a straight line with an arrow marked on it. Single light ray bears no practical significance as no real light source can be there that emits just a single ray of light. There is always a bunch of light rays travelling together.

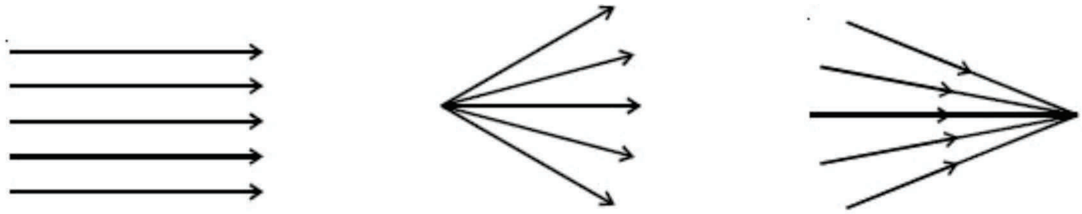
A collection of light rays travelling together is known as a light beam. A beam of light may be parallel, convergent or divergent.

Parallel beam of light: It is a bundle of light rays which are parallel to one another when coming from the source of light, Figure 11 (a). If the source of light is located at very large distance from the region of observation (like the sun), then in that case the beam obtained is effectively parallel.

> **KEY TERMS**
> - A ray can be defined as an imaginary straight line drawn in the direction in which light travels.
> - A collection of light rays travelling together is known as a light beam.

Divergent beam of light: In a divergent beam, the light rays disperse away from a source of light. The rays going out from a point source of light is the most relevant example of a divergent beam, Figure 11 (b).

Convergent beam of light: It is the beam of light in which the rays from a source of light converge to a point. A real light source does not emit a converging beam directly, Figure 11 (c).



(a) Parallel beam *(b) Divergent beam* *(c) Convergent beam*

Figure 11. Types of beams

Shadows

> **KEY TERMS**
> - The dark region behind an opaque object where light cannot reach is known as a shadow.

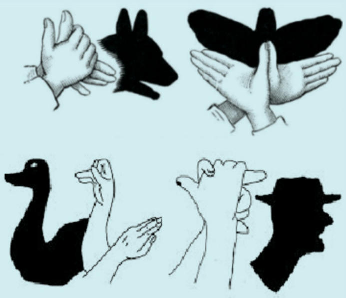
Light rays travel from a source in a straight line. If an opaque (solid) object gets in the way, it stops light rays from traveling through it. An opaque object is an object through which no light can pass. One such example is your body. Since light cannot pass through your body, it forms a dark region behind. This dark region where the light cannot reach is known as a **shadow**. Light travels in straight lights and so cannot bend around your body, which is why you see a shadow. The size and shape of a shadow depend on the position and size of the light source with respect to the object.

Opaque objects form clear dark shadows. A transparent object does not make any shadow as light passes straight through it. Translucent objects create faint shadows as light is able to pass partially through them.

ACTIVITY 3

1. Discuss in group why your shadow is longest in the early morning and in the late afternoon but there is little or no shadow at noon.
2. Shadow play

The figures below shows a few shadows that we can create with our hands and make-believe that they are shadows of different animals. Have fun!



Parts of a shadow

A shadow have three regions: the **umbra**, **penumbra** and **antumbra**.

Umbra

The **umbra** (Latin for «shadow») is the innermost and darkest part of a shadow, where the light source is completely blocked by the opaque body. It gets no light at all.

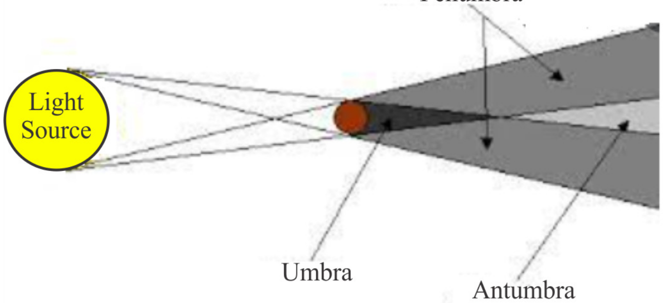


Figure 12. Umbra, Penumbra and Antumbra of a shadow

Penumbra

The **penumbra** (from the Latin *paene* «almost, nearly») is the region in which it gets light from some parts of the source of light. It is the region of partial shadow.

Antumbra

The **antumbra** (from Latin *ante*, "before") is the lighter area of a shadow that appears beyond the umbra, at a certain distance from the object casting the shadow. It only exists if the light source has a larger diameter than the object.

KEY TERMS

- The umbra is the innermost and darkest part of a shadow, where the light source is completely blocked by the opaque body. .
- The penumbra is the region in which it gets light from some parts of the source of light.
- The antumbra is the lighter area of a shadow that appears beyond the umbra, at a certain distance from the object casting the shadow.

Eclipses

An eclipse takes place when one heavenly body such as a moon or Earth moves into the shadow of another heavenly body. There are two types of eclipses on Earth: an eclipse of the moon (lunar eclipse) and an eclipse of the sun (solar eclipse).

Solar Eclipse (Eclipse of the Sun)

A solar eclipse happens when the Sun, Moon and Earth are perfectly aligned, and the moon comes between the earth and the sun during their motion. In this case the Moon blocks light from the Sun and casts a shadow on Earth. There are three main types of solar eclipses:

1. **Total solar eclipse:** A total solar eclipse takes place on the part of the Earth where the Umbra of the shadow of the moon is formed when it hits the Earth. It is visible from a small area on Earth. During total eclipse, the sky becomes very dark, as if it were night.
2. **Partial solar eclipse**: This happens on the part of the Earth where the penumbra of shadow of the Moon is formed. The Sun appears to have a dark shadow on a small part of its surface.
3. **Annular solar eclipse**: An annular eclipse is seen on the part of the Earth where an antumbra is formed. An annular eclipse happens when the Moon is farthest from Earth. Because the Moon is farther away, it seems smaller, it does not block the entire view of the Sun. The Moon in front of the Sun looks like a dark disk on top of a larger Sun-colored disk. This creates what looks like a ring around the Moon.

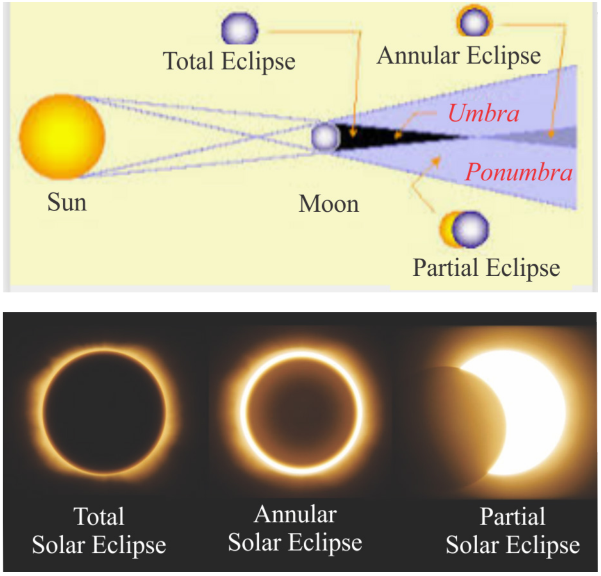


Figure 13. Total, Partial and Annular Solar Eclipse

Hybrid Eclipse: A relatively rare event, a hybrid eclipse is when one part of the Earth sees an annular eclipse, while another part of the Earth sees a total eclipse.

Solar eclipses happen every 18 months somewhere on Earth. Solar eclipses last only few minutes.

Warning!! Never look directly at the Sun. Doing so can permanently damage your eyes! You must use proper safety equipment to look at any type of solar eclipse.

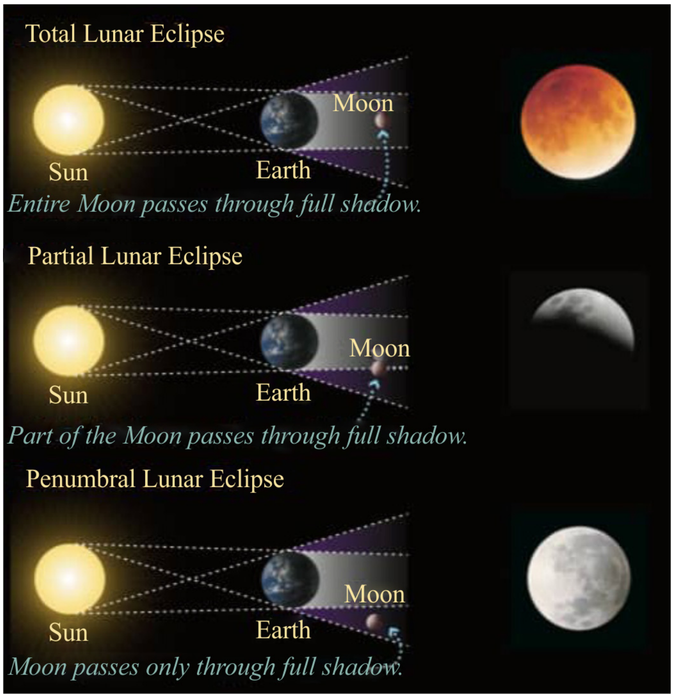
Lunar Eclipse (Eclipse of the Moon)

KEY TERMS

- A solar eclipse happens when the Sun, Moon and Earth are perfectly aligned, and the moon comes between the earth and the sun.
- The Lunar Eclipse occurs when the Earth, the Sun and the Moon come in a straight line, and the Earth come in-between the Sun and the Moon

The Lunar Eclipse occurs when the Earth, the Sun and the Moon come in a straight line, and the Earth comes in-between the Sun and the Moon. In this position, there won't be any sunlight falling onto the Moon. As Moon presents in the darkness, that portion of the moon will have an eclipse. A lunar eclipse occurs when the moon crosses the earth`s shadow, causing the moon to disappear from the sky. There are three kinds of Lunar Eclipses.

Total Lunar Eclipse: This occurs when the earth's umbra will be able to darken the whole of the moon's area so that the moon becomes completely invisible. Although the Moon is in Earth's shadow, some sunlight reaches the Moon. The sunlight passes through Earth's atmosphere, which filters out most of the blue light. This makes the Moon appear red to people on Earth.



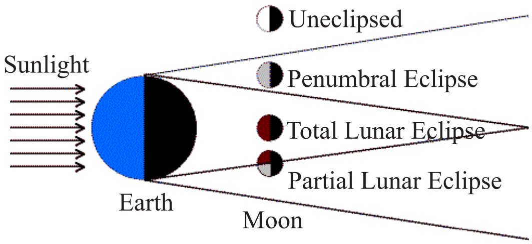


Figure 14. Total, Partial and Penumbral Lunar Eclipse

Partial Lunar Eclipse: A partial lunar eclipse happens when part of the Moon enters Earth's shadow. In a partial eclipse, Earth's shadow appears very dark on the side of the Moon facing Earth. What people see from Earth during a partial lunar eclipse depends on how the Sun, Earth and Moon align.

Penumbral Lunar Eclipse: This occurs when the moon travels through the faint penumbral portion of earth's shadow.

There are no annular lunar eclipses because Earth is much bigger than the Moon, and its shadow will never be small enough to leave a ring.

A lunar eclipse usually lasts for 2 to 3 hours. At least two partial lunar eclipses happen every year, but total lunar eclipses are rare. A lunar eclipse can occur only when the Moon is full. A lunar eclipse can be seen from Earth at night. It is safe to look at a lunar eclipse.

Demonstration: The Formation of Shadows and Eclipses

Aim of the demonstration: Using a torch light and ball demonstrate how shadows and eclipses are produced.

Materials: A flashlight to represent the Sun, a basket (or other ball of a similar size) to represent the Earth, and a table tennis ball (or a ball of similar size) to represent the moon. Note that this activity works best in a darkened room.

Procedure:

1. Place the "Earth" (basketball) on a table.
2. Place the flashlight on the table about 30 cm away from the Earth so that it shines on the Earth.

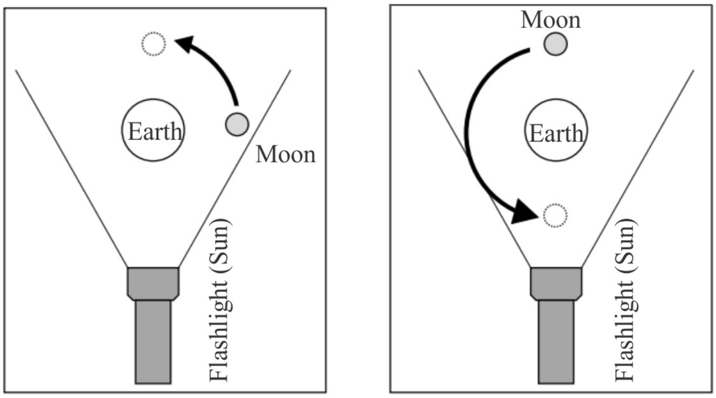


Figure 15. Demonstration of a) lunar eclipse b) solar eclipse

3. Place the moon (table tennis) on one side of the Earth, about 10 cm away, see Fig 15 (a).
4. Begin to move the moon around the Earth until it is on the opposite side of the Earth from the sun.
   - Watch as the Earth's shadow begins to fall across the moon.
   - This is a model of a lunar eclipse.
5. Now start moving the moon around the Earth again until it is between the sun and the Earth, see Figure 15 (b).

- Watch for a shadow on the Earth as the moon begins to move between the sun and Earth.
- This is a model of a solar eclipse.

6. Using your model, explain the difference between a lunar eclipse and a solar eclipse to your partner.
7. In both cases differentiate the region where Umbra and Penumbra is formed.

ACTIVITY 4

A student had a ball, a screen and a torch in working condition. He tried to form a shadow of the ball on the screen by placing them at different positions. Sometimes the shadow was not seen. Explain why.

A Pin-hole Camera

KEY TERM

- A pinhole camera is a simple camera without a lens but with a hole of pin size.

A **pinhole camera**, also called camera obscura, which is Latin for 'dark room' is a simple camera without a lens but with a hole of pin size. It is a simple device that consists of a small box that is black on the inside and has a tiny hole on one end. On the other end there is a screen for viewing. If you turn the side with the hole towards a distant tree and then look at the screen on the opposite end, you will notice an image.

The image may be fuzzy depending on how far away or near the object is. But the interesting thing about this image is that it is upside down (inverted). This inversion is a proof that light travels in a straight line.

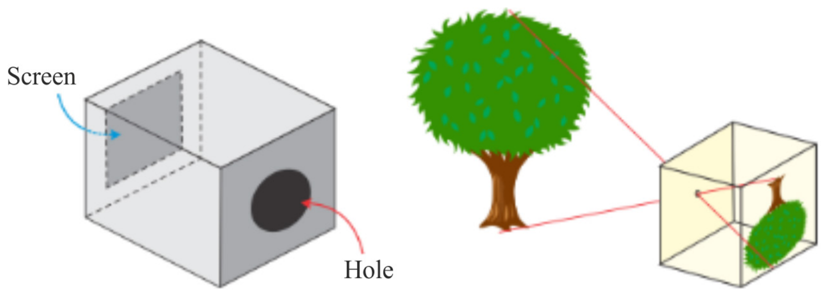


Figure 16. Pin hole cameras show that light travels in a straight line

As we can see in the diagram, the light rays from the top of the tree that passes through the pin hole hit the bottom of the screen. Similarly, the light rays from the bottom of the tree that pass through the pin hole hit the top of screen. If light did not travel in a straight line, this would not be a case and the image would not be inverted.

The characteristics of the image formed by a Pinhole Camera

- A real image is obtained as the image is obtained on the screen.
- The size of the image obtained is comparatively smaller than the actual object.
- The image is inverted.

ACTIVITY 5

You can make a simple pinhole camera with simple materials probably available at your own home. The stepwise procedure on how to make a pinhole camera has been described below.

1. Take a small box like a shoe-box.
2. Paint the box entirely in black to light-proof it.
3. Make your circular pinhole on the bottom of the box.
4. Make a shutter by cutting a piece of thick black chart paper (5 x 5 cm ideal)
5. Use a tape to hold the shutter in place.
6. Use a light adhesive to control shutter flap and light entering the box.
7. Make a viewfinder out of cardboard.

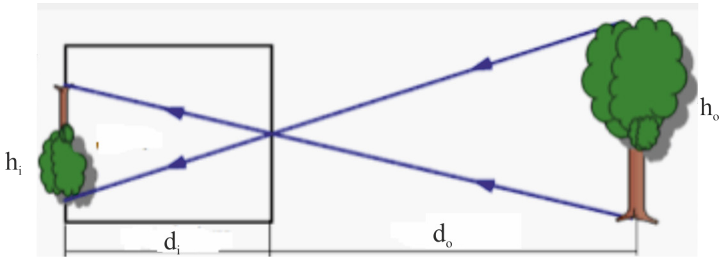
Calculations in a pin-hole camera

Using magnification formula:

$$\frac{\text{Height of image}}{\text{Height of object}} = \frac{\text{Distance of image}}{\text{Distance of object}}$$

Referring to the figure

$$\frac{h_i}{h_o} = \frac{d_i}{d_o}$$



Examples

In a pinhole camera, the distance between the pinhole and the screen is 25cm,

(a) what is the height of an object placed 100cm from the pinhole when the image in the camera is 10cm high?

(b) What is the magnification produced?

Solution

$d_i = 25cm, d_o = 100cm, h_i = 10cm$

From the equation for magnification:

$$\frac{d_i}{d_o} = \frac{h_i}{h_o}$$

$$\Rightarrow h_o = \frac{d_i}{d_o} h_i = \frac{100cm}{25cm} \times 10cm = 40cm$$

Did you know?

Invention of the pinhole camera: Scottish inventor Sir David Brewster is credited with describing the first pinhole camera in 1856.

Exercises

1. Distinguish between shadow and eclipse.
2. What does rectilinear propagation mean?
3. Why is shadow formed when an opaque object comes in path of light?
4. What is an eclipse?
5. What is a solar eclipse?
6. What are the three types of solar eclipse?
7. What is a lunar eclipse?
8. The length of a pinhole camera is 25.0 cm. An object 2.0 cm tall is placed 10.0 m from the pinhole. Calculate the height of the image.
9. What happens to image while we increase the size of hole of pinhole camera?
10. What are the effects of an image formed on the screen of a pinhole camera when distance between object and pinhole is increased?

6.4 Reflection of Light by Plane and Spherical Mirrors

Key terms

- Reflection of light is the phenomenon in which light traveling in one medium, falls on the surface of another medium and returns back to the first medium.
- Regular reflection is reflection from smooth and shiny surfaces.
- Diffuse reflection is reflection rough, unpolished surface.

In this section, we will be studying the types of reflection and the laws of reflection. We will also learn in detail about the types and uses of, and differences between concave and convex mirrors, and the characteristics of images formed by plane and spherical mirrors in different cases.

Reflection of Light

When light strikes the surface of a material, some of the light is reflected from the surface. The rest is either absorbed or transmitted through the material. Reflection of light is the phenomenon in which light traveling in one medium, falls on the surface of another medium and returns back to the first medium.

Types of Reflection

Depending on the nature of the reflecting surface, reflection could be either regular or diffused.

Regular reflection, also known as specular reflection, happens when light is reflected from smooth and shiny surface. Reflection from a mirror forms regular reflection.

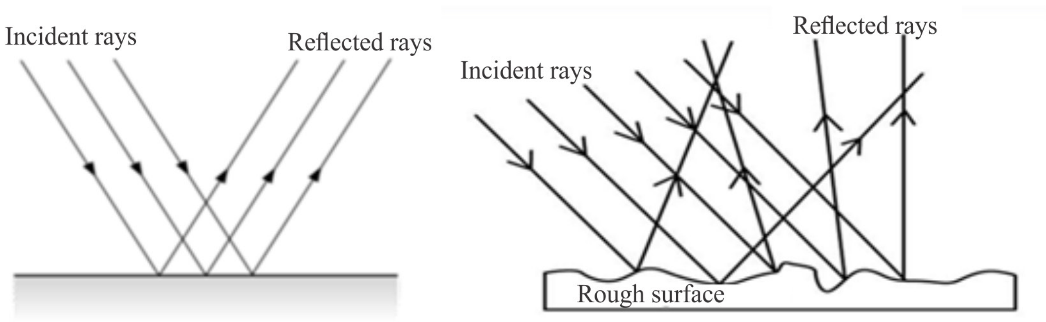


Figure 17. a) Regular reflection b) Diffused reflection

Diffuse reflection happens when light is reflected from a rough, unpolished surface. Reflection from a wall, wood, paper etc. forms diffused reflection. The surface may appear or feel smooth, like a piece of paper, but it is actually rough at the microscopic level. This causes the beams of light to reflect at different angles.

In regular reflection, all the rays are reflected in the same direction. This explains why regular reflection forms a clear image that can be seen. In diffused reflection, the rays are reflected in many different directions. This is why diffused reflection forms, a blurry image or no image.

Activity 6

Discuss in group about real life situation where we use reflection of light.

Laws of Reflection

To explain the law of reflection, let us define some of the terms we are going to use in this part:

Incident Ray is the ray striking the surface.

Reflected Ray is the ray reflected from the surface.

Point of Incidence is the point at which the incident ray strikes the surface.

Normal (N) is a line drawn perpendicular to the surface at a point where the incident ray strikes the surface.

Angle of Incidence (i) is the angle between the incident ray and the normal to the surface.

Angle of Reflection (r) is the angle between the reflected ray and the normal.

The law of reflection is stated as follows:

1. The incident ray, the reflected ray and the normal to the surface lie in the same plane.
2. The angle of incidence is equal to the angle of reflection, i.e., i = r.

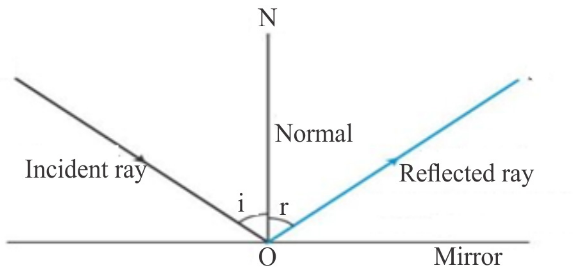
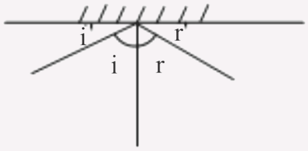


Figure 18. Reflection by a plane mirror

Examples

A ray of light strikes a reflecting surface at an angle of 20°. What is the (a) angle of incidence (b) the angle of reflection and (c) the angle between the incident ray and the reflected ray?

Solution



$i' = 20^\circ$, $\Rightarrow$ $r^1 = 20^\circ$

(a) $i' + i = 90^\circ$

$i = 90^\circ - i' = 90^\circ - 20^\circ = 70^\circ$

The angle of incidence (i) = 70°

(b) The angle of reflection = the angle of incidence = 70°

(c) The angle of incidence ray and the reflected ray is (i + r) = 140°

Images formed by Mirrors

A mirror is a reflective surface through which light does not pass through, but bounces off and produces an image. There are two major types of mirrors. One is the flat plane mirror, and the other is a curved or spherical mirror. A mirror with a flat surface is called a **plane mirror**, and a mirror with a curved surface is called a **spherical mirror**.

Image is a visual representation of an object which is placed somewhere in front of or behind a mirror or lens. When an object is placed in front of a mirror, its image is seen in the mirror. The object is the source of the incident rays and the image is formed by the reflected rays. Based on the intersection of light rays, the images are classified as either (a) a real image or a virtual image, (b) erect image or inverted image, (c) magnified, diminished or the same size as the object image.

Real vs. Virtual images

In real image, the rays of light actually meet after reflection while in a virtual image it appears to meet but not actually meet. A real image can be obtained on screen but a virtual image cannot. Real images are formed inverted but virtual images are erect.

ACTIVITY 7

(a) Standing in front of a plane mirror, raise your left hand. Which hand of your image is raised?

(b) Lower the left hand and raise the right hand. What change occurs to the image?

When our right side appear left and our left side appear right, such a shift of the lateral side of the images in the opposite direction is called lateral inversion.

Activity 8

Place a mirror perpendicularly on a table. Hold different objects like pen, pencil etc. in front of the mirror. Observe their images. Keeping a ruler in front of the mirror, place these objects at different positions and observe.

(a) Is the size of the object and the image the same?

(b) Does the distance of the image change when the distance of the object from the mirror changes?

The characteristics of the image formed by a plane mirror

The following are the characteristics of the image formed by a plane mirror.

1. The image is virtual and erect.
2. The image is of the same size as that of the object ($h_o = h_i$).

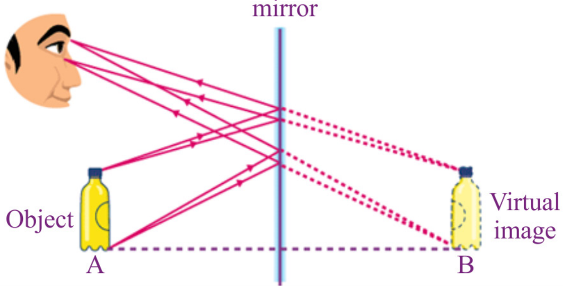


Figure 19. Images Formed By Plane Mirrors

3. The image is laterally inverted. Right side appear to be left and left side appeared to be right.
4. The image is as far behind the mirror as the object is in front of it ($s_o = s_i$).

Examples

If you stand 10 cm in front of a plane mirror, what is the distance between you and your image?

Solution

Since image distance = object distance = 10 cm, the distance between you and your image = 10 cm + 10 cm = 20 cm.

Did you know?

The earliest known manufactured mirrors were polished stone pieces.

Spherical Mirrors

A spherical mirror is a mirror with a curved surface that are painted on one of the sides. There are two types of spherical mirrors: concave, and convex.

Concave Mirror

A spherical mirror of which the reflecting surface is curved inwards is known to be a **concave mirror**. The back of the mirrors is always shaded so that reflection can take place only from the inward bulged surface. Concave mirrors are also called converging mirrors, because the ray of light after bouncing back from it appears to converge to a focus. The surface of the steel spoon which is curved inwards can acts like a concave mirror.

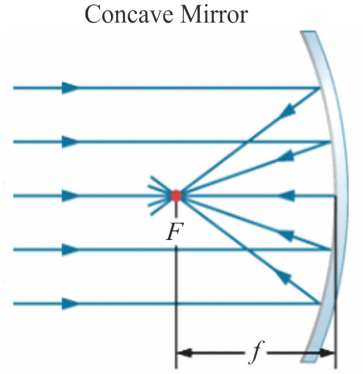


Figure 20. Concave mirror

Uses of Concave Mirror

- Concave mirrors are most widely used in shaving because they have reflective and curved surfaces.
- Concave mirrors are also widely used in making astronomical telescopes.
- Concave mirrors are widely used in headlights of automobiles and in motor vehicles, torchlights, railway engines, etc. as reflectors.
- The dentists use concave mirrors to see large images of the teeth of patients.
- Large concave mirrors are used to concentrate sunlight to produce heat in solar furnaces.
- Concave mirrors are also used as satellite dishes. The main purpose of this mirror is to gather weaker signals coming from large areas and concentrate them at one point.

Figure 21. Uses of Concave mirrors

Convex Mirror

A spherical mirror having its reflecting surface curved outwards is known to be a **convex mirror**. The inside of the mirror is shaded so that reflection only takes place from the outward bulged part. It is also known as a diverging mirror as the light after reflecting through its surface diverges to many directions. The outer surface of a steel spoon acts like a convex mirror.

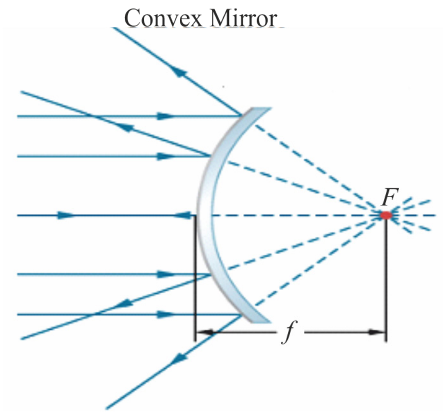


Figure 22. Convex mirror

Uses of Convex Mirror

Figure 23. Rear View mirror

- Convex mirrors are commonly used as rear-view (wing) mirrors in vehicles, enabling the driver to see traffic behind him/her to facilitate safe driving.
- Convex mirrors used inside buildings so that people can see all around the building at once.
- Convex mirrors are mostly used for constructing magnifying glasses.
- Convex mirrors are also used for security purposes in many places. They are placed near Automatic Teller machines (ATMs) to let the bank customers check whether someone is behind them or not.

Activity 9

Metal spoon as concave and convex mirror

Pick up a well-polished metal spoon and you can see an example of each type of curvature. The side of the spoon that holds the food is a concave mirror; the back of the spoon is a convex mirror.

Figure 24. Metallic spoon as concave and convex mirror

The following are some of the most commonly used terms when talking about spherical mirrors:

Principal axis: A spherical mirror is always part of a bigger virtual sphere. A line passing the center of this sphere which touches the reflecting surface at its center is called the **principal axis**.

Vertex (Pole): The point where the principal axis meets the reflective surface is the vertex or pole which is the center of the mirror.

Center of Curvature: The center of the sphere, from which the spherical mirror has been cut is the center of curvature.

Radius of Curvature: The distance from the vertex to the center of curvature is the radius of curvature (R).

Principal Focus (Focal Point): The point which lies midway between the vertex and the center of curvature is the focal point, marked as 'F' in the Figure 25.

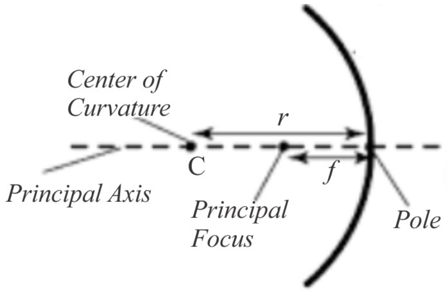


Figure 25. Terminologies used in spherical mirrors

Focal length: The distance between the vertex and the focal point of the mirror is called the focal length of the mirror (f).

Although the Figure 25 above is that of a concave mirror, the properties remain the same for convex mirrors as well. The difference between them is based on the way they reflect light rays.

Image Formation by Spherical Mirrors

KEY TERMS

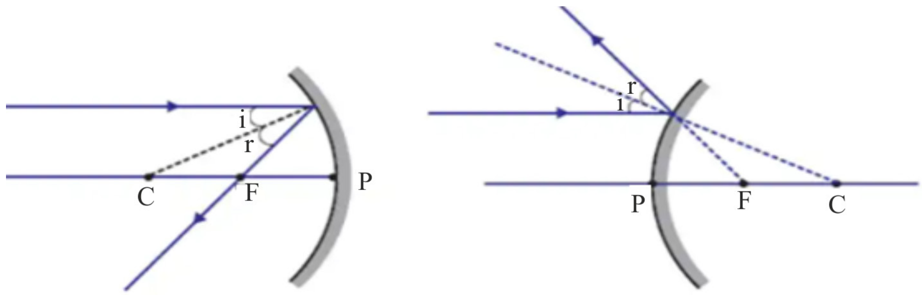
- A spherical mirror of which the reflecting surface is curved inwards is known to be a concave mirror.
- A spherical mirror having its reflecting surface curved outwards is known to be a convex mirror.

Image properties: Dependent on the position of the object, there are four basic properties of images (real or virtual, erect or inverted, magnified or diminished, in front of the mirror or behind the mirror). These properties can be verified either graphically or by using the mirror equation and the definition of magnification.

Ray diagram

Using any two of the following three rays, we can easily determine the position, characteristics and size of the image formed by spherical mirrors for different position of the object on the principal axis.

Parallel Rays: When a ray, parallel to the principal axis strikes concave or convex mirrors, the reflected ray passes through the focal point upon reflection for concave and appear as if they came from the focal point for convex mirror.



(a) concave Mirror (b) convex Mirror

Figure 26. Ray diagram of parallel rays

Radial Rays: Rays passing through the center of curvature are reflected back along their original path -**concave** (or a ray which would have passed through the center of curvature is reflected back along itself - **convex**).

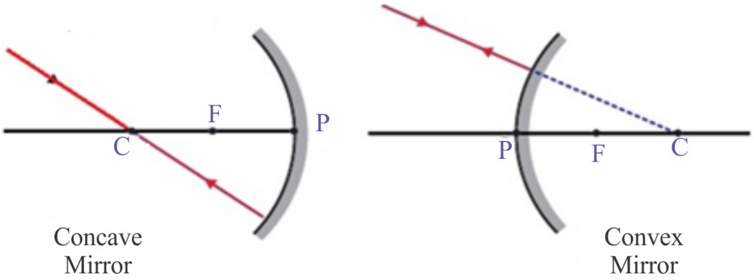


Figure 27. Ray diagram of radial rays

Focal Rays: Rays passing through the focal point are reflected parallel to the principal axis - **concave** (for **convex** mirrors a ray that would have passed through the focal point is reflected parallel to the axis).

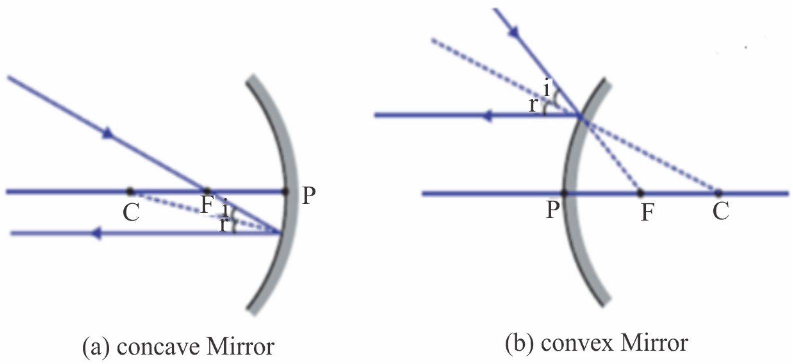


Figure 28. Ray diagram of focal rays

ACTIVITY 10

The passenger side rear-view mirror in new cars often have a warning label that reads, "OBJECTS IN MIRROR ARE CLOSER THAN THEY APPEAR" Are these rear-view mirrors concave or convex?

Image Formation by Concave Mirror

When an object is placed in front of a mirror, we see its image somewhere in front of the mirror or behind the mirror. The general characteristics of the image depend on the location of the object with respect to the center of curvature and the focal point. The image formation in a concave mirror for the following six possible object positions can be observed using a ray diagram.

Concave Mirror Ray Diagram

Case 1: When an object is placed at infinity, a real and inverted image is formed at the focus. The size of the image is much smaller compared to that of the object.

Case 2: When an object is placed behind the center of curvature, a real image is formed between the center of curvature and focus. The size of the image is smaller than that of the object.

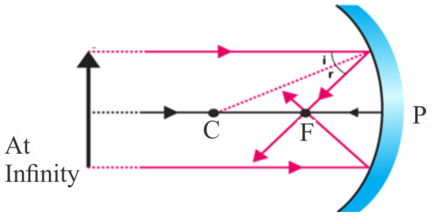


Figure 29. Ray diagram of object at infinity

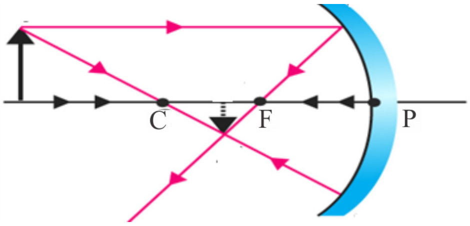


Figure 30. Ray diagram of object beyond C

Case 3: When an object is placed at the center of curvature and focus, the real image is formed at the center of curvature. The size of the image is the same as that of the object.

Case 4: When an object is placed in between the center of curvature and focus, the real image is formed behind the center of curvature. The size of the image is larger than that of the object.

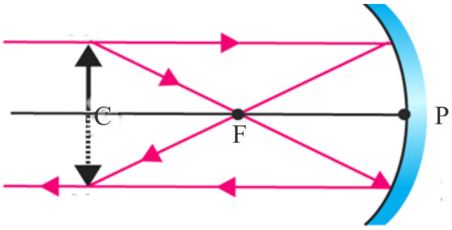


Figure 31. Ray diagram of object at C

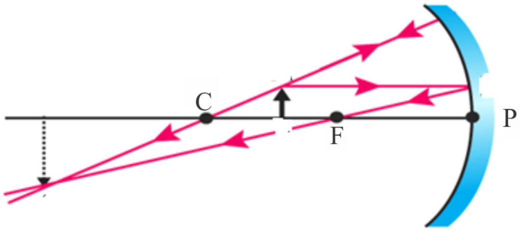


Figure 32. Ray diagram of object between C and F

Case 5: When an object is placed at the focus, the real image is formed at infinity. The size of the image is much larger than that of the object.

Case 6: When an object is placed in between focus and pole, a virtual and erect image is formed. The size of the image is larger than that of the object.

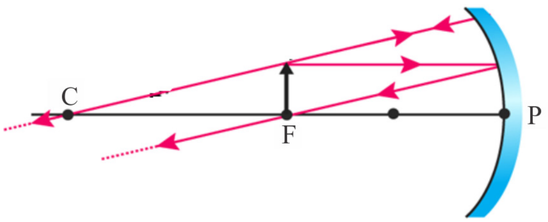


Figure 33. Ray diagram of object at F

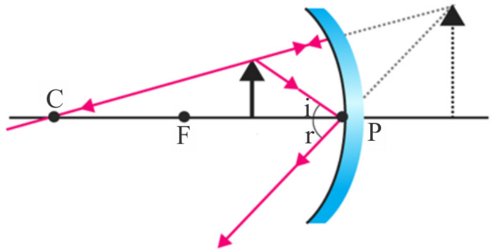


Figure 34. Ray diagram of object between F and the mirror

Summary

S. No	Position of Object	Position of Image	Size of Image	Nature of Image
1	At infinity	At the focus F	Highly Diminished	Real and Inverted
2	Beyond the center of curvature C	Between F and C	Diminished	Real and Inverted
3	At the center of curvature C	At C	Same Size	Real and Inverted
4	Between C and F	Beyond C	Enlarged	Real and Inverted
5	At focus F	At Infinity	Enlarged	Real and Inverted
6	Between F and the mirror	behind the mirror	Enlarged	virtual and erect

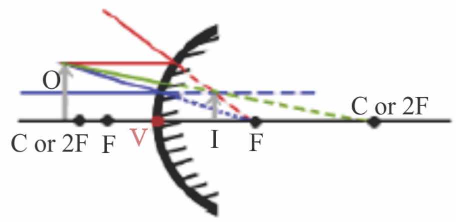
Activity 11

1. Look at yourself in the concave mirror and move the mirror forward and backward. Note down your observations for the following questions:
   (a) Is the image inverted or upright?
   (b) Is the image upright when the mirror is closer to your face or farther away?
   (c) Is the image of your face (when it is upright) smaller or bigger than your face?
   (d) Is the image of your face (when it is inverted) smaller or bigger than your face?
2. Repeat the same procedure using a convex mirror and record your observations.

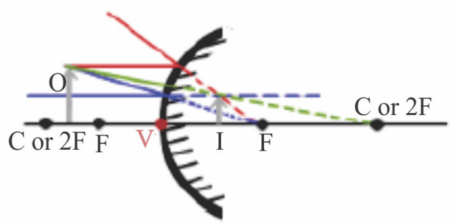
Image Formation by Convex Mirror

The general characteristics of images in convex mirrors are independent of the location of the object. The image formed in a convex mirror is always virtual and erect, whatever be the position of the object. In this section, let us look at the types of images formed by a convex mirror by placing the object at different distances from the object.

1. Object beyond C



2. Object between C and F



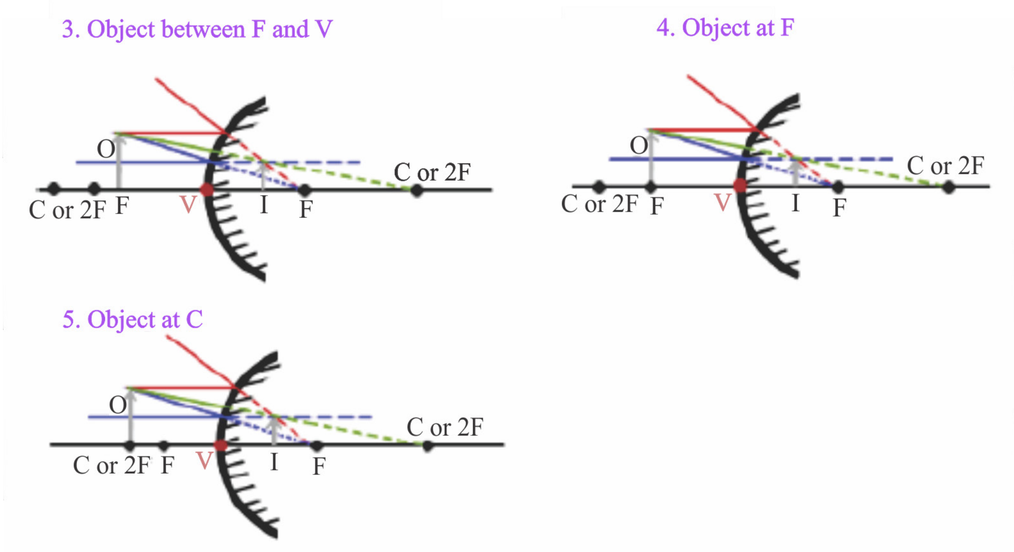


Figure 35. Ray diagram of image formed by convex mirror

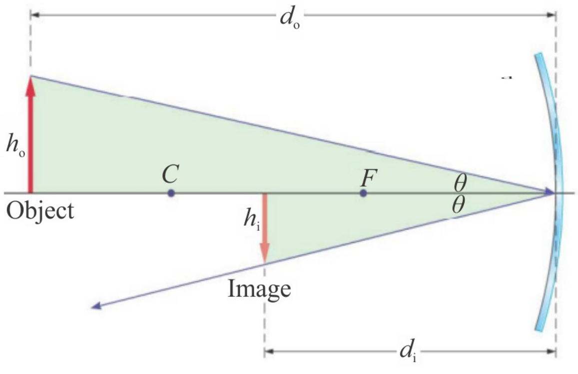
From Figure we see that the nature of the image formed by a convex mirror is independent of the position of the object. It is always, virtual, erect and diminished.

ACTIVITY 12

Suppose you would like to use the Sun to start a fire. Which type of mirror, concave or convex, would you work best?

The Mirror Equation

The mirror equation is used to determine the nature, position and size of the image formed by spherical mirrors. The derivation of mirror equation:



a)

Since the two triangles are similar:

$$\frac{h_o}{h_i} = \frac{d_o}{d_i}$$

Since the two yellow triangles are similar:

$$\frac{h_o}{h_i} = \frac{d_o - R}{R - d_i}$$

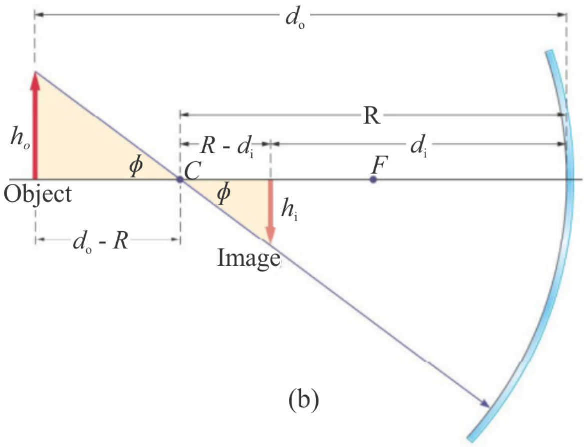
Combining these two equations we have

$$\frac{d_o}{d_i} = \frac{d_o - R}{R - d_i} \; or \; 1 = \frac{1 - R/d_o}{R/d_i - 1}$$

Rearranging gives

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

Where f is the focal length of the mirror, d_o is the distance of the object from the mirror and d_i is the distance of the image from the mirror.



(b)

Magnification (m) is defined as the ratio of the image height to object height.

$$Magnification\,(m) = \frac{h_i}{h_o} = \frac{d_i}{d_o}$$

Sign conventions for mirrors equations

In order to make use of the above formulae the following sign conventions must be followed. The sign conventions for the given quantities in the mirror equation and magnification equations are as follows:

- f is positive (+) if the mirror is a concave and f is negative (-) if the mirror is a convex
- d_o is positive (+) if the object is a real, and is negative (-) if the object is a virtual.
- d_i is positive (+) if the image is a real and located on the object's side of the mirror, and is negative (-) if the image is a virtual and located behind the mirror.
- h_i is positive (+) if the image is an upright (and therefore, virtual) and is negative (-) if the image an inverted image (and therefore, real)
- m is positive for virtual or erect images, and m is negative for real or inverted images.

Examples

An object is placed 10cm in front of a concave mirror of focal length 15cm. Find the image distance and magnification.

Solution

$d_o = 10cm$, $f = 15$ cm

Using the mirror equation,

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{15cm} - \frac{1}{10cm} = \frac{2\text{-}3}{30cm} = -\frac{1}{30cm}$$

$d_i = -30cm$

The negative sign indicates that the image is virtual.

magnification is: $m = \frac{d_i}{d_o} = \frac{30cm}{10cm} = \underline{\underline{3}}$

The image is 3 times larger (magnified).

Examples

An object is placed 75cm in front of a convex mirror of focal length 50cm. What are (a) the position, and (b) the nature (characteristic) of the image?

Solution

$d_o = 75cm$

$f = -50cm$ (convex mirror)

(a) $\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = -\frac{1}{50cm} - \frac{1}{75cm}$

$$\frac{1}{d_i} = \frac{-75cm - 50cm}{50cm \times 75cm} = \frac{-125cm}{50cm \times 75cm} = -\frac{1}{30cm}$$

$d_i = -30cm$

(b) $m = \frac{d_i}{d_o} = \frac{30cm}{75cm} = \frac{2}{5} = 0.4$

The image is virtual, erect and diminished ($m < 1$).

Examples

When an object is placed at a distance of 20cm from a concave mirror, the resulting image is erect and five times the size of the object. What is the focal length of the mirror?

Solution

$d_o = 20cm$, $m = 5$

Using the magnification equation, we obtain the image position d_i.

$$m = \frac{d_i}{d_o} \text{ or } d_i = m \times d_o = 5 \times 20cm = 100cm$$

Since the image is erect, $d_i = -100cm$

$$\text{Thus, } \frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{20cm} + \frac{1}{-100cm} = \frac{1}{20cm} - \frac{1}{100cm}$$

$$\frac{1}{f} = \frac{5-1}{100cm} = \frac{4}{100cm} = \frac{1}{25cm}$$

$$f = 25cm$$

Examples

A pencil is placed 8cm away from a concave mirror. A virtual image is formed at 12cm away from the mirror. What is the focal length of the mirror?

Solution

$d_o = 8cm$, $d_i = -12cm$ (virtual)

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{8cm} + \frac{1}{(-12cm)}$$

$$\frac{1}{f} = \frac{1}{8cm} - \frac{1}{12cm} = \frac{3-2}{24cm} = \frac{1}{24cm}$$

$$f = 24 \text{ cm}$$

Demonstration: Image Formed by Spherical Mirrors

Aim: To study the nature of images formed by convex and concave mirrors.

Materials needed: Concave mirror, convex mirror, a sheet of white paper, a well-lit window, and adhesive tape.

Method:

1. Take the sheet of white paper and fold it about two or three times to make a screen.
2. Stick the screen (folded paper) on the window grill of an open window.
3. Take the concave mirror and hold it facing the window (and the screen).
4. Move it back and forth till you get a clear image of the objects outside the window (such as leaves, trees, and sky) on the screen.
5. Note down your observations for the following questions:
   (a) Is it a real or a virtual image?
   (b) Is the image inverted or upright?
   (c) Is the image smaller or bigger than the object?

Experiment: Image Formation by Concave Mirror

Aim: To study the nature and size of the image formed by a concave mirror on a screen by using a candle and a screen (for different distances of the candle from the mirror).

Materials required

An optical bench, candle, match box, a small candle stand, concave mirror of nearly 25 cm focal length, a screen, three uprights (with clamps), meter scale, adhesive tape and spirit level.

Procedure

1. Obtain the focal length of the concave mirror by focusing the image of a distant object (Sun). A sharp image of the Sun or a tree will be formed at the focal point.
2. Place the optical bench on a rigid platform and using the spirit level make it horizontal with the help of leveling screws provided at the base of the bench.
3. Clamp the concave mirror on an upright and fix it near one end of the optical bench such that its principal axis is horizontal and parallel to the optical bench. In this position the mirror would lie in a plane perpendicular to the principal axis of the mirror.
4. Mount a lighted candle vertically on a small candle stand at different positions, and observe the nature of the image on a screen.

5. Read the positions of the mirror, screen and candle uprights on the optical bench meter scale and record the readings in the observation table.
6. Measure height of the lighted candle using the meter scale. Also measure the height of the image formed on screen.

Observations

Approximate focal length of the concave mirror =...cm.

1. Nature, size and position of image with different positions of object

No.	Distance of object	Distance of image	Height of object	Height of image on the screen	Nature of image	Focal length of the mirror by calculation
1						
2						
3						
4						
5						
6						

Questions

1. Compare the calculated focal length with what you have determined by measurement.
2. Compare the nature of the image you have observed with what you know in theory.

Precaution

1. This experiment should be performed at a shaded place where no direct light reaches (preferably in a dark room) otherwise the images may not be distinctly visible.
2. The aperture of mirror should be small otherwise the image formed will not be distinct.
3. Eye should be placed at a distance more than 25 cm from the image being formed on the screen.

Exercises

1. Define the following terms in the context of spherical mirrors: (a) Pole, (b) Center of curvature, (c) Principal axis, (d) Principal focus, (e) focal length, (a) Radius of curvature.

2. The focal length of a concave mirror is 20 cm, what is its radius of curvature?
3. When an object is placed at 30 cm in front of a concave mirror, image of the same size is formed. What is the focal length of the mirror?
4. Focal length of a concave mirror is 30 cm. What will be the characteristics of the image formed, when the object is placed at a distance of 40 cm in front of the mirror?
5. Name the type of mirrors used in the following situations: (a) Headlights of a car, (b) Rear-view mirror of vehicle, (c) Solar furnace
6. If the image formed by a mirror for all positions of the object placed in front of it is always virtual and diminished, state the type of the mirror. Draw any ray diagram in support of your answer.
7. The linear magnification produced by a spherical mirror is +1/3. Analyzing this value, state the type of mirror and the position of the object with respect to the pole of the mirror. Draw any diagram to justify your answer.
8. A spherical mirror produces a magnification of -1 on a screen placed at a distance of 50 cm, from the mirror. (a) Write the type of the mirror. (b) Find the distance of the image from the object. (c) What is the focal length of the mirror?
9. An object is kept at a distance of 4m, in front of a spherical mirror, which forms its erect image at a distance of 1m, from the mirror. What is the magnification? Is the mirror concave or convex?
10. A concave mirror produces a three times magnified image on a screen. If the object is placed 20 cm in front of the mirror, how far is the screen from the object?
11. The image of a candle flame placed at a distance of 30 cm from a mirror is formed on a screen placed in front of the mirror at a distance of 60 cm from its pole. (a) What is the nature of the mirror? (b) Find its focal length.
12. Suppose you want to observe an erect image of a candle flame using a concave mirror of focal length 20 cm. State the range of distance of the candle flame from the mirror. List two other characteristics of the observed image.

6.5 Electromagnetic Spectrum

More generally, **electromagnetic waves** (of which visible light is one example) are transmitted as a signal that our radios pick up so we can listen to music. Pulses of infrared light are transmitted as signals so we can communicate with our TVs. This section is about the range of all wavelengths or frequencies (including the visible light) over which electromangeitc radiation extends.

White light is actually made up of a rainbow of different colors: violet, indigo, blue, green, yellow, orange and red. There are many other "colors" of light that we cannot see. These include ultraviolet (UV) and infrared (IR) light that is just beyond the range of human vision. They also include more exotic forms of radiation such as X-rays, gamma rays, and radio waves. Together, all of these different "colors" of light are called the **electromagnetic spectrum**. Thus, the electromagnetic spectrum is the range of all possible frequencies of electromagnetic radiation including Visible light, together with these invisible types of lights.

Key terms

- The electromagnetic spectrum is the range of all possible frequencies of electromagnetic radiation.
- The seven members of electromagnetic family are: radio waves, microwaves, infrared light, visible light, ultraviolet light, X-rays and gamma-rays.

There are seven members of electromagnetic family. Arranged in order of decreasing wavelength (and increasing frequency): radio waves, microwaves, infrared light, visible light, ultraviolet light, X-rays and gamma-rays, see Figure 6.6. The types of radiation that occur in different parts of the spectrum have different uses and dangers - depending on their wavelength and frequency.

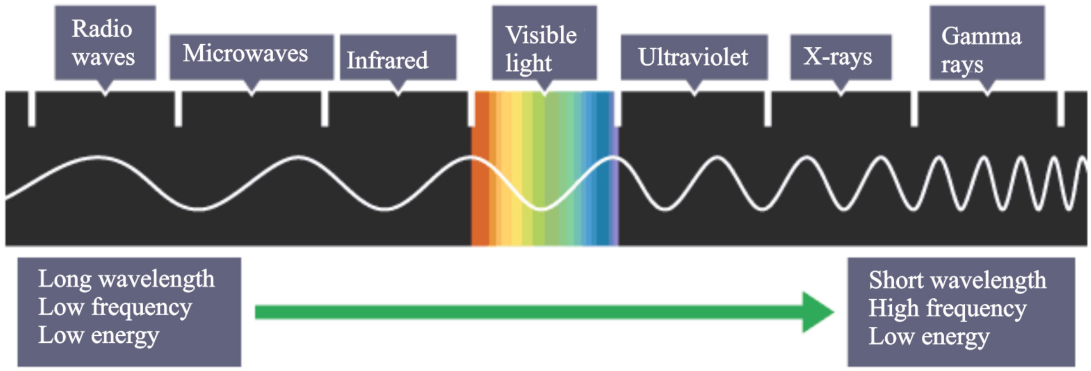


Figure 36. The electromagnetic spectrum

Radio waves

Radio waves have the longest wavelengths of all the electromagnetic waves. They range from about 1m to 300m. Radio waves are used primarily for communications, to transmit data such as television and radio, mobile phones, radar, Wi-Fi, Bluetooth, satellites, and computer networks. They are generated by electronic devices and are used in radio and communication system.

Microwaves

Microwaves fall in the range of the electromagnetic spectrum between radio and IR. They have frequencies from about 3 GHz to 30 THz, and wavelengths of about 10-1000mm. We use microwaves to cook food, satellite communication, and in radar that helps to predict the weather.

Infrared

Beyond the red end of the visible spectrum is infrared radiation. This ranges from 700nm down to 0.1cm and has frequencies from about 30 to 400 THz. We feel such radiation from a heat lamp but we cannot see this radiation. It is used for photography, medicine, military techniques, optical fiber communication, TV remote control and others. Anything that gives off heat radiates infrared waves.

Visible light

The visible light spectrum covers the wavelengths that can be seen by the human eye. This is the range of wavelengths from 380 to 700 nm which corresponds to the frequencies 430×10^{12} Hz - 790×10^{12}Hz.

Our eyes detect visible light. Visible light is used for seeing, photography, optical fiber and communication. Fireflies, light bulbs, and stars all emit visible light.

Ultraviolet

Ultraviolet (UV) light is the range of the electromagnetic spectrum between visible light and X-rays. It has frequencies of about 8×10^{14} to 3×10^{16} Hz and wavelengths of about 10 to 380 nanometers. This light is used in medicine and industrial applications. It is also used in detecting forged bank notes, helps to make vitamin D, hardening some types of dental filling and in nightclubs. UV is also dangerous for living things (plants or animals). Ultraviolet rays from the Sun cause sunburns. We are protected from the Sun's ultraviolet rays by the ozone layer in Earth's atmosphere..

X-rays

X – rays are produced by X – ray tube or machines. The wavelength of X – rays are in the interval 10 - 0.01nm. It is used in medicine and testing of materials. They can penetrate soft tissue like skin and muscle and are used to take X-ray pictures of bones in medicine. A dentist uses X-rays to image your teeth, and airport security uses them to see through your bag.

Gamma (γ) rays are electromagnetic waves having wavelengths from 0.1 – 10nm. Doctors use gamma-ray imaging to see inside your body. Gamma rays are used to Kill cancer cells, sterilizing medical equipment, killing bacteria to prolong shelf life of fruit. Gamma radiation causes damage to living tissue, which makes it useful for killing cancer cells when applied in carefully measured doses to small regions. Uncontrolled exposure, though, is extremely dangerous to humans. Gamma rays are produced in high energy nuclear explosions.

As the wavelengths of electromagnetic waves get shorter, their frequency and energy increases. Gamma rays are the shortest waves in the spectrum and, as a result, have the most energy.

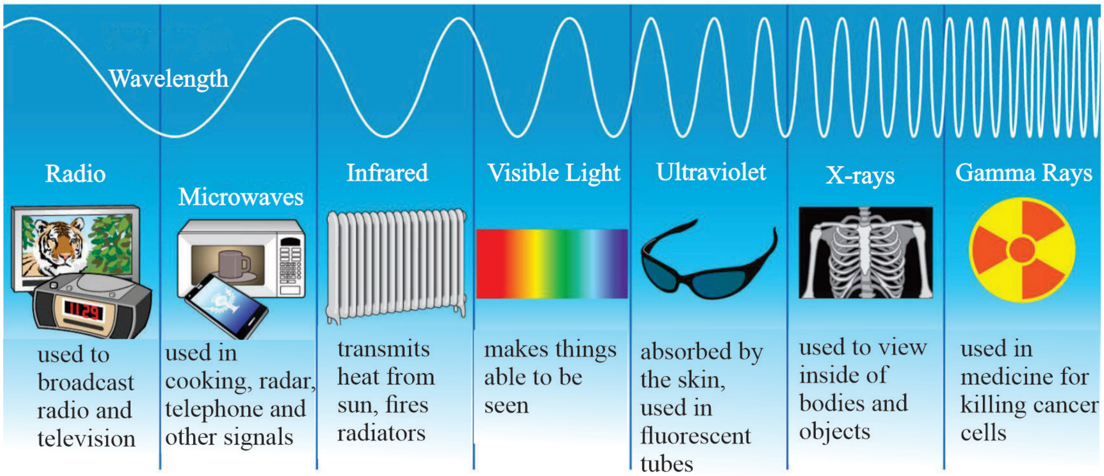


Figure 37. Types of electromagnetic radiation

Hazards of electromagnetic radiation

Over-exposure to certain types of electromagnetic radiation can be harmful. The higher the frequency of the radiation, the more energy it carries and the more damage it is likely to cause to the body:

Radio waves: one of the few known effects of radio waves on the human body is a very small rise in temperature of up to 0.2 ^{o}C. Some people claim the very low frequency radio waves from overhead power cables and mobile phone base stations near their homes has affected their health, although this has not been reliably proven;

Microwaves can cause internal heating of body tissues;

Infrared radiation is felt as heat and causes skin to burn;

Visible light from a laser which is very intense can damage the retina at the back of the eye;

Ultraviolet can damage skin cells and lead to skin cancer and damage the eyes, it can cause skin to age prematurely;

X-rays damage cells inside the body. They cause dangerous ionization and when this happens with molecules in living cells, the genetic material of a cell, the DNA is damaged. This can lead to cancer. That is why doctors and dentists stand behind protective screens when taking lots of X-rays;

Gamma rays also damage cells inside the body causing dangerous ionization in living cells which damages DNA. This can lead to cell death and cancer.

Did you know?

X-rays were discovered by German scientist Wilhelm Roentgen.

What is a Laser?

KEY TERMS

- Laser is an acronym for "Light Amplification by Stimulated Emission of Radiation", which produces a light of a certain wavelength only.

"Laser" is an acronym for "Light Amplification by Stimulated Emission of Radiation", which describes the theory of laser operation. Albert Einstein published the theoretical basis for the laser in 1917, but it was only in 1960 that the first functioning laser was constructed by Theodore Maiman in California, using a ruby crystal to produce laser light. A Laser is a device that stimulates atoms or molecules to emit light at particular wavelengths and amplifies that light, typically producing a very narrow beam of radiation.

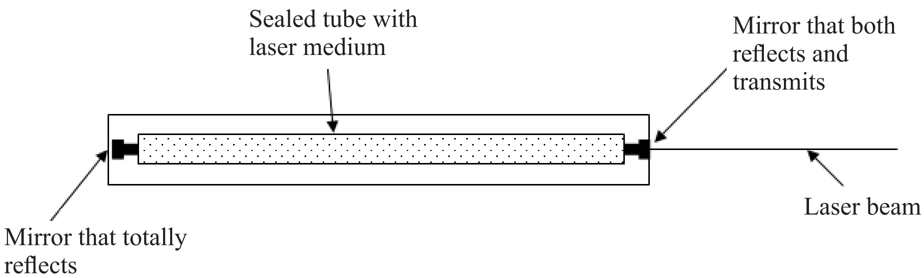


Figure 38. Laser

Properties of laser light

Unlike other forms of light, laser light has special properties which make it significantly more effective and dangerous than conventional light of the same power. The laser light particles (photons) are usually:

- **Monochromatic:** consisting of a single wavelength or color.
- **Coherent:** photons are in phase (like marching soldiers).
- **Collimated**: photons are almost in parallel (aligned), with little divergence from the point of origin.

Safety Aspects

Improperly used laser devices are potentially dangerous. Effects can range from mild skin burns to irreversible injury to the skin and eye can also cause fire. So, when using a laser source wear laser safety glasses and use warning signs.

KEY TERMS

- Photoelectric effect is the process of emitting the electrons from the surface of a metal when the metal surface is exposed to an electromagnetic radiation of sufficiently high frequency.
- The electrons emitted in this process are known as photoelectrons and the current constituted by these electrons is known as photoelectric current.

Laser Applications

Particularly important application areas of laser are laser material processing, optical data transmission and storage and optical metrology. Further development of this technology led lasers to become widely used in medical practice.

Photoelectric Effect

Photoelectric effect is the process of emitting the electrons from the surface of a metal when the metal

surface is exposed to an electromagnetic radiation of sufficiently high frequency. The electrons emitted in this process known as photoelectrons and the current constituted by these electrons known as photoelectric current. The process through which photoelectrons are ejected from the surface of the metal due to the action of light is commonly referred to as photoemission. An illustration detailing the emission of photoelectrons as a result of the photoelectric effect is shown in the figure.

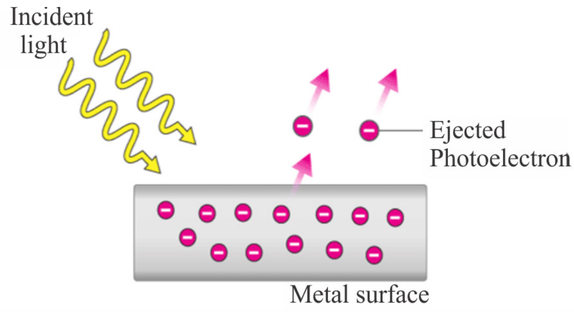


Figure 39. Photoelectric effect

Exercises

1. Compare the speed, wavelength, and frequency of the components of the electromangeitc spectrum.
2. In which part of the electromagnetic spectrum are each of these waves: (a) $f = 10.0$ kHz, (b) λ=750nm, (c) f = 1.25×10^8Hz
3. Is visible light a type of electromagnetic wave? Explain
4. How are electromagnetic waves created?
5. What are photons?
6. What is an electromagnetic spectrum?
7. Consider the electromagnetic spectrum as you answer these three questions.
   (a) Which region of the electromagnetic spectrum has the highest frequency?
   (b) Which region of the electromagnetic spectrum has the longest wavelength?
   (c) Which region of the electromagnetic spectrum will travel with the fastest speed?
8. For the same monochromatic light source, would the photoelectric effect occur for all metals?
9. What are the conditions for the photoelectric effect?
10. What is threshold frequency?
11. What is the energy of the photons in the beam of light of wavelength 240nm?

Summary

- Light is a form of energy that enables us to see objects surrounding us, it causes the sensation of vision.
- The speed of light in vacuum is 300,000,000m/s. The speed of light is different in different media.
- Light is dualistic in nature, it behaves both as a particle and as a wave

- Objects which are capable of emitting their own light are called Source of light.
- Objects which do not emit light but reflect the light from luminous objects are called non-luminous objects.
- Natural light sources produce light naturally without any human involvement, but artificial sources of light are constructed by humans.
- Reflection is the phenomenon in which light travelling in one medium, incident on the surface of another returns to the first medium.
- Refraction is a phenomenon in which there is a change in the speed of light as it travels from one medium to another and there is a bending of the ray of light.
- When light travels from a denser medium to a lighter medium, it speeds up and the light bends *away* from the normal.
- When light travels from a lighter medium to a denser medium it slows down and the refracted ray bends towards the normal.
- The process of splitting white light into its constituent colors is termed as dispersion.
- The bending of light when it passes around an edge or through a small opening is called diffraction.
- The process of transforming unpolarized light into the polarized light is known as polarization.
- When two or more light waves having the same frequency and wavelength traveling in the same medium meet together at a point, they cancel or enhance the effect of each other at that point. This phenomenon is called interference of light waves.
- The property of light travelling in a straight line is known as rectilinear propagation of light.
- A ray can be defined as an imaginary straight line drawn in the direction in which light travels.
- A collection of light rays travelling together is known as a light beam.
- The dark region behind an opaque object where the light cannot reach is known as shadow.
- The umbra is the innermost and darkest part of a shadow, where the light source is completely blocked by the opaque body.
- The penumbra is the region in which it gets light from some parts of the source of light.
- The antumbra is the lighter area of a shadow that appears beyond the umbra, at a certain distance from the object casting the shadow..
- Solar eclipse happens when the Sun, Moon and Earth are perfectly aligned, and the moon comes between the earth and the sun during their motion. In this case the Moon blocks light from the Sun and casts a shadow on Earth.

- The Lunar Eclipse occurs when the Earth, the Sun and the Moon come in a straight line, and the Earth come in-between the Sun and the Moon.
- A pinhole camera is a simple camera without a lens but with a hole of pin size.
- The image formed by a Pinhole Camera is real, smaller than the object, and inverted.
- In a pinhole camera

$$\frac{\text{Height of image}}{\text{Height of object}} = \frac{\text{Distance of image}}{\text{Distance of object}}$$

- Regular reflection is reflection from smooth and shiny surfaces.
- Diffuse reflection is reflection from rough, unpolished surface.
- The law of reflection is stated as follows:
- The incident ray, the reflected ray and the normal to the surface lie in the same plane.
- The angle of incidence is equal to the angle of reflection, i.e., i = r.
- The image formed by a plane mirror is virtual, erect, the same size as that of the object, laterally inverted and is formed behind the mirror as the object is in front of it.
- A spherical mirror of which the reflecting surface is curved inwards is known to be concave mirror.
- A spherical mirror having its reflecting surface curved outwards is known to be convex mirror.
- Depending on the position of the object, concave mirror can produce an image of any characteristics (real or virtual, magnified or diminished)
- Independent of the position of the object, the nature of the image formed by a convex mirror is always, virtual, erect and diminished.
- The Mirror Equation: $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$
- Magnification(m) = $\frac{h_i}{h_o} = \frac{d_i}{d_o}$
- The electromagnetic spectrum is the range of all possible frequencies of electromagnetic radiation.
- The seven members of electromagnetic family are: radio waves, microwaves, infrared light, visible light, ultraviolet light, X-rays and gamma-rays.
- Laser is an acronym for “Light Amplification by Stimulated Emission of Radiation”, which produces a light of a certain wavelength only.

- Photoelectric effect is the process of emitting the electrons from the surface of a metal when the metal surface is exposed to an electromagnetic radiation of sufficiently high frequency.
- The electrons emitted in this process are known as photoelectrons and the current constituted by these electrons is known as photoelectric current.
- Einstein's Photoelectric Equation: $E_{photon} = \mathbf{\Phi} + E_{electron} \Rightarrow hf = hf_{th} + \frac{1}{2}\mathbf{m}_e v^2$
- The minimum amount of energy required to remove an electron from the metal is called the threshold energy or work function.

Review Exercises

1. If the angle between the incident ray and reflected ray of light striking a mirror is 120^o. What is (a) the angle of incidence? (b) the angle between the reflected ray and the surface?
2. What is the purpose of light?
3. Look at the picture. What is the reason for writing AMBULANCE in that way?
4. Calculate the frequency of the longest and shortest wave length of light that are visible to the eye. Speed of light = 3×10^8m/s .
5. The frequency of ultraviolet light is in the range of 7.7×10^{14} to 3×10^{16} Hz. Calculate the corresponding wavelength range.
6. An observer walks toward a plane mirror at a speed of 2m/s. With what speed does he approach his image?
7. Calculate the height of the image formed by a pinhole camera of length 12cm used to photograph an object 60cm away from the hole and 70cm high.
8. Find out the focal length of concave mirror with a radius of curvature 30cm.
9. Determine the image distance and image height for a 5.00-cm tall object placed at the following distances from a concave mirror having a focal length of 15.0 cm. (a) 45cm (b) 30cm, 9 c) 20cm, (d) 10cm (e) For each case state the nature of the image.
10. A magnified, inverted image is located a distance of 32.0 cm from a concave mirror with a focal length of 12.0 cm. Determine the object distance and tell whether the image is real or virtual.
11. An inverted image is magnified by 2 when the object is placed 22 cm in front of a concave mirror. Determine the image distance and the focal length of the mirror.

Health Related Caution

What are the ways to avoid dengue and malaria fever?

- Time your outings.
- Reduce mosquito habitat.
- Sleep under mosquito-net.
- Put screens on windows and doors.
- Keep your house airy and well-lit.
- Do not let water stagnate anywhere.
- Wear long pants and long sleeves to cover your body.
- Apply mosquito repellent with DEET (diethyltoluamide) to exposed skin.
- Treat clothing, mosquito nets, tents, sleeping bags and other fabrics with an insect repellent called permethrin.

How can a person reduce the risk of getting HIV?

- Get tested for HIV.
- Do not inject drugs.
- Choose less risky sexual behaviors.
- Use condoms every time you have sex.
- Limit your number of sexual partners.
- Get tested and treated for STDs.
- Talk to your health care provider about pre-exposure prophylaxis (PrEP).

What is Bullying?

Any unwanted written, verbal, graphic, or physical act by an individual or group toward another person(s) that causes harm or distress.

Types of Bullying

- Physical
- Verbal
- Social
- Emotional
- Cyber

Signs of Bullying

- Headaches
- Depression
- Loss of friends
- School absenteeism
- Academic problems

What You Can Do

PREVENT

- Be a role model for positive communication, healthy relationships, and self-care.
- Reinforce acts of kindness, respect, and inclusion.
- Set policies and rules about bullying.

RECOGNIZE

- Know the definition of bullying and its many forms.
- Talk with and actively listen to the youth who confide in you.
- Watch for warning signs of bullying.

INTERVENE

- If you witness bullying behavior
- Respond quickly and consistently to send the message that it is not acceptable.
- Separate the students involved.
- Meet any immediate medical or mental health needs.
- Stay calm and model respectful behavior.

Source: Teacher's Diary on *Cyber-Crime Awareness* by UNODC, Cybercrime and MoE, Republic of Liberia

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MINISTRY OF EDUCATION

TEXTBOOK MANAGEMENT
Student Textbook Assignment Form

School : ____

District : ______________________Country: ______________________

Teacher : ______________________Subject: ______________________

Grade : ____

No.	Name of the Student	Name of the Textbook	Barcode Number	Condition of the TB when received	Date Received	Date Returned	Condition of TB when returned	Signature of Student
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								